Two level production inventory model with exponential demand and time dependent deterioration rate

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**Abstract**

In this paper, a production inventory model is developed by considering two different rates of production with exponential demand rate. It is assumed that rate of deterioration is linear function of time and shortages are not allowed. Production rates are different in two different phases of production. Likewise demand rate also changed. Optimal production inventory model is developed to determine the optimal production quantity and minimized the total cost, i.e. minimized the sum of production cost, setup cost, holding cost and deterioration cost. A numerical example for the verification of the problem is also given. Sensitivity analysis is also carried out for the different parameters.

**Keywords**

EPQ, time dependent deterioration rate, exponentially increasing demand, and different production.

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**1. Introduction**

Inventory level decrease with the demand rates and also, one of the factor decrease inventory level as well as profit which called deterioration and we cannot be ignored it. In general, deterioration is defined as spoilage, decay, damage, loss of utility, obsolescence etc. like fruits, fertilizer, food items, medicine. Some items have low rate of deterioration such as hardware, toys, glassware etc. Therefore, rate of deterioration is also different for different items. Shah and Jaiswal \(^1\) discussed an inventory model by assuming constant rate of deterioration. Misra \(^2\) investigated a deterministic model in which rate of replenishment is finite, shortage not allowed and rate of deterioration considered both variable and constant. Chaudhari and Chaudhari \(^3\) developed an inventory model. They considered finite rate of replenishment and constant rate of deterioration. Balki and Benkherouf \(^4\) developed a production lot size inventory model in their studied they considered arbitrary production rate and time dependent demand. Bhunia and Maiti \(^5\) investigated deterministic inventory model for variables production. They also assumed production rate depends on either demand or on hand inventory. Chao and Chang \(^6\) developed a production inventory model, they assumed that variables demand rate, production is proportional to demand without shortage and no deteriorating items. Wee and Wang \(^7\) introduced a production inventory model by assuming finite rate of production, time dependent demand, deteriorating items and shortages. Sana et. al. \(^8\) presented a production inventory model for deteriorating items over finite planning horizon with finite rate of production, time varying linear demand and constant deterioration rate. They also considered completely backlogged shortages. Yang and Wee \(^9\) investigated an integrated multi-lot-size production inventory model in which rate of production and demand are constant with deteriorating items. Avikar et al. \(^10\) presented
a production inventory model for deteriorating items with exponentially increasing demand over a fixed time horizon, rate of production and deterioration both constant.

In the present study, we consider two different rates of production. Sivashankari and Panayappan [11] developed a production inventory model for two level production with deteriorative items and shortages. They considered rate of production, deterioration and demand was constant over time horizon. Islam et al. [12] presented a production model with constant production rate and assumed that market demands is increasing exponentially and rate of deterioration is also constant production rate and assumed that market demands are exponentially decreasing. Patra and Maji [13] discussed a production inventory model in which variable production rate, deterioration and two type of demand rates for defective and non-defective items are considered.

In this paper, presented two level production both rate of production is constant, demand is the function of time which is increasing exponentially and rate of deterioration is also linear function of time.

2. Assumptions and Notations

**Assumptions**

(a) The demand rate is \(a \exp(bt)\), where \(a\) and \(b\) are both constant such that \(a > 0\) and \(b > 0\).
(b) Two rates of production are considered.
(c) Production run only single product.
(d) The production rate is always greater than or equal to sum of the demand rate and deterioration rate.
(e) Rate of deterioration is linear function of time.
(f) Shortages are not allowed.

**Notations**

(1) \(P\) is initial production rate in units per unit time.
(2) \(I_1\): On-hand inventory level at time \(T_1\).
(3) \(I_2\): On-hand inventory level at time \(T_2\).
(4) \(I^*\) Optimal inventory.
(5) \(C_p\) production cost per unit time.
(6) \(C_h\) Holding cost per unit per year.
(7) \(C_s\) Setup cost per set up.
(8) \(\alpha + \beta t\) Rate of deterioration where \(0 < \alpha, \beta < 1\).
(9) \(T\) cycle time.
(10) \(T_i\): unit time in period, \(i = 1, 2, \ldots\)
(11) \(T_c\): Total cost.

