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On solving fuzzy assignment problem based On distance method for ranking of generalized trapezoidal fuzzy numbers using centroid of incenters and an index of modality

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Abstract
This paper proposes new algorithms based on proposed ranking methods using centroid for finding an optimum solution of fuzzy cost based fuzzy assignment problem. The fuzzy cost which is involved in the fuzzy assignment problems is measured as generalized trapezoidal fuzzy number. The proposed algorithms which are developed from classical crisp algorithms based on LPP and Hungarian method are easy to calculate the optimal fuzzy cost in uncertain real life situations. Finally, the numerical example is given for illustrating the capability of the proposed algorithms for finding the optimum solution.

Keywords
Fuzzy Assignment Problem, Fuzzy Linear Programming Problem, Hungarian Algorithm, Generalized Trapezoidal Fuzzy Number, Fuzzy Ranking Method.

AMS Subject Classification

03E72, 90BXX.

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1. Introduction

The assignment problem is a special type of transportation problem and also a linear programming problem. It is the well known optimization problem and is widely applied in both manufacturing and service systems. The two components of assignment problem are the assignments which refer underlying combinatorial structure and the objective function which refers the desires to be optimized. The main objective of this is to find an optimal assignment to a given number of persons to equal number of jobs on one to one basis in such way to minimize total cost of performing all jobs or to maximize the total profit.

All the models and algorithms developed to find the optimal solution of transportation problems are applicable to assignment problems in the deterministic environment. One of the first such algorithms was the Hungarian algorithm, developed and published in 1955 by Harold Kuhn [1] and it is reviewed by James Munkres in 1957 [3]. Over the past years; many variations of the classical assignment problems have been proposed. Though, in an uncertain environment, classical assignment problems could not be successfully applied for real life problems. So the concept of fuzzy sets was introduced by Zadeh in 1965 to deal with imprecision and vagueness. In recent years, many researchers began to investigate the Fuzzy Assignment Problem. In the fuzzy assignment problem, all the parameters are considered as fuzzy numbers.

Mostly fuzzy assignment problems are solved with the help of fuzzy ranking methods based on classical procedures under uncertainty [6, 10, 11, 13]. Fuzzy ranking method for ranking of fuzzy numbers is an important procedure in many applications of fuzzy optimization techniques. From the beginning itself many authors are involved in ranking of fuzzy numbers, but most of the methods are based on centroid. In 1976, first method was introduced by Jain [2], then a large number of methods have been developing by many authors. Recently, Hair Ganesh and Phani Bushan Rao et. al. have introduced a method for ranking of generalized trapezoidal fuzzy numbers based on centroid using radious of gyration and various centres of triangle [4, 7–9, 12].

In this work we have proposed methods for ranking of generalized trapezoidal fuzzy numbers based on centroid of incentres. But the ultimate aim of this work is the utilization of the proposed methods of ranking of generalized trapezoidal fuzzy numbers for finding the optimum solution of Assignment Problem.

2. Preliminaries

This section summarizes some basic definitions and operations of fuzzy numbers.

Definition 2.1. *A fuzzy number is a fuzzy subset defined on the universal real number setR, with the membership function* $\mu_{\tilde{A}}(x)$ *if it satisfying the properties given below:*

1. $\tilde{A}$ *is convex*
2. $\tilde{A}$ *is normal, i.e., there is a* $x_0 \in \tilde{A}$ *such that* $\mu_{\tilde{A}}(x_0) = 1$
3. $\mu_{\tilde{A}}(x)$ *is a piecewise continuous in its domain.*
4. $\tilde{A}$ *is convex, i.e.,* $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$, $\forall x_1, x_2 \in X$

Definition 2.2. *If the fuzzy number has a trapezoidal shape with four vertices* (a,b,c,d) *and it is depicted graphically as in Fig. 1, then the fuzzy number* $\tilde{A} = (a,b,c,d)$ *is called a trapezoidal fuzzy number. Theoretically it possesses membership function given below.*

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a < x \leq b \\ 1, & b \leq x \leq c \\ \frac{c-x}{d-c}, & c < x \leq d \\ 0, & x \geq d \end{cases}$$

Definition 2.3. *A fuzzy number* $\tilde{A} = (a,b,c,d:w)$ *is called a generalized trapezoidal fuzzy number if it possesses a following membership function theoretically. Graphically it is depicted in Fig. 2.*

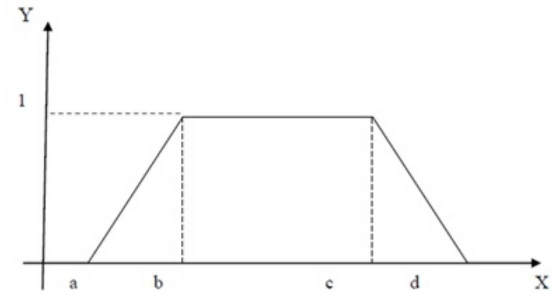

Figure 1. Trapezoidal Fuzzy Number

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a < x \leq b \\ w, & b \leq x \leq c \\ \frac{c-x}{d-c}, & c < x \leq d \\ 0, & x \geq d \end{cases}$$

