Nano minimal and maximal $\delta \mathcal{S}$-open sets

S. Parimala$^1$ and V. Chandrasekar$^2$

Abstract
The class of sets namely nano minimal $\delta$-semiopen and nano maximal $\delta$-semiopen sets and their topological properties are introduced and discussed in this paper.

Keywords
$\text{NMi}_\delta \mathcal{S}o$, $\text{NMi}_\delta \mathcal{S}c$, $\text{NMax}_\delta \mathcal{S}o$ and $\text{NMax}_\delta \mathcal{S}c$ sets.

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1,2 Department of Mathematics, Kandaswamy Kandar’s College, P-velur, Tamil Nadu-638182, India.

*Corresponding author: pspmaths@gmail.com; vokkc3895@gmail.com

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1. Introduction and Preliminaries

Lellis Thivagar [2] introduced the Nano topology (briefly, $\mathcal{N}$) and also defined Nano open (briefly, $\mathcal{N}o$) and Nano closed (briefly, $\mathcal{N}c$) sets, Nano-interior (briefly, $\mathcal{N}nt$) and Nano-closure (briefly, $\mathcal{N}cl$) in a nano topological spaces (briefly, $\mathcal{N}ts$). The class of sets namely, $\delta$-open sets are playing more important role in topological spaces, because of their applications in various fields of Mathematics and other real fields. The authors in [3, 7] studied various kinds of $\delta$-open sets and their properties in topological spaces. The class of sets, namely nano $\delta$-preopen, nano $\delta$-semiopen and nano $\delta\alpha$-open sets using nano $\delta$-open sets were introduced in [4], Parimala et. al. The purpose of this paper is to discuss the concept of nano minimal $\delta$- semiopen and nano maximal $\delta$-semiopen sets and study their properties and applications in $\mathcal{N}ts$.

Definition 1.1. [2, 8] Let $(\mathcal{N}, \tau_{\mathcal{N}}(P))$ be a $\mathcal{N}ts$ and a subset $L \subseteq \mathcal{N}$ is said to be nano regular open (briefly, $\mathcal{N}ro$) if $L = \mathcal{N}nt(\mathcal{N}cl(L))$.

Definition 1.2. [5] Let $(\mathcal{N}, \tau_{\mathcal{N}}(P))$ be a $\mathcal{N}ts$ and let $L \subseteq \mathcal{N}$ then the nano $\delta$-interior (resp. nano $\delta$-closure) of $L$ is defined and denoted by $\mathcal{N}nt_{\delta}(L) = \bigcup \{M : M$ is a $\mathcal{N}ro$ set and $M \subseteq L\}$ (resp. $\mathcal{N}cl_{\delta}(L) = \bigcup \{x \in \mathcal{N} : \mathcal{N}nt(\mathcal{N}cl(M)) \cap L \neq \phi, M$ is a $\mathcal{N}ro$ set and $x \in M\}$.

Definition 1.3. [5] A subset $L$ of $P$ is said to be nano $\delta$-open (resp. nano $\delta$-closed) (briefly, $\mathcal{N}do$ (resp. $\mathcal{N}dc$)) set if $L = \mathcal{N}nt_{\delta}(L)$ (resp. $L^c$ is a nano $\delta$-open set).

Definition 1.4. [4] A subset $L \subseteq (\mathcal{N}, \tau_{\mathcal{N}}(P))$ is called as a nano $\delta$-semi open (briefly, $\mathcal{N}_\delta \mathcal{S}o$) set if $L \subseteq \mathcal{N}nt(\mathcal{N}nt_{\delta}(L))$.

The complements of the above respective open sets are their respective closed sets.

The family of all $\mathcal{N}do$ sets is denoted by $\mathcal{N}do(\mathcal{N}, \tau_{\mathcal{N}}(P))$ and the family of all nano $\delta$-semi closed (briefly, $\mathcal{N}_\delta \mathcal{S}c$ sets) is denoted by $\mathcal{N}dc(\mathcal{N}, \tau_{\mathcal{N}}(P))$.