3. Mathematical Formulation and it’s Solutions

In mathematical formulation of production inventory model for two level production with above described assumptions and notations is depicted in Figure 1. Let the cycle starts at time \(t = 0\), and during the time interval \([0 \ T_1]\), the production rate is \(P\) and the demand rate is \(ae^{bt}\) such that \(P > ae^{bt}\) and the inventory level increases at the rate \(P - ae^{bt}\). At time \(T_1\) suppose that \(I_1\) is maximum inventory level. During the time interval \([T_1 \ T_2]\), the production rate is \(\lambda P\) and demand rate is \(\lambda ae^{bt}\) where ‘\(\lambda\)’ is constant and \(\lambda > 0\) and hence, inventory level increases in this interval at the rate \(\lambda (P - ae^{bt})\). At time \(T_2\) production stopped and suppose that \(I_2\) is maximum inventory at that time. Due to demand and deterioration inventory level starts to decrease and after time \(T\) inventory becomes zero.

Let \(I(t)\) be the inventory level at any time \(t\) where \(0 \leq t \leq T\). Then the model is governing by the following differential equations

\[
\frac{dI}{dt} + (\alpha + \beta t)I = P - ae^{bt} \quad 0 \leq t \leq T_1 \quad (3.1)
\]
\[
\frac{dI}{dt} + (\alpha + \beta t)I = \lambda(P - ae^{bt}) \quad T_1 \leq t \leq T_2 \quad (3.2)
\]
\[
\frac{dI}{dt} + (\alpha + \beta t)I = -ae^{bt} \quad T_2 \leq t \leq T \quad (3.3)
\]

The boundary conditions of above Differential equations are given by

\[
I(0) = 0, \quad I(T_1) = I_1, \quad I(T_2) = I_2, \quad I(T) = 0 \quad (3.4)
\]

![Figure 1](image)

Using the boundary condition (3.4) and neglecting second and higher power of \(\alpha\) and \(\beta\), being \(\alpha\) and \(\beta\) are very small then, the solution of equation (3.1), (3.2) and (3.3) are given by respectively

\[
I = \frac{a}{b} \left(1 - e^{bt}\right) + P \left(\frac{t}{2} - \frac{\beta t^3}{3}\right) - \frac{a a}{b^2} (1 + bt - e^{bt})
\]
\[
+ \frac{a \beta}{b^2} \left(\frac{1}{2} - \frac{t^2}{2} + \frac{t}{b} - \frac{1}{b^2} e^{bt}\right) \quad 0 \leq t \leq T_1 \quad (3.5)
\]
\[
I = \frac{a \lambda}{b} \left(1 - e^{bt}\right) + \lambda P \left(\frac{t}{2} - \frac{\beta t^3}{3}\right)
\]
\[
- \frac{a \alpha \lambda e^{bt}}{b} \left(1 - \frac{\alpha + \beta t^2}{b} + \frac{\beta}{b^2}\right)
\]
\[
- \frac{\alpha}{b} \left[\frac{a}{b} \left(1 - e^{bt}\right) + PT_i (1 - \lambda) + \frac{a \alpha e^{bt_i}}{b}\right]
\]
\[
+ \frac{a (1 - e^{bt_i})}{b} + \frac{a \lambda e^{bt_i}}{b} \left(1 - \frac{\alpha + \beta T_i}{b} + \frac{\beta}{b^2}\right)
\]
\[
+ P(1 - \lambda) \left(T_i - \frac{\alpha T_i^2}{2} - \frac{\beta T_i^3}{3}\right) - \frac{a a (1 + b T_i - e^{bt_i})}{b^2}
\]
The total cost is the sum of the production cost, setup cost, holding cost and deterioration cost.