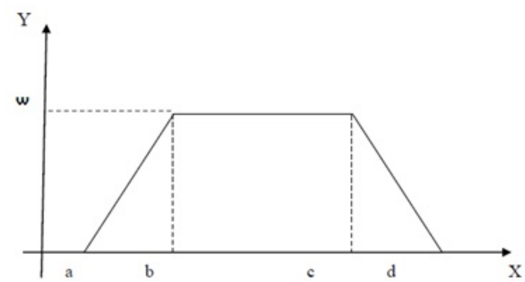

Figure 2. Generalized Trapezoidal Fuzzy Number

Definition 2.4. *The underlying arithmetic operations between two generalized trapezoidal fuzzy numbers* $\tilde{A} = (a,b,c,d:w_A)$ *and* $\tilde{B} = (e,f,g,h:w_B)$ *defined and recapitulated as follows:*

1. $\tilde{A} + \tilde{B} = (a+e, b+f, c+g, d+h : \min(w_A, w_B))$
2. $\tilde{A} - \tilde{B} = (a-h, b-g, c-f, d-e : \min(w_A, w_B))$
3. $\tilde{A} \times \tilde{B} = (a \times e, b \times f, c \times g, d \times h : \min(w_A, w_B))$
4. *Proposed Ranking Method based on centroid of incenters*

3. Proposed Ranking Method Based on Centroid of Incenters

Definition 3.1 ([5]). *The gravity point of any plane figure is centroid, so that the centroid of a trapezoid might be considered as the balancing point of the trapezoid (Fig. 1). Divide the trapezoid into three triangles. These three triangles are* $\triangle APB$, $\triangle CQD$ *and* $\triangle ADC$. *The centroid of incenters of these three triangles is taken as the point of reference to define the ranking of generalized trapezoidal fuzzy numbers. The reason*

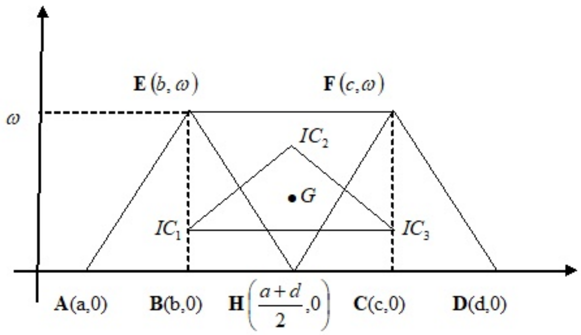

Figure 3. Centroid of Incenters of Trapezoidal Fuzzy Number

for selecting this point as a reference point is that each incenter point (G_1 of $\triangle AEH$, G_2 of $\triangle EHF$ and G_3 of $\triangle HFD$) are balancing points of each triangle, and the centroid of these incenter points is equistant from each centroids. Thus, this point would be gravity point (better reference point) than the other center point of the trapezoid.

The incenters of the three triangles are

$$IC_1 = (x_{I_1}, y_{I_1}) = \left(\frac{a\alpha_1 + b\beta_1 + \left(\frac{a+d}{2}\right)\gamma_1}{\alpha_1+\beta_1+\gamma_1}, \frac{w\beta_1}{\alpha_1+\beta_1+\gamma_1}\right),$$

$$IC_2 = (x_{I_2}, y_{I_2}) = \left(\frac{b\alpha_2 + c\beta_2 + \left(\frac{a+d}{2}\right)\gamma_2}{\alpha_2+\beta_2+\gamma_2}, \frac{w(\alpha_2+\beta_2)}{\alpha_2+\beta_2+\gamma_2}\right)$$

$$IC_3 = (x_{I_3}, y_{I_3}) = \left(\frac{c\alpha_3 + d\beta_3 + \left(\frac{a+d}{2}\right)\gamma_3}{\alpha_3+\beta_3+\gamma_3}, \frac{w\alpha_3}{\alpha_3+\beta_3+\gamma_3}\right)$$

where

$$\alpha_1 = \sqrt{\left(b - \left(\frac{a+d}{2}\right)\right)^2 + w^2},$$

$$\beta_1 = \sqrt{\left(a - \left(\frac{a+d}{2}\right)\right)^2},$$

$$\gamma_1 = \sqrt{(a-b)^2 + w^2},$$

$$\alpha_2 = \sqrt{\left(c - \left(\frac{a+d}{2}\right)\right)^2 + w^2},$$

$$\beta_2 = \sqrt{\left(b - \left(\frac{a+d}{2}\right)\right)^2 + w^2},$$

$$\gamma_2 = \sqrt{(b-c)^2},$$

$$\alpha_3 = \sqrt{\left(d - \left(\frac{a+d}{2}\right)\right)^2},$$

$$\beta_3 = \sqrt{\left(c - \left(\frac{a+d}{2}\right)\right)^2 + w^2},$$

$$\gamma_3 = \sqrt{(c-d)^2 + w^2}$$

The point IC_3 does not lie in the line $\overline{IC_1IC_2}$. Therefore, IC_1, IC_2 and IC_3 are non collinear and they could form a triangle.