Definition 1.5. [4] Let $(\mathcal{N}, \tau_{\mathcal{N}}(P))$ be a $\mathcal{N}ts$ and let $L \subseteq \mathcal{N}$ then the nano $\delta$-semi interior (resp. nano $\delta$-semi closure) of $L$ is the union (resp. intersection) of all $\mathcal{N}do$ (resp. $\mathcal{N}dc$) sets contained in (resp. containing) $L$ and denoted by $\mathcal{N}nt_{\delta}(L)$ (resp. $\mathcal{N}cl_{\delta}(L)$).

Throughout this paper, $(\mathcal{N}, \tau_{\mathcal{N}}(P))$ is a $\mathcal{N}ts$ with respect to $P$ where $P \subseteq \mathcal{N}$, $R$ is an equivalence relation on $\mathcal{N}$. Then $\mathcal{N}/R$ denotes the family of equivalence classes of $\mathcal{N}$ by $R$. All other undefined notions from [1,2,6].

2. Nano minimal $\delta \mathcal{S}$ open sets

Definition 2.1. A non-empty subset $L$ of a $\mathcal{N}ts$, $(\mathcal{N}, \tau_{\mathcal{N}}(P))$ is said to be Nano minimal $\delta \mathcal{S}$ open set (resp. Nano minimal $\delta \mathcal{S}$ closed) (briefly, $\text{NMin}_\delta \mathcal{S}o$ (resp. $\text{NMin}_\delta \mathcal{S}c$)) if any $\mathcal{N}do$ (resp. $\mathcal{N}dc$) set which is contained in $L$ is either empty or $L$. 
The family of all \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(P)\) sets will be denoted by \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(P)\). We set \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(P,x) = \{F \mid x \in F \in \mathfrak{Mm}_i\delta_{\mathcal{C}}(P)\}\) (resp. \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(C,P,x) = \{F \mid x \in F \in \mathfrak{Mm}_i\delta_{\mathcal{C}}(C,P)\}\).

Example 2.2. Let \(\mathcal{U} = \{p, q, r, s, t\}\) with \(\mathcal{U}/R = \{\{r\}, \{p, q\}, \{s, t\}\}\) and \(P = \{p, r\}\). Then the \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(P) = \{\mathcal{U}, \phi, \{r\}, \{p, q\}, \{p, q, r\}\}\) are \(\mathfrak{N}_0\) sets; the \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(C,P)\) sets are \(\{r\}, \{p, q\}, \{r, t\}, \{p, r, t\}, \{p, q, r, t\}\); \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(C)\) sets are \(\{r\}, \{p, q\}\).

Theorem 2.3. For any \(\mathfrak{N}_0\) sets, \(\langle \mathcal{U}, \tau_{\mathcal{U}}(P) \rangle\).

(i) Let \(L\) be a \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(P)\) (resp. \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(C)\)) and \(M\) be an \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(C)\) (resp. \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(P)\)) set. Then \(L \cap M = \emptyset\) if \(M \subseteq L\).

(ii) Let \(L\) and \(M\) be \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(P)\) (resp. \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(C)\)) sets. Then \(L \cap M = \emptyset\) if \(L \subseteq M\).

Proof. (i) Suppose \(L \cap M \neq \emptyset\). Since \(L\) is a \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(P)\) (resp. \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(C)\)), \(M\) be an \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(P)\) (resp. \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(C)\)) with \(L \cap M \neq \emptyset\). Since \(L \cap M \neq \emptyset\), \(L \subseteq M\) and hence \(L \cap M \subseteq \emptyset\).

(ii) Suppose that \(\bigcup_{\lambda \in \Lambda} L_{\lambda} \cap L = \emptyset\). Then \(\exists \lambda \in \Lambda\) \(L_{\lambda} \cap L = \emptyset\).

\[\bigcup_{\lambda \in \Lambda} L_{\lambda} \cap L = \emptyset.\]

\[\exists \lambda \in \Lambda\] \(L_{\lambda} \cap L = \emptyset\).

3. Nano Maximal \(\delta_{\mathcal{C}}\)-open sets

Definition 3.1. A \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(P)\) set \(L\) of \(\mathcal{U}\) such that \(\emptyset \neq L \subseteq \mathcal{U}\) is called a nano maximal \(\delta_{\mathcal{C}}\)-semiopen (briefly, \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(P)\)) set if any \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(P)\) set which contains \(L\) is either \(\mathcal{U}\) or \(L\).