Let us consider $T_1 = \theta T_2$, where $\theta > 0$.

Total Cost Function

\[
T_c = \frac{aC_p}{bT}{[(1-\lambda)e^{\beta T_2}+(1+\lambda)e^{\beta T_2}-e^{\beta T}]} + C_0\frac{T}{T}
\]

\[
+a(T_1+\beta T_2)\left[\frac{a}{b}\left(1-e^{\beta T_1}\right)+PT_1(1-\lambda)\right]
\]

\[
+a\lambda e^{\beta T_1} T_1 \leq t \leq T_2
\]

\[
I = \frac{a}{b}\left[\beta\left(1-\frac{1}{b^2}\right)+\alpha-1\right]e^{\beta T_1} - \frac{ae^{\beta T_1}}{b}(\alpha + \beta T_2)
\]

\[
+ \frac{ae^{\beta T_1}}{b}(1+\alpha)\left(T - \frac{1}{b}\right) + \beta\left(1 - \frac{T_2}{b^2} + \frac{T_2}{2} - T_1\right)\right]
\]

\[
T_2 \leq t \leq T
\]

**Total cost:** The total cost is the sum of the production cost, setup cost, holding cost and deterioration cost.

On the basis of above cost parameters data, we obtain the optimal inventory $I^*$, production time $(T_1$ and $T_2$), maximum inventory $I_1$ and total cost $T_c$ decreases but maximum inventory $I_2$ increases.

4. Numerical Example

Let us consider an inventory model with the following data: $P = 4000$, $a = 600$, $b = 0.3$, $\alpha = 0.01$, $\beta = 0.1$, $\theta = 0.4$, $\lambda = 2$, $C_0 = 80$, $C_p = 40$, $C_h = 2$.

On the basis of above cost parameters data, we obtain the optimal solution as: $T = 5.831$, $T_1 = 1.461$, $T_2 = 3.6524$, $I_1 = 4218.8$, $I_2 = 1177.8$, $T_c = 31119.1$ Production cost = 22958.4, Setup cost = 13.72, Holding cost = 6706.1, Deterioration cost = 1440.8.

5. Sensitivity analysis

(1) Increasing of deterioration, cycle time $T$, optimal inventory $I^*$, production time $(T_1$ and $T_2)$, maximum inventory $I_1$ and total cost $T_c$ decreases but maximum inventory $I_2$ increases.

(2) Increasing of setup cost, cycle time $T$, optimal inventory $I^*$, production time $(T_1$ and $T_2)$, maximum inventory $I_1$ and $I_2$ and total cost $T_c$ remain unchanged.

(3) Increasing of holding cost, cycle time $T$, optimal inventory $I^*$, production time $(T_1$ and $T_2)$, and maximum inventory $I_1$ decreases but total cost $T_c$ and maximum inventory $I_2$ increases.

(4) Increasing of production cost, cycle time $T$, optimal inventory $I^*$, production time $(T_1$ and $T_2)$, maximum inventory $I_1$ and total cost $T_c$ increases but maximum inventory $I_2$ decreases.
Table 1. Variation in rate of deteriorating items with inventory and total cost

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$T$</th>
<th>$I^*$</th>
<th>Production cost</th>
<th>Setup cost</th>
<th>Holding cost</th>
<th>Deteriorating cost</th>
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Table 2. Effect of demand and cost parameters on optimal values

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<th>$I^*$</th>
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6. Conclusions

In this paper, we proposed two level production inventory models with exponential demand and time dependent deterioration rate. The objective is to determine optimal production policy and minimize the total inventory cost. The variation in production rate provides a way resulting consumer satisfaction and earning potential profit. The present model differs from the existing models, as two-level production and exponentially increasing demand are considered here. A numerical example is provided to demonstrate means practical usage. This model can be extended by taking more realistic assumptions such as shortage, other types of demand patterns etc.

References