The centroid of incenters IC_1, IC_2 and IC_3 of the generalized trapezoidal fuzzy number $A = (a,b,c,d;w)$ is defined as

$$G = (\bar{x}_0, \bar{y}_0) = \left(\frac{x_{I_1}+x_{I_2}+x_{I_3}}{3}, \frac{y_{I_1}+y_{I_2}+y_{I_3}}{3}\right) \quad (3.1)$$

The centroid of incenters for the triangular fuzzy number $A = (a,b,c;w)$, i.e., $c = b$ as a special case is given by

$$G = (\bar{x}_0, \bar{y}_0) = \left(\frac{x_{I_1}+x_{I_2}+x_{I_3}}{3}, \frac{y_{I_1}+y_{I_2}+y_{I_3}}{3}\right) \quad (3.2)$$

Definition 3.2. *The index of optimism associated with the ranking which represents the degree of optimism of a decision maker is defined as $I_\alpha(\tilde{A}) = \alpha\bar{y}_0 + (1-\alpha)\bar{x}_0$ where $\alpha \in [0,1]$ for a generalized trapezoidal fuzzy number $\tilde{A} = (a,b,c,d;w)$ with centroid of incenters $CI_{\tilde{A}}(\bar{x}_0,\bar{y}_0)$. We will have a pessimistic decision maker's view point when $\alpha = 0$ and which is equal to the distance of centroid of incenters from y-axis. We will have an optimistic decision maker's view point when $\alpha = 1$ and which is equal to the distance of centroid of incenters from x-axis and we will have a moderate decision maker's view point when $\alpha = 0.5$ and which is equal to the mean of distances of centroid of incenters from x and y axes. This method uses an index of modality that represents the neutrality of decision maker.*

Definition 3.3. *The ranking function of generalized trapezoidal fuzzy number $\tilde{A} = (a,b,c,d;w)$based on the Euclidean distance from the centroid of incenters of trapezoid to original point (origin) (1) for any type of decision makers whether they are optimistic ($\alpha = 1$), neutral ($\alpha = 0.5$) or pessimistic ($\alpha = 0$) is defined as $I_{Distance}(\tilde{A}) = \sqrt{\bar{x}_0^2 + \bar{y}_0^2}$. This function maps the every element in the set of all fuzzy numbers into a crisp one of the set of real numbers $R = (-\infty, +\infty)$.*

4. Mathematical Formulation of the Fuzzy Assignment Problem

Consider the fuzzy problems of assignment of n resources (workers) to n activities (jobs) so as to minimize the overall fuzzy cost or fuzzy time in such a way that each resource can associate with one and only one job. The fuzzy cost matrix $(\tilde{c}_{ij})$ is given as under:

$$\begin{array}{cc|cccc|c} & & \multicolumn{4}{c}{\text{Activity}} & \text{Availability} \\ & & A_1 & A_2 & \dots & A_n & \\ \hline \multirow{4}{*}{\text{Resource}} & R_1 & \tilde{c}_{11} & \tilde{c}_{12} & \dots & \tilde{c}_{1n} & 1 \\ & R_2 & \tilde{c}_{21} & \tilde{c}_{22} & \dots & \tilde{c}_{2n} & 1 \\ & \dots & \dots & \dots & \dots & \dots & \dots \\ & R_n & \tilde{c}_{n1} & \tilde{c}_{n2} & \dots & \tilde{c}_{nn} & 1 \\ \hline & \text{Required} & 1 & 1 & \dots & 1 & \end{array}$$

This fuzzy cost matrix is same as that of fuzzy transportation problem except that availability at each of the resources and the requirement at each of the destinations is unity.

Let x_{ij} denote the assignment of ith resource of jth activity, such that

$$x_{ij} = \begin{cases} 1, & \text{if resource } i \text{ is asssigned to activity } j \\ 0, & \text{otherwise} \end{cases}$$

Then the mathematical formulation of the fuzzy assignment problem is

$$\text{Minimize } \tilde{Z} = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij}x_{ij} \tag{4.1}$$

subject to the constraints:

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ and } \sum_{j=1}^{n} x_{ij} = 1;\ x_{ij} = 0 \text{ or } 1 \text{ for all}$$
$$i = 1,2,\dots,n \text{ and } j = 1,2,\dots,n$$

5. Proposed Algorithms for finding optimum solution of Assignment Problem based on proposed ranking indices

Algorithm 5.1

Step 1: First the given cost matrix for a Fuzzy Assignment Problem to be checked whether it is a balanced or unbalanced. If it is unbalanced one, then the problem to be changed as a balanced one by adding the dummy row(s) / column(s) with zero entries. If it a balanced assignment problem then step 2 to be executed.

