On two dimensional new integral transformation and its applications

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Abstract
In this paper, we have find out the relationship of generalized Elzaki transformation with new integral transformation and defined two dimensional new integral transformation and its relationship with two dimensional Laplace Transformation. Also, we have find out the conditions for convergence and uniform convergence of two dimensional new integral transformation. Also, as an application we have solved new integral transform of fractional derivative.

Keywords
Elzaki – Tarig Transform, Two – dimensional Laplace Transform, fractional derivative .

AMS Subject Classification
35A22, 44A10,26A33.

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1. Introduction
The theory of modern integral transform [4], [9],[10],[11],[14] which includes Fourier, Mellin, Laplace, Wavelet, Hilbert, Weirstrass, Chirplet, Abel’s, Laplace-Stieltjes , Laplace-Carson, L2- transform and zz transformation which plays an important role in the theory of fractional calculus.

Recently, the fractional differential equation and hence fractional partial differential equation were solved by Manjarekar and Bhadane [16], [17] for non – local and non – singular kernel with new definition of fractional derivative [2],[3]. The generalized definition of integral transform [18] has been defined in well manner under new conditions with its inversion and convolution property, relationship with other integral transform.

The paper mainly divided into three parts, in the first part the generalized definition and its relationship with one – dimensional new integral transformation were given. Second part consist definition of two dimensional Generalized Elzaki transformation and its relationship with two dimensional new integral transformation and find out new integral transformation of some functions. In the last part, we have proved the conditions for convergence and Uniform convergence of new integral transformation.

2. Preliminaries
We present some basic definitions needed in proving the main results.

2.1 Laplace type Integral Transform
Consider a function \( f(x) \) which is piecewise continuous and of exponential order then the Laplace type integral [4] transform
is defined as follows
\[ \mathcal{L}_\varepsilon \{ f(x) \} = \int_0^\infty e^{-\Phi(p) \varepsilon} f(x) \, dx \]  
(2.1)

where \( \Phi(p), a(x) \) are invertible functions with \( \varepsilon(x) = \int e^{-a(x)} \, dx \) is an exponential function.

### 2.2 Elzaki – Tarig Transformation

Consider the set \( S \) defined as follows:
\[ S = \{ f(x) : \exists k_1, k_2 > 0, |f(x)| < Me^{k_2}, x \in (-1)^j X [0, \infty), M > 0 \} \]
then for the given function which satisfies the condition of the set \( S \), the Elzaki – Tarig transformation of \( f(x) \) is defined as [10,14]
\[ \mathbb{S}_\varepsilon \{ f(x) \} = \int_0^\infty pe^{-\Phi(p) x} f(x) \, dx \]  
(2.2)

with \( p \neq 0 \).

### 2.3 Tarig Transformation

Given function which satisfies the condition of the set \( S \), the Tarig transformation of \( f(t) \) is defined as [10,14]
\[ \mathbb{T}_\varepsilon \{ f(x) \} = \int_0^\infty \frac{1}{p} e^{-\Phi(p) x} f(x) \, dx \]  
(2.3)

with \( p \neq 0 \).

### 2.4 New Integral Transform

The new integral transform of a function \( f(t) \) at a point \( u \) is defined by [13]
\[ \tilde{f}(u) = \mathcal{T}[f(t)](u) = \int_0^u f(t) e^{-t} \, dt \]  
(2.4)

where, \( u \) is a real number for those value of \( u \) for which the improper integral converges. The existence and convergence has been proved [13]. If we put \( u = s \) in the above definition then it gives us,
\[ f(u) = \mathcal{T}[f(t)](u) = u F(u) \]
where, \( F(u) \) is nothing but the Laplace transformation of \( f(t) \).