The family of all \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(P)\) sets denoted as \(\mathfrak{Mm}_{\delta_{\mathcal{C}}}(P)\). We set \(\mathfrak{Mm}_{\delta_{\mathcal{C}}}(P,x) = \{L \mid x \in \mathfrak{Mm}_{\delta_{\mathcal{C}}}(P)\}\).

Example 3.2. Let \(\mathcal{U} = \{p, q, r, s, t\}\) with \(\mathcal{U}/R = \{\{r\}, \{p, q\}, \{s, t\}\}\) and \(P = \{p, r\}\). Then the \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(P) = \{\mathcal{U}, \phi, \{r\}, \{p, q\}, \{p, q, r\}\}\) are \(\mathfrak{N}_0\) sets; the \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(C,P)\) sets are \(\{r\}, \{p, q\}, \{r, t\}, \{p, r, t\}, \{p, q, r, t\}\); \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(C)\) sets are \(\{r\}, \{p, q\}\).

Theorem 3.3. Let a subset \(L\) of \(\mathcal{U}\) such that \(\emptyset \neq L \subseteq \mathcal{U}\). Then \(L\) is \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(P)\) if \(L = \mathcal{U}\) or \(L = \emptyset\).

Proof. Necessity. Let \(L\) be a \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(P)\) set. Then \(L \subseteq \mathcal{U}\) or \(L = \emptyset\). Hence by Definition 3.1, \(\mathcal{U}\) or \(L = \emptyset\).

Sufficiency. Let \(\mathcal{U}\) or \(L \subseteq \mathcal{U}\). Then \(\emptyset \neq \mathcal{U}\) or \(L \subseteq \mathcal{U}\). Hence \(\emptyset \neq \mathcal{U}\) or \(L = \emptyset\).

Theorem 3.4. Let \(L\) be a \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(P)\) set and if \(M\) be a \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(P)\) (resp. \(\mathfrak{Mm}_{\delta_{\mathcal{C}}}(P)\)) set. Then \(L \cup M = \mathcal{U}\) or \(M \subseteq L\) (resp. \(M = \mathcal{U}\)).

Theorem 3.5. Let \(L\) be a \(\mathfrak{Mm}_i\delta_{\mathcal{C}}(P)\)

(i) If \(x\) be an element of \(\mathcal{U}\), then \(\mathcal{U}/L \subseteq \mathcal{U}\) for \(\mathfrak{Mm}_{\delta_{\mathcal{C}}}(P)\) set containing \(x\).

(ii) either (i) or (ii) holds

(a) \(\forall x \in \mathcal{U}\) and each \(\mathfrak{Mm}_{\delta_{\mathcal{C}}}(P)\) set \(M\) containing \(x\), \(M = \mathcal{U}\).

(b) \(\exists \mathfrak{Mm}_{\delta_{\mathcal{C}}}(P)\) set \(M \ni \mathcal{U} \subseteq L \subseteq \mathcal{U}\) and \(M \subseteq \mathcal{U}\).
Theorem 3.6. Let \( L, M, N \) be \( \mathfrak{NMa}\delta \mathcal{F}o \) \( \ni \) \( L \neq M \). If \( L \cap M \subset N \), then either \( L = N = M \) or \( N \subseteq M \).

Theorem 3.7. Let \( L, M, N \) be mutually disjoint \( \mathfrak{NMa}\delta \mathcal{F}o \) sets. Then \( (L \cap M) \notin (L \cap N) \).

We call a set cofinite if its complement is finite.

Theorem 3.8. Let a cofinite \( \mathfrak{N}\delta \mathcal{F}o \) set \( L \) of \( \mathcal{U} \) such that \( \phi \neq L \neq \mathcal{U} \), then \( \exists \) (cofinite) \( \mathfrak{NMa}\delta \mathcal{F}o \) set \( M \subset L \). 

Proof. If \( L \) is a \( \mathfrak{NMa}\delta \mathcal{F}o \) set, we may set \( L = M \). If \( L \) is not a \( \mathfrak{NMa}\delta \mathcal{F}o \) set, then there exists \( x \in \mathcal{U} \setminus \mathfrak{N}\delta \mathcal{F}cl(L) \). Hence \( \exists a \mathfrak{N}\delta \mathcal{F}o \) set \( M \ni x \in M \) and \( M \cap L = \phi \). Therefore, \( M \subset \mathcal{U} \setminus L \). On the other hand, by Theorem 3.6(c), \( \mathcal{U} \setminus L \subset M \) and \( M = \mathcal{U} \setminus L \). Hence \( L \subset \mathfrak{N}\delta \mathcal{F}cl(L) \). 