Step 2: The fuzzy cost matrix to be defuzzified by using the proposed ranking method.

Step 3: Hungarian Algorithm to be applied to assign each machine to only one job and each job requires only one machine so as to minimize the total assignment cost.

Algorithm 5.2

Step 1: In the above mathematical form of fuzzy assignment problem, the fuzzy cost coefficients to be defuzzified into the following crisp ones by the proposed ranking method

$$\text{Minimize } z = \sum_{i=1}^{n}\sum_{j=1}^{n} I(\tilde{c}_{ij})x_{ij} \tag{5.1}$$

subject to the constraints:
$\sum_{i=1}^{n} x_{ij} = 1$ and $\sum_{j=1}^{n} x_{ij} = 1$; $x_{ij} = 0$ or 1 for all
$i = 1,2,\dots,n$ and $j = 1,2,\dots,n$

Step 2: The crisp assignment problem (5.1) to be solved by any of the conventional methods like simplex method etc.

Step 3: The optimum solution obtained from step 2 would assign each machine to only one job and each job requires only one machine so as to minimize the total assignment cost.

6. Numerical Example

A company wishes to assign 4 jobs to 4 machines in such a way that each job is assigned to some machine and no machine works on more than one job. The cost of assigning jobs i to machine j is given by the following matrix:

Jobs	Machine			
	1	2	3	4
A	(13,16,19,21: 0.2)	(23,24,27,28: 0.6)	(14,15,18,20: 0.4)	(7,10,12,15: 0.3)
B	(10,11,14,16: 0.3)	(23,26,29,30: 0.2)	(11,14,15,18: 0.1)	(24,25,28,29: 0.5)
C	(33,37,39,41: 0.1)	(16,18,21,22: 0.2)	(14,18,19,21: 0.4)	(12,13,17,18: 0.3)
D	(16,17,19,21: 0.4)	(24,25,27,30: 0.3)	(22,23,24,27: 0.2)	(7,10,11,13: 0.1)

Solution using Algorithm 5.1

The given trapezoidal fuzzy matrix is a balance one, so it can be solved by Hungarian Algorithm by converting it into crisp matrix using the proposed ranking techniques.

By using Definition 3.1 for fuzzy ranking technique, the given trapezoidal fuzzy matrices for pessimistic, moderate and optimistic decision maker's ($\alpha = 0$, 0.5 and 1) become

α	Jobs	Machine			
		1	2	3	4
0	A	17.3328	25.5000	16.6729	11.0000
	B	12.6703	27.1645	14.5000	26.5000
	C	37.6665	19.3317	18.1634	15.0000
	D	18.1729	26.3382	23.8355	10.3332
0.5	A	8.9163	12.9942	8.5849	5.7506
	B	6.5842	13.8320	7.5005	13.4958
	C	19.0842	9.9154	9.3499	7.7486
	D	9.3353	13.4202	12.1815	5.4193
1	A	0.4999	0.4884	0.4968	0.5012
	B	0.4982	0.4996	0.5011	0.4916
	C	0.5020	0.4992	0.5364	0.4973
	D	0.4977	0.5023	0.5275	0.5053

Proceeding the above three matrices by Hungarian Method, the optimal allocations are as follows:

α	Jobs	1	2	3	4
0	A	2.4896	6.3251	[0]	0
	B	[0]	10.1625	0	17.6729
	C	22.6665	[0]	1.3337	3.8432
	D	3.9965	7.8301	7.8294	[0]
Optimal allocation: $A \to 3,\ B \to 1,\ C \to 2,\ D \to 4$					
0.5	A	1.2477	3.1588	[0]	0
	B	[0]	5.0810	0	8.8296
	C	11.3356	[0]	0.6850	1.9180
	D	1.998	3.9161	3.9279	[0]
Optimal allocation: $A \to 3,\ B \to 1,\ C \to 2,\ D \to 4$					
0.5	A	0.0115	[0]	0	0.0139
	B	0.0055	0.0069	[0]	0
	C	0.0036	0.0008	0.0296	[0]
	D	[0]	0.0046	0.0214	0.0087
Optimal allocation: $A \to 2,\ B \to 3,\ C \to 4,\ D \to 1$					