### 3. Generalized one dimensional Elzaki – Tarig transformation

We have define, Generalized Elzaki – Tarig Transformation by using the definition (2.1), (2.2) and (2.3) as [18],
\[ \mathbb{S}_\varepsilon f(x) = \int_0^\infty \Phi \left( \frac{1}{p} \right) \Phi_1(p) e^{-\Phi(p) \varepsilon} f(x) \, dx \]  
(3.1)

where, \( f(x) \in S \), defined by
\[ S = \{ f(x) : \exists k_1, k_2 > 0, |f(x)| < Me^{k_2}, x \in (-1)^j X [0, \infty), M > 0 \} \]

and \( \Phi \left( \frac{1}{p} \right), \Phi_1(p) \) are invertible functions of \( p \) with \( \varepsilon(x) = \int e^{-a(x)} \, dx \) an exponential function and \( a(x) \) is an invertible function, thus from the definitions above it can be seen that it is the generalization of Elzaki – Tarig transformation.

The existence of inverse, convolution property and derivative property has been [18] proved.

### 4. Relation with other Transformations

#### [4.1] Elzaki Tarig Transformation:

The Elzaki - Tarig Transformations [18] of a function \( f(x) \) can be obtained by taking \( \Phi(p) = \frac{1}{p}, \Phi_1(p) = 1, \varepsilon(x) = x \) in equation (3.1)
\[ \mathbb{S}_\varepsilon \{ f(x) \} = \int_0^\infty pe^{-\Phi(p) x} f(x) \, dx, p \neq 0 \]

#### [4.2] Tarig Transformation:

The Tarig Transformation [18] of a function \( f(x) \) can be obtained by taking \( \Phi(p) = \frac{1}{p}, \Phi_1(p) = 1, \varepsilon(x) = x \) in equation (3.1)
\[ \mathbb{T}_\varepsilon \{ f(x) \} = \int_0^\infty \frac{1}{p} e^{-\Phi(p) x} f(x) \, dx, p \neq 0 \]

#### [4.3] Laplace Transformation:

The Laplace Transformation [18] of a function \( f(x) \) can be obtained by taking \( \Phi(p) = p, \Phi_1(p) = p, \varepsilon(x) = x \) in equation (3.1) with \( \text{Re.}(p) > 0 \)
\[ \mathbb{L}_\varepsilon \{ f(x) \} = \int_0^\infty pe^{-\Phi(p) x} f(x) \, dx, \text{Re.}(p) > 0 \]

#### [4.4] New Integral Transformation:

The new Integral Transformation [13] of a function \( f(x) \) can be obtained by taking \( \Phi(p) = p, \Phi_1(p) = p^2, \varepsilon(x) = x \) and \( p > 0 \) in equation (3.1)we get
\[ \mathbb{N}_\varepsilon \{ f(x) \} = \int_0^\infty p e^{-\Phi(p) x} f(x) \, dx \]
\[ \Rightarrow \mathbb{N}_\varepsilon \{ f(x) \} = \int_0^\infty p e^{-\Phi(p) x} f(x) \, dx \]
\[ \Rightarrow \mathbb{N}_\varepsilon \{ f(x) \} = pF(p) \]

where, \( F(p) \) is Laplace Transform of a function \( f(x) \) at a point ‘p’.

### 5. Generalized two dimensional Elzaki – Tarig transformation

We extend this definition of two dimensional generalized Elzaki – Tarig Transformation by using the definition [12] as;
\[ \mathbb{S}_\varepsilon \[ f(x,t) \] = \int_0^\infty \int_0^\infty \Phi \left( \frac{1}{p} \right) \Phi_1(p) e^{-\Phi(p) \varepsilon(x,t)} f(x,t) \, dx \, dt \]  
(5.1)

where, \( p \neq 0 \) and \( q \neq 0 \) and \( f(x,t) \in S \) for a.e. ‘t’, defined by
\[ S = \{ f(x,t) : \exists k_1, k_2 > 0, |f(x,t)| < Me^{k_2}, x \in (-1)^j X [0, \infty), M > 0 \} \]
which true for \( a.e.\ t \in (-1)^{1/2} X [0, \infty) \) and vice versa, such that \( \Phi(x) = \frac{1}{2} x \Phi'(x) \) and \( \Phi(x, y) = \Phi(x) \Phi(y) \) both are invertible functions of \( p \) and \( q \) with:

\[
\varepsilon(p, q, x, t) = [\Phi(p) \varepsilon(x) + \Phi(q) \varepsilon(t)]
\]

are invertible functions, thus from the definitions above it can be seen that it is the generalization of two dimensional [12] Elzaki – Tarig transformation.