Theorem 3.9. Let \( L \) be a \( \mathfrak{NMa}\delta \mathcal{F}o \) set of \( \mathcal{U} \). Then either \( \mathfrak{N}\delta \mathcal{F}cl(L) = \mathcal{U} \) or \( \mathfrak{N}\delta \mathcal{F}cl(L) = L \).

Proof. Since \( L \) is a \( \mathfrak{NMa}\delta \mathcal{F}o \) set, only the two cases are possible by Theorem 3.5(c). Let \( \mathfrak{N}\delta \mathcal{F}cl(L) \neq \mathcal{U} \). Then there exists \( x \in \mathcal{U} \setminus \mathfrak{N}\delta \mathcal{F}cl(L) \). Hence \( \exists a \mathfrak{N}\delta \mathcal{F}o \) set \( M \ni x \in M \) and \( M \cap L = \phi \). Therefore, \( M \subset \mathcal{U} \setminus L \). On the other hand, by Theorem 3.5(a), \( \mathcal{U} \setminus L \subset M \) and \( M = \mathcal{U} \setminus L \). Hence \( L \subset \mathfrak{N}\delta \mathcal{F}cl(L) \). 

Theorem 3.10. Let \( L \) be a \( \mathfrak{NMa}\delta \mathcal{F}o \) set of \( \mathcal{U} \). Then either \( \mathfrak{N}\delta \mathcal{F}int(\mathcal{U} \setminus L) = \mathcal{U} \setminus L \) or \( \mathfrak{N}\delta \mathcal{F}int(\mathcal{U} \setminus L) = \phi \).

Proof. By Theorem 3.9, we have \( \mathfrak{N}\delta \mathcal{F}cl(L) = L \) or \( \mathfrak{N}\delta \mathcal{F}cl(L) = \mathcal{U} \). Hence \( \mathfrak{N}\delta \mathcal{F}int(\mathcal{U} \setminus L) = \mathcal{U} \setminus L \) or \( \mathfrak{N}\delta \mathcal{F}int(\mathcal{U} \setminus L) = \phi \).

Theorem 3.11. For any \( \mathfrak{N} \mathfrak{N} \),

1. Let \( L \) be a \( \mathfrak{NMa}\delta \mathcal{F}o \) set of \( \mathcal{U} \) and \( M \) a nonempty subset of \( \mathcal{U} \). Then \( \mathfrak{N}\delta \mathcal{F}cl(M) = \mathcal{U} \setminus L \).

2. Let \( L \) be a \( \mathfrak{NMa}\delta \mathcal{F}o \) set of \( \mathcal{U} \) and \( G \) a proper subset of \( \mathcal{U} \) with \( L \subset G \). Then \( \mathfrak{N}\delta \mathcal{F}int(G) = L \).

Proof. (1) Since \( \phi \neq M \subset \mathcal{U} \setminus L \), by Theorem 3.4(c) we have that \( \mathfrak{N}\delta \mathcal{F}cl(M) = \mathcal{U} \setminus L \).

(ii) either (i) or (ii) holds
(a) \( \forall x \in \mathcal{U} \setminus L \) and each \( \mathfrak{N}\delta \mathcal{F}o \) set \( M \) containing \( x \) we have \( \mathcal{U} \setminus L \subset M \).
(b) \( \exists a \mathfrak{N}\delta \mathcal{F}o \) set \( M \) \ni \( \mathcal{U} \setminus L = M \neq \mathcal{U} \).

By Theorem 3.9, we have \( \mathfrak{N}\delta \mathcal{F}int(G) = \mathcal{U} \setminus L = \phi \). Therefore, \( \mathfrak{N}\delta \mathcal{F}int(G) = L \).

(2) If \( G = L \), then \( \mathfrak{N}\delta \mathcal{F}int(G) = \mathfrak{N}\delta \mathcal{F}int(L) = L \). If \( G \neq L \), then we have \( L \subset G \). Thus \( L \subset \mathfrak{N}\delta \mathcal{F}int(G) \). Since \( L \)

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