The fuzzy optimal total costs for the three cases are

$$\begin{aligned}
Z_{\alpha=0} &= Z_{\alpha=0.5}\\
&= (14,15,18,20:0.4)+(10,11,14,16:0.3)\\
&\quad +(16,18,21,22:0.2)+(7,10,11,13:0.1)\\
&= (47,54,64,71:0.1)\\
Z_{\alpha=1} &= (23,24,27,28:0.6)+(11,14,15,18:0.1)\\
&\quad +(12,13,17,18:0.3)+(16,17,19,21:0.4)\\
&= (60,68,78,85:0.1)
\end{aligned}$$

Now, by using Definition 3.1 for $\alpha = 0$ and by using Definition 3.2, we have the crisp optimal total costs

$$\begin{aligned}
Z_{\alpha=0} &= Z_{\alpha=0.5} = 59.0000 \text{ and}\\
Z_{\alpha=0} &= Z_{\alpha=0.5} = 59.0021\\
Z_{\alpha=1} &= 72.8333 \text{ and}\\
Z_{\alpha=1} &= 72.8350
\end{aligned}$$

Similarly by using Definition 3.2 for fuzzy ranking technique, the given trapezoidal fuzzy matrix becomes

Jobs	Machine			
	1	2	3	4
A	17.3400	25.5047	16.6803	11.0114
B	12.6801	27.1691	14.5087	26.5046
C	37.6698	19.3381	18.1713	15.0082
D	18.1797	26.3430	23.8414	10.3455

Proceeding the above matrix by Hungarian Method, the optimal allocation is as follows:

	1	2	3	4
A	2.4883	6.3231	[0]	0
B	[0]	10.1591	0	17.6648
C	22.6616	[0]	1.3345	3.8403
D	3.9939	7.8273	7.8270	[0]

$$A \to 3,\ B \to 1,\ C \to 2,\ D \to 4 \tag{6.1}$$

The fuzzy optimal total costs is

$$\begin{aligned}
Z &= (14,15,18,20:0.4)+(10,11,14,16:0.3)\\
&\quad +(16,18,21,22:0.2)+(7,10,11,13:0.1)\\
&= (47,54,64,71:0.1)
\end{aligned}$$

Now, by using Definition 3.1 for $\alpha = 0$ and by using Definition 3.2, we have the crisp optimal total costs $Z = 59.0000$ and $Z = 59.0021$.

Solution using Algorithm 2

The given fuzzy assignment problem can be formulated in the form of the following mathematical programming problem using the linear model (4.1).

Minimize Z

$$\begin{aligned}
&= (13,16,19,21)x_{11}+(23,24,27,28)x_{12}\\
&\quad +(14,15,18,20)x_{13}+(7,10,12,15)x_{14}\\
&\quad +(10,11,14,16)x_{21}+(23,26,29,30)x_{22}\\
&\quad +(11,14,15,18)x_{23}+(24,25,28,29)x_{24}\\
&\quad +(33,37,39,41)x_{31}+(16,18,21,22)x_{32}\\
&\quad +(14,18,19,21)x_{33}+(12,13,17,18)x_{34}\\
&\quad +(16,17,19,21)x_{41}+(24,25,27,30)x_{42}\\
&\quad +(22,23,24,27)x_{43}+(7,10,11,13)x_{44}
\end{aligned}$$

subject to the constraints

$$\begin{aligned}
&x_{11}+x_{12}+x_{13}+x_{14}=1\\
&x_{11}+x_{21}+x_{31}+x_{41}=1\\
&x_{21}+x_{22}+x_{23}+x_{24}=1\\
&x_{12}+x_{22}+x_{32}+x_{42}=1\\
&x_{31}+x_{32}+x_{33}+x_{34}=1\\
&x_{13}+x_{23}+x_{33}+x_{43}=1\\
&x_{41}+x_{42}+x_{43}+x_{44}=1\\
&x_{14}+x_{24}+x_{34}+x_{44}=1
\end{aligned}$$

The above fuzzy mathematical programming problem becomes the following crisp mathematical programming problems by our ranking methods based on proposed centroid of trapezoidal fuzzy number.

By the Definition 3.1, we have the crisp linear programming problems for $\alpha = 0, 0.5, 1$ as follows:

The crisp LPP for $\alpha = 0$ is

Minimize Z

$$\begin{aligned}
&= 17.3328x_{11}+25.5000x_{12}+16.6729x_{13}\\
&\quad +11.0000x_{14}+12.6703x_{21}+27.1645x_{22}\\
&\quad +14.5000x_{23}+26.5000x_{24}+37.6665x_{31}\\
&\quad +19.3317x_{32}+18.1634x_{33}+15.0000x_{34}\\
&\quad ++18.1729x_{41}+26.3382x_{42}23.8355x_{43}\\
&\quad +10.3332x_{44}
\end{aligned}$$

subject to the constraints

$$x_{11}+x_{12}+x_{13}+x_{14}=1$$

$$
\begin{aligned}
&x_{11}+x_{21}+x_{31}+x_{41}=1\\
&x_{21}+x_{22}+x_{23}+x_{24}=1\\
&x_{12}+x_{22}+x_{32}+x_{42}=1\\
&x_{31}+x_{32}+x_{33}+x_{34}=1\\
&x_{13}+x_{23}+x_{33}+x_{43}=1\\
&x_{41}+x_{42}+x_{43}+x_{44}=1\\
&x_{14}+x_{24}+x_{34}+x_{44}=1
\end{aligned}
$$