### 6. Relationship with other two dimensional transformations

#### [6.1] 2 – D Elzaki – Tarig Transformation

The two dimensional Elzaki – Tarig Transformations from above definition of a function \( f(x, t) \) can be obtained by taking \( \Phi(\frac{1}{p}, \frac{1}{q}) = pq \) and \( \Phi_1(p, q) = pq, \varepsilon(p) = \frac{1}{p}, \varepsilon(q) = \frac{1}{q} \)

\[
\varepsilon(x, \varepsilon(t)) = \varepsilon(p, q) = t \text{ in (5.1), we get}
\]

\[
\Theta_{p+\frac{q}{2}} \left[ (f(x, t); (p, q)) \right] = \int_0^\infty \int_0^\infty \frac{d^2}{dtdx} \left[ f(x, t) dt dx \right], \ p \neq 0, q \neq 0
\]

#### [6.2] 2 – D Tarig Transformation

The two dimensional Elzaki – Tarig Transforms from above definition of a function \( f(x, t) \) can be obtained by taking \( \Phi(\frac{1}{p}, \frac{1}{q}) = p^2 q^2 \) and \( \Phi(p, q) = \frac{1}{p}, \varepsilon(p) = \frac{1}{p}, \varepsilon(q) = \frac{1}{q} \)

\[
\varepsilon(x, \varepsilon(t)) = t \text{ in (5.1), we get}
\]

\[
\Theta_{p+\frac{q}{2}} \left[ (f(x, t); (p, q)) \right] = \int_0^\infty \int_0^\infty \frac{d^2}{dtdx} \left[ f(x, t) dt dx \right], \ p \neq 0, q \neq 0
\]

#### [6.3] 2 – D Finite Elzaki-Tarig Transform

The two dimensional finite Elzaki-Tarig transformation can be defined by using (5.1) as:

\[
\tau_{p+\frac{q}{2}} \left[ (f(x, t); (p, q); a, b) \right] = \int_0^\infty \int_0^\infty \frac{d^2}{dtdx} \left[ f(x, t) dt dx \right], \ p \neq 0
\]

where \( p, q \neq 0 \) and \( \tau_{p+\frac{q}{2}} \left[ (f(x, t); (p, q); a, b) \right] \mid M \) is positive constant which is independent of \( a \) and \( b \) such that,

\[
\lim_{(a, b, \epsilon \to 0)} \tau_{p+\frac{q}{2}} \left[ (f(x, t); (p, q); a, b) \right] = \tau_{p+\frac{q}{2}} \left[ (f(x, t); (p, q)) \right]
\]

exists and finite value \( \forall a, b > 0 \)

#### [6.4] 2 – D New Integral Transform

The two dimensional New Integral Transform can be defined using (3.1) and by taking \( \Phi_1(p, q) = p^2 q^2 \) and \( \Phi(\frac{1}{p}, \frac{1}{q}) \)

\[
= \frac{1}{pq} \Phi(p, q) = pq \varepsilon(x) = x, \varepsilon(t) = t \text{ in equation (5.1), we get}
\]

\[
\tau_{p+\frac{q}{2}} \left[ (f(x, t); (p, q)) \right] = \int_0^\infty \frac{d^2}{dtdx} \left[ f(x, t) dt dx \right], \ p > 0
\]