The conventional assignment problem in the above form of LPP is solved with the help of TORA software. We get the solution with optimal objective value are as follows:

$$
\begin{aligned}
&x_{13}=x_{21}=x_{32}=x_{44}=1,\\
&x_{11}=x_{12}=x_{14}=x_{22}=x_{23}\\
&\quad=x_{24}=x_{31}=x_{33}=x_{34}\\
&\quad=x_{41}=x_{42}=x_{43}\\
&\quad=0
\end{aligned}
$$

The optimal objective value

$$
\begin{aligned}
&=(14,15,18,20:0.4)+(10,11,14,16:0.3)\\
&\quad+(16,18,21,22:0.2)+(7,10,11,13:0.1)\\
&=(47,54,64,71:0.1)
\end{aligned}
$$

Now, by using Definition 3.1 for $\alpha=0$ and by using Definition 3.2, we have the crisp optimal total costs $Z=59.0000$ and $Z=59.0021$.

The above optimal objective value represents the optimal total cost. Moreover, the optimal assignment is
$A\rightarrow 3,\ B\rightarrow 1,\ C\rightarrow 2,\ D\rightarrow 4$.

The crisp LPP for $\alpha=0.5$ is
Minimize Z

$$
\begin{aligned}
&=8.9163x_{11}+12.9942x_{12}+8.5849x_{13}+5.7506x_{14}\\
&\quad+6.5842x_{21}+13.8320x_{22}+7.5005x_{23}+13.4958x_{24}\\
&\quad+19.0842x_{31}+9.9154x_{32}+9.3499x_{33}+7.7486x_{34}\\
&\quad+9.3353x_{41}+13.4202x_{42}+12.1815x_{43}+5.4193x_{44}
\end{aligned}
$$

subject to the constraints

$$
\begin{aligned}
&x_{11}+x_{12}+x_{13}+x_{14}=1\\
&x_{11}+x_{21}+x_{31}+x_{41}=1\\
&x_{21}+x_{22}+x_{23}+x_{24}=1\\
&x_{12}+x_{22}+x_{32}+x_{42}=1\\
&x_{31}+x_{32}+x_{33}+x_{34}=1\\
&x_{13}+x_{23}+x_{33}+x_{43}=1\\
&x_{41}+x_{42}+x_{43}+x_{44}=1\\
&x_{14}+x_{24}+x_{34}+x_{44}=1
\end{aligned}
$$

The conventional assignment problem in the above form of LPP is solved with the help of TORA software. We get the solution with optimal objective value are as follows:

$$
\begin{aligned}
&x_{13}=x_{21}=x_{32}=x_{44}=1,\\
&x_{11}=x_{12}=x_{14}=x_{22}=x_{23}\\
&\quad=x_{24}=x_{31}=x_{33}=x_{34}\\
&\quad=x_{41}=x_{42}=x_{43}\\
&\quad=0
\end{aligned}
$$

The optimal objective value

$$
\begin{aligned}
&=(14,15,18,20:0.4)+(10,11,14,16:0.3)\\
&\quad+(16,18,21,22:0.2)+(7,10,11,13:0.1)\\
&=(47,54,64,71:0.1)
\end{aligned}
$$

Now, by using Definition 3.1 for $\alpha=0$ and by using Definition 3.2, we have the crisp optimal total costs $Z=59.0000$ and $Z=59.0021$.

The above optimal objective value represents the optimal total cost. Moreover, the optimal assignment is
$A\rightarrow 3,\ B\rightarrow 1,\ C\rightarrow 2,\ D\rightarrow 4$.

The crisp LPP for $\alpha=1$ is
Minimize Z

$$
\begin{aligned}
&=0.4999x_{11}+0.4884x_{12}+0.4968x_{13}\\
&\quad+0.5012x_{14}+0.4982x_{21}+0.4996x_{22}\\
&\quad+0.5011x_{23}+0.4916x_{24}+0.5020x_{31}\\
&\quad+0.4992x_{32}+0.5364x_{33}+0.4973x_{34}\\
&\quad+0.4977x_{41}+0.5023x_{42}+0.5275x_{43}\\
&\quad+0.5053x_{44}
\end{aligned}
$$

subject to the constraints

$$
\begin{aligned}
&x_{11}+x_{12}+x_{13}+x_{14}=1\\
&x_{11}+x_{21}+x_{31}+x_{41}=1\\
&x_{21}+x_{22}+x_{23}+x_{24}=1\\
&x_{12}+x_{22}+x_{32}+x_{42}=1\\
&x_{31}+x_{32}+x_{33}+x_{34}=1\\
&x_{13}+x_{23}+x_{33}+x_{43}=1\\
&x_{41}+x_{42}+x_{43}+x_{44}=1\\
&x_{14}+x_{24}+x_{34}+x_{44}=1
\end{aligned}
$$