### 7. Theory and Applications of New Integral Transformation

**Lemma:** Let \( X \geq 0 \) be an absolutely continuous r.v. and \( f(t) \) be its density if \( \tilde{f}(u) = \mathbb{E} \{ f(t) \} \) in New Integral transformation with \( \tilde{f}(0) = 1, \tilde{f}(u) > 0 \) then the expected value has the property

\[
E(X^\alpha) = \int_0^\infty t^\alpha f(t) dt = \frac{\alpha}{1-\alpha} \int_0^\infty \int_0^\infty \frac{d^2}{dtdx} \left[ f(t) dt dx \right], 0 < \alpha < 1
\]

**Proof:** We prove this property by using the relation of Laplace transform and new integral transform along with property if \( F(s) = \mathbb{L} \{ f(t) \} \), then by [11],

\[
\frac{\alpha}{1-\alpha} \int_0^\infty \frac{1-\alpha}{\tau_{p+\frac{q}{2}}(t)} \int_0^\infty \frac{d^2}{dtdx} \left[ f(t) dt dx \right], 0 < \alpha < 1
\]

By using the relation, \( \mathbb{T} \{ f(t) \} = s F(s), s > 0 \)

\[
\Rightarrow \int_0^\infty \frac{s}{\tau_{p+\frac{q}{2}}(t)} ds = \frac{\alpha}{1-\alpha} \int_0^\infty \frac{d^2}{dtdx} \left[ f(t) dt dx \right], 0 < \alpha < 1
\]

\[
\Rightarrow \int_0^\infty \frac{s}{\tau_{p+\frac{q}{2}}(t)} ds = \frac{\alpha}{1-\alpha} \int_0^\infty t^\alpha f(t) dt = E(X^\alpha)
\]

### 8. Convergence and Uniform Convergence theorem for double new integral transformation

In this section, we prove the convergence, uniform convergence theorem for double new integral transformation with its condition;

**Theorem 8.1:** If \( f(x, t) \) is continuous on \([0, \infty) \times [0, \infty) \) and integral converges at \( p = p_0 \) and \( q = q_0 \) then the two-dimensional new integral transform of \( f(x, t) \) converges on for \( p > p_0 \) and \( q > q_0 \) where \( \varepsilon(x, q, x, t) \geq 0 \) in the positive quadrant.

We will use the following lemmas to prove the Theorem.

**Lemma 8.1.1:** If \( \mathbb{T} \{ f(x, t); q \} = \int_0^t \int_0^s f(x, t) e^{-q \eta} dt d\eta \) converges at \( q = q_0 \) then the integral converges for \( q > q_0 > 0 \).

**Proof:** Consider the set of following function:

\[
\alpha(x, t) = \int_0^t \Phi(x, u) e^{-q \eta} du, 0 < t < \infty \tag{A}
\]

then clearly by definition \( \alpha(x, 0) = 0 \) and \( \lim_{t \to +\infty} \int_0^t \Phi(x, t) e^{-q \eta} dt \)

converges at \( q = q_0 \) and by fundamental theorem of calculus from above equation we get,

\[
\alpha(x, t) = q_0 e^{q_0 t} \Phi(x, t)
\]

\[
\Rightarrow \Phi(x, t) = \frac{1}{q_0} \Phi(x, t)
\]

Choose \( \varepsilon_1 \) and \( R_1 \) such that \( 0 < \varepsilon_1 < R_1 \), then the integral (A) becomes,

\[
\int_{\varepsilon_1}^{R_1} \Phi(x, t) e^{-q \alpha} dt = \int_{\varepsilon_1}^{R_1} \frac{1}{q_0} \Phi(x, t) e^{q_0 t} e^{-q \alpha} dt
\]
Applying integration by parts, we get,
\[ \int_{\varepsilon_1}^{R} q\phi(x,t)e^{-q\phi} \, dt = \int_{\varepsilon_1}^{R} \frac{q}{q_0} \alpha(x,t)e^{-(q-q_0)t} \, dt \]

Applying integration by parts, we get,
\[ \int_{\varepsilon_1}^{R} q\phi(x,t)e^{-q\phi} \, dt = \left. \frac{q}{q_0} e^{-(q-q_0)t}\alpha(x,t) \right|_{\varepsilon_1}^{R} + \int_{\varepsilon_1}^{R} (q-q_0)\alpha(x,t)e^{-(q-q_0)t} \, dt \]
\[ = \frac{q}{q_0} \left[ e^{-(q-q_0)t}\alpha(x,t) \right]_{\varepsilon_1}^{R} - \int_{\varepsilon_1}^{R} (q-q_0)\alpha(x,t)e^{-(q-q_0)t} \, dt \]
\[ = \frac{q}{q_0} \left[ e^{-(q-q_0)t}\alpha(x,t) \right]_{\varepsilon_1}^{R} - \int_{\varepsilon_1}^{R} (q-q_0)\alpha(x,t)e^{-(q-q_0)t} \, dt \]
\[ + (q-q_0) \int_{\varepsilon_1}^{R} \alpha(x,t)e^{-(q-q_0)t} \, dt \]

Taking \( \varepsilon_1 \to 0 \), we get,
\[ \int_{\varepsilon_1}^{R} q\phi(x,t)e^{-q\phi} \, dt = \left. \frac{q}{q_0} e^{-(q-q_0)t}\alpha(x,t) \right|_{0}^{R} + \int_{0}^{R} (q-q_0)\alpha(x,t)e^{-(q-q_0)t} \, dt \]
\[ = \left. \frac{q}{q_0} e^{-(q-q_0)t}\alpha(x,t) \right|_{0}^{R} + \int_{0}^{R} (q-q_0)\alpha(x,t)e^{-(q-q_0)t} \, dt \]
\[ = \int_{0}^{R} (q-q_0)\alpha(x,t)e^{-(q-q_0)t} \, dt \]

Now by applying, \( R_1 \to \infty \), if \( q > q_0 \), then the first term inside the bracket on RHS tends to zero.

So that the above integral becomes,
\[ \int_{0}^{\infty} p\phi(x,t)e^{-pt} \, dt = \left. \frac{p}{p_0} e^{-(p-p_0)t}\alpha(x,R_1) \right|_{0}^{\infty} + \int_{0}^{\infty} (p-p_0)\alpha(x,t)e^{-(p-p_0)t} \, dt \]
\[ = \left. \frac{p}{p_0} e^{-(p-p_0)t}\alpha(x,R_1) \right|_{0}^{\infty} + \int_{0}^{\infty} (p-p_0)\alpha(x,t)e^{-(p-p_0)t} \, dt \]
\[ = \frac{p}{p_0} \left[ e^{-(p-p_0)t}\alpha(x,R_1) \right]_{0}^{\infty} + \int_{0}^{\infty} (p-p_0)\alpha(x,t)e^{-(p-p_0)t} \, dt \]
\[ = \frac{p}{p_0} \left[ e^{-(p-p_0)t}\alpha(x,R_1) \right]_{0}^{\infty} + \int_{0}^{\infty} (p-p_0)\alpha(x,t)e^{-(p-p_0)t} \, dt \]

Taking \( \varepsilon_2 \to 0 \), we get,
\[ \int_{\varepsilon_2}^{R_2} q\phi(x,t)e^{-q\phi} \, dx = \int_{\varepsilon_2}^{R_2} \frac{q}{q_0} e^{-(q-q_0)x}\alpha(x,t) \, dx + \int_{\varepsilon_2}^{R_2} (q-q_0)\alpha(x,t)e^{-(q-q_0)x} \, dx \]
\[ = \frac{q}{q_0} \left[ e^{-(q-q_0)x}\alpha(x,t) \right]_{\varepsilon_2}^{R_2} - \int_{\varepsilon_2}^{R_2} (q-q_0)\alpha(x,t)e^{-(q-q_0)x} \, dx \]
\[ + (q-q_0) \int_{\varepsilon_2}^{R_2} \alpha(x,t)e^{-(q-q_0)x} \, dx \]

Now by applying, \( R_2 \to \infty \), if \( p > p_0 \), then the first term inside the bracket on RHS tends to zero. So that the above integral becomes,
\[ \int_{0}^{\infty} p\phi(x,t)e^{-pt} \, dx = \frac{p}{p_0} \int_{0}^{\infty} \alpha(x,t)e^{-(p-p_0)t} \, dx \]

Thus by combining Lemma (8.1.1) and (8.2.1) the two dimensional new integral transformation of \( f(x,t) \) converges uniformly on \([p,\infty) \times [q,\infty]\) if \( p > p_0 \) and \( q > q_0 \).