The conventional assignment problem in the above form of LPP is solved with the help of TORA software. We get the solution with optimal objective value are as follows:

$$
\begin{aligned}
&x_{12}=x_{23}=x_{34}=x_{41}=1,\\
&x_{11}=x_{13}=x_{14}=x_{21}=x_{22}\\
&\quad=x_{24}=x_{31}=x_{32}=x_{33}\\
&\quad=x_{42}=x_{43}=x_{44}\\
&\quad=0
\end{aligned}
$$

Table 1. Comparative Analysis

Method	Definition	Decision Maker	Optimum Assignment	Fuzzy Total Cost	Ranking Method	Crisp Total Cost
Algorithm 5.1	Defn. 3.1	$\alpha = 0$	$A \to 3,\ B \to 1,\ C \to 2,\ D \to 4$	(47,54,64,71: 0.1)	Defn. 3.1, $\alpha = 0$	59.0000
		$\alpha = 0.5$			Defn. 3.2	59.0021
		$\alpha = 1$	$A \to 2B \to 3,\ C \to 4,\ D \to 1$	(60,68,78,85: 0.1)	Defn. 3.1, $\alpha = 0$	72.8333
					Defn. 3.2	72.8350
	Defn. 3.2		$A \to 3,\ B \to 1,\ C \to 2,\ D \to 4$	(47,54,64,71: 0.1)	Defn. 3.1, $\alpha = 0$	59.0000
					Defn. 3.2	59.0021
Algorithm 5.2	Defn. 3.1	$\alpha = 0$	$A \to 3,\ B \to 1,\ C \to 2,\ D \to 4$	(47,54,64,71: 0.1)	Defn. 3.1, $\alpha = 0$	59.0000
		$\alpha = 0.5$			Defn. 3.2	59.0021
		$\alpha = 1$	$A \to 2, B \to 3,\ C \to 4,\ D \to 1$	(60,68,78,85: 0.1)	Defn. 3.1, $\alpha = 0$	72.8333
					Defn. 3.2	72.8350
	Defn. 3.2		$A \to 3,\ B \to 1,\ C \to 2,\ D \to 4$	(47,54,64,71: 0.1)	Defn. 3.1, $\alpha = 0$	59.0000
					Defn. 3.2	59.0021

The optimal objective value

$$\begin{aligned} &= (23,24,27,28:0.6)+(11,14,15,18:0.1) \\ &\quad +(12,13,17,18:0.3)+(16,17,19,21:0.4) \\ &= (60,68,78,85:0.1) \end{aligned}$$

Now, by using Definition 3.1 for $\alpha = 0$ and by using Definition 3.2, we have the crisp optimal total costs $Z = 72.8333$ and $Z = 72.8350$.

The above optimal objective value represents the optimal total cost. Moreover, the optimal assignment is
$A \to 2, B \to 3,\ C \to 4,\ D \to 1$.

By the Definition 3.2, we have the crisp mathematical programming as
Minimize Z

$$\begin{aligned} &= 17.3400x_{11}+25.5047x_{12}+16.6803x_{13} \\ &\quad +11.0114x_{14}+12.6801x_{21}+27.1691x_{22} \\ &\quad +14.5087x_{23}+26.5046x_{24}+37.6698x_{31} \\ &\quad +19.3381x_{32}+18.1713x_{33}+15.0082x_{34} \\ &\quad +18.1797x_{41}+26.3430x_{42}+23.8414x_{43} \\ &\quad +10.3455x_{44} \end{aligned}$$

subject to the constraints

$$\begin{aligned} x_{11}+x_{12}+x_{13}+x_{14} &= 1 \\ x_{11}+x_{21}+x_{31}+x_{41} &= 1 \\ x_{21}+x_{22}+x_{23}+x_{24} &= 1 \\ x_{12}+x_{22}+x_{32}+x_{42} &= 1 \\ x_{31}+x_{32}+x_{33}+x_{34} &= 1 \\ x_{13}+x_{23}+x_{33}+x_{43} &= 1 \\ x_{41}+x_{42}+x_{43}+x_{44} &= 1 \\ x_{14}+x_{24}+x_{34}+x_{44} &= 1 \end{aligned}$$

The conventional assignment problem in the above form of LPP is solved with the help of TORA software. We get the solution with optimal objective value are as follows:

$$\begin{aligned} & x_{13}=x_{21}=x_{32}=x_{44}=1, \\ & x_{11}=x_{12}=x_{14}=x_{22}=x_{23} \\ &\quad =x_{24}=x_{31}=x_{33}=x_{34} \\ &\quad =x_{41}=x_{42}=x_{43} \\ &\quad =0 \end{aligned}$$

The optimal objective value

$$\begin{aligned} &= (14,15,18,20:0.4)+(10,11,14,16:0.3) \\ &\quad +(16,18,21,22:0.2)+(7,10,11,13:0.1) \\ &= (47,54,64,71:0.1) \end{aligned}$$

Now, by using Definition 3.1 for $\alpha = 0$ and by using Definition 3.2, we have the crisp optimal total costs $Z = 59.0000$ and $Z = 59.0021$.

The above optimal objective value represents the optimal total cost. Moreover, the optimal assignment is
$A \to 3,\ B \to 1,\ C \to 2,\ D \to 4$.

The solutions obtained from the two proposed algorithms based on Definitions 3.1 and 3.2 are summarized in Table 1.

7. Conclusion

In this paper, we have introduced two algorithms which have been converted from crisp techniques based on proposed ranking methods using centroid of incenters. In order to analyze the proposed algorithms based on new ranking indices, a numerical example has been given to find its optimum solution using the proposed algorithms. Subsequently, the different optimum solutions and its allocations have been arrived based on proposed distance based ranking index and the index of modality. From these solutions, we can observe that the optimum allocations of pessimistic and moderate decision makers' view point provide a minimum total cost rather than the total cost calculated from optimum allocations of optimistic view point. Moreover, in calculating crisp total cost, first minimum cost 59.0000 is given by pessimistic view point index and the next minimum cost 59.0021 is given by distance based ranking index. Finally, the pessimistic view point index gives an acceptable minimum total cost of Rs. 59 than the total cost

59.0021 calculated by the distance based index which may be considered as a view point of decision makers whether they are optimistic ($\alpha = 1$), neutral ($\alpha = 0.5$) or pessimistic ($\alpha = 0$).

References

[1] Harold W. Kuhn, The Hungarian Method for the assignment problem, *Naval Research Logistics Quarterly*, 2(1955), 83–97.

[2] R. Jain, Decision making in the presence of fuzzy variablem, *IEEE Transactions on Systems Man, and Cybrenctics*, 6(1976), 698–703.

[3] James Munkres, Algorithms for the Assignment and Transportation Problems, *Journal of the Society for Industrial and Applied Mathematics*, 5(1)(1957), 32–38.

[4] S. Jayakumar and A. Hari Ganesh, Fuzzy Multi Criteria Group Decision Making (Mcgdm) Approach for Variety Selection in Rice Farming, *Australian Journal of Basic and Applied Sciences*, 6(12)(2012), 308–318.

[5] P. Maheswari and M. Vijaya, On Initial Basic Feasible Solution (IBFS) of Fuzzy Transportation Problem Based on Ranking of Fuzzy Numbers using Centroid of Incenters, *International Journal of Applied Engineering Research*, *(Special Issue)*, 14(4) (2019), 155–164.

[6] A. Nagoor Gani and V. N. Mohamed, Solution of a Fuzzy Assignment Problem by Using a New Ranking Method, *International Journal of Fuzzy Mathematical Archive*, 2(2013), 8–16.

[7] Phani Bushan Rao and N. Ravi Shankar, Ranking Fuzzy Numbers with a Distance Method using Circumcenter of centroids and an Index Modality, *Advances in Fuzzy Systems*, (2011), 1–7.

[8] N. Ravi Shankar, Fuzzy Risk Analysis based on A New Approach of Ranking Fuzzy Numbers using Orthocenter of Centroids, *International Journal of Computer Applications*, 42(3)(2012), 24–36.

[9] S. Suresh Babu, Ranking Generalized Fuzzy Numbers using centroid of centroids, *International Journal of Fuzzy Logic Systems*, 2(3)(2012), 17–32.

[10] Surapati Pramanik and Pranab Biswas, Fuzzy Ranking Method to Assignment Problem with Fuzzy Costs, *International Journal of Mathematical Archive*, 2(12)(2011), 2549–2560.

[11] K. Thangavelu, G. Uthra and R. M. Umamageswari, Solution of Fuzzy Assignment Problem with Ranking of Generalized Trapezoidal Fuzzy Numbers, *International Journal of Pure and Applied Mathematics*, 106(6)(2016), 9–16.

[12] Y. L. P. Thorani, Ordering Generalized Trapezoidal Fuzzy Numbers, *International Journal of Contemporary Mathematical Sciences*, 7(12)(2012), 555–573.

[13] Y. L. P. Thorani and Ravi N. Shankar, Fuzzy Assignment Problem with Generalized Fuzzy Numbers, *Applied Mathematical Sciences*, 7(71)(2013), 3511–3537.

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