8.2 Uniform Convergence for double new integral transformation

Theorem 8.2: If \( f(x,t) \) is continuous on \([0, \infty) \times [0, \infty)\) and integral \( \mathbb{E}(x,t) = p_0q_0 \int_{0}^{\infty} \int_{0}^{\infty} f(u,v)e^{-p_0u}e^{-q_0v}dudv \)

is bounded on \([0, \infty) \times [0, \infty)\) then the two-dimensional new integral transform of \( f \) converges uniformly \([p,\infty) \times [q,\infty)\) if \( p > p_0 \) and \( q > q_0 \).

We will use the following lemmas to prove the Theorem.

Lemma 8.2.1: If \( g(x,t) = q_0 \int_{0}^{t} f(x,v)e^{-q_0v}dv \) is bounded on \([q_0,\infty)\) then the integral converges uniformly on \([q,\infty)\) for \( q > q_0 \).

Proof: If \( 0 < R < R_1 \) then consider the integral,
\[ \int_{R}^{R_1} f(x,t)e^{-q\phi} \, dt \]
\[ = q \int_{R}^{R_1} e^{-(q-q_0)t}e^{q_0\phi}f(x,t) \, dt \]
\[ = q \int_{R}^{R_1} e^{-(q-q_0)t}e^{q_0\phi}f(x,t) \, dt \]
\[ - \int_{R}^{R_1} e^{-(q-q_0)t}e^{q_0\phi}f(x,t) \, dt \] (By using Fundamental Theorem of Calculus)

Now by applying integration by parts, we have
\[ \int_{R}^{R_1} f(x,t)e^{-q\phi} \, dt \]
\[ = \left. \frac{q}{q_0} e^{-(q-q_0)t}\alpha(x,R_1) \right|_{R}^{R_1} + \int_{R}^{R_1} (q-q_0)\alpha(x,t)e^{-(q-q_0)t} \, dt \]
\[ = \left. \frac{q}{q_0} e^{-(q-q_0)t}\alpha(x,R_1) \right|_{R}^{R_1} + \int_{R}^{R_1} (q-q_0)\alpha(x,t)e^{-(q-q_0)t} \, dt \]

So that if \( |g(x,t)| \leq M \) the above integral gives us,
\[ \left| q \int_{R}^{R_1} f(x,t)e^{-q\phi} \, dt \right| \]
\[ \leq \frac{Mq_0}{q_0} \left[ e^{-(q-q_0)t}\alpha(x,R_1) \right|_{R}^{R_1} + \int_{R}^{R_1} (q-q_0)\alpha(x,t)e^{-(q-q_0)t} \, dt \]

hence by cauchy’s criteria for uniform convergence on the integral \( I = q \int_{R}^{R_1} f(x,t)e^{-q\phi} \, dt \) converges uniformly on \([q,\infty)\), if \( q > q_0 \).
Lemma 8.2.2: If \( h(x,t) = p_0 \int f(u,t)e^{-pu}du \) is bounded on \([p_0,\infty)\) then the integral converges uniformly on \([p,\infty)\) for \( p > p_0 > 0 \).

Proof: If \( 0 < R < R_1 \) then consider the integral,

\[
\int_{R}^{R_1} p f(x,t)e^{-px}dx = \frac{p}{p_0} \int_{R}^{R_1} e^{-(p-p_0)x}f(x,t)dx
\]

Similarly, if \( p > p_0 > 0 \), then the integral converges uniformly on \([p_0,\infty)\) for \( p > p_0 > 0 \).

Condition 1:
If the function \( f(x,t) \) satisfies [5] condition of boundedness i.e. \( |f(x,t)| \leq M e^{\alpha x + \beta y} \) for all \( x \leq 0, y \leq 0 \) where \( M, \alpha, \beta \) are positive constants and \( f(x,t) \in S = \{(p,q): p > \alpha, q > \beta\} \)

Condition 2:
If the function \( f(x,t) = f_1(t) \) and the integrals;

\[
\mathcal{T}_1(p) = \int_{0}^{\infty} pf_1(x)e^{-px}dx
\]

and finite then the integral \( \mathcal{T}_2(p,q) = \int_{0}^{\infty} q f_2(t)e^{-qt}dt \) exists and finite, Moreover [5];

\[
\mathcal{T}_1(p) = \left\{ (p,q) : \mathcal{T}_1(p,q) = \mathcal{T}_1(p) \mathcal{T}_2(q) \right\}
\]

Condition 3:
If we define [5], \( \mathcal{T}_3(p,q,a,b) = \int_{0}^{\infty} pq e^{-px} e^{-qt} f(x,t)dxdy \) and if there exists \( M > 0 \) independent of \( a \) and \( b \) such that,

\[
\lim_{(a,b) \to (m,0)} \mathcal{T}_3(p,q,a,b) = \mathcal{T}_3(p,q)
\]

exists and finite \( \forall a,b > 0 \), then \( \mathcal{T}_3(p,q) \) exist and finite.

Then by using the above condition the following result [5] can be proved.

Theorem 8.3: \( \mathcal{T}_4(p,q) = \int_{0}^{\infty} pq e^{-px} e^{-qt} f(x,t)dxdy \) converges at a point \((p_0,q_0)\) then the integral converges for all the points \((p,q)\) for all \( p > p_0 \) and \( q > q_0 \) with the property that \( f(x,0) = 0 = f(0,t) \).

Proof: Given the integral

\[
\mathcal{T}_4(p,q) = \int_{0}^{\infty} pq e^{-px} e^{-qt} f(x,t)dxdy
\]

By applying Fubini's theorem to the above equation, we get,

\[
\mathcal{T}_4(p,q) = \int_{0}^{\infty} \int_{0}^{\infty} pq e^{-px} e^{-qt} f(x,t)dxdy
\]

Define

\[
\mathcal{T}_5(p,q,a,b) = \int_{0}^{\infty} pq e^{-px} e^{-qt} f(x,t)dxdy
\]

By applying Fubini's theorem to the above equation, we get,

\[
\mathcal{T}_5(p,q,a,b) = \int_{0}^{\infty} \int_{0}^{\infty} pq e^{-px} e^{-qt} f(x,t)dxdy
\]

Define \( \phi(x,t) = \int_{0}^{\infty} e^{-pt} dt \) so that the above equation becomes

\[
\mathcal{T}_6(p,q,a,b) = \int_{0}^{\infty} pq e^{-px} e^{-qt} f(x,t)dxdy
\]

Define \( p - p_0 = \alpha, q - q_0 = \beta \) with \( |\phi(x,t)| < M \)
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\[ |\mathcal{T}_x(p, q; a, b)| \leq pq \]

Now, if \( \alpha > 0 \) and \( \beta > 0 \)

\[ \lim_{(a, b) \to (\infty, \infty)} \mathcal{T}_x(p, q; a, b) \text{ exists and finite. Moreover,} \]

\[ \lim_{(a, b) \to (\infty, \infty)} \mathcal{T}_x(p, q; a, b) = pq(p - p_0)(q - q_0) \int_0^\infty e^{-\alpha x} \phi(x, t) \, dx \, dt \]

\section{9. Conclusion}

We have proved the relationship between generalized Elzaki-Tarig transformation and define the new two dimensional integral transformation. Besides that, convergence, uniform convergence, and existence of two dimensional new integral transformation under some conditions are also been proved.

\begin{thebibliography}{99}


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