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Non-instantaneous deteriorating inventory optimization in green supply chain for environment savvy customer with learning effect

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Abstract

In this paper, we establish a single item inventory model for deteriorating items in a Green Supply Chain under learning effect. We consider a single manufacturer and single retailer model having one manufacturing cycle followed by multiple retailer cycle. Customer is environment savvy and prefers products with low carbon emission. It is considered that product maintains its value for a period of time before there is a loss of value. Thus the deterioration is assumed to be non-instantaneous. This is a more optimal modeling as it allows retailer to utilize full profit on the product selling before it starts offering discounts. It is assumed that the used products as well as unsold products are transferred back to the manufacturer where they are recycled and remanufactured. Learning is taken into account in estimating the total average cost. Learning is a natural phenomenon that occurs everywhere. Naturally, a person doing task repetitively will perform better over period of time. This leads to reduction in various costs. Numerical analysis is carried out at the end to validate model.

Keywords

Green Supply Chain, Non-instantaneous deterioration, Learning Effect, Remanufacturing, Carbon concerned demand.

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Contents

1. Introduction

Over the last two decades, there is a paradigm shift in the approach of companies, government and customers towards environment. People have realized that environment degradation is seriously affecting their health and day to day life and it is their responsibility to work towards reducing environment pollution, reducing waste etc. and thus contribute to the betterment of environment. Green Supply Chain has emerged as the new standard of supply chain. This includes green manufacturing, green operations, remanufacturing, reverse logistics, recycling and waste management. This results in reduced carbon emissions, lesser waste, low pollution and conservation of natural resources. This has led to companies investing on green processes and moving whatever processes can be moved to 'Green'. It has been observed that customers are concerned about the environment and prefer products with low carbon emission. Further, customers are ready to pay premium for products which have been produced in environmental friendly manner.

Kelle and Silver (1989) developed an optimal system to forecast the returns of reusable containers. Pohlen and Farris (1992) developed a reverse logistic model for plastic industry with recycling. Crainic et al. (1993) developed a comprehensive green supply chain model for transporting from land to sea and vice versa. Walton et al. (1998) studied a number of furniture companies and identify environment friendly practices for greening the supply chain. Yeh and Chuang (2006) developed a multi-objective genetic algorithm for partner selection in green supply chain problems. Kannan et al. (2013) developed an inventory model integrating fuzzy multi criteria decision making method and multi-objective programming approach for supplier selection and order allocation in a green supply chain. Hovelaque and Bironneau (2015) developed a carbon-constraint EOQ model taking demand as carbon dependent.

In most of the businesses, deterioration has a substantial impact on the profitability and hence cannot be ignored. Deterioration is the loss of value of the product over a period of time. This can be due to spoilage, expiry, fashion, upgraded launches etc. The classical inventory model didn't take deterioration into account. Deterioration can have a significant impact depending on the nature of the product. Ghare and Schrader (1963) first model a deteriorating inventory considering exponential decay. Shah and Jaiswal (1977) developed an order-level inventory model with constant deterioration. In this model, we assume that there is a time period during which product doesn't loss its value after which product starts deteriorating. This is generally referred as non-instantaneous deterioration. This allows retailer to utilize full benefits on the product selling before accounting for loss of value. Wu et al. (2006) considered non-instantaneous deterioration and developed inventory model considering stock-dependent demand and partial backlogging. Jaggi et al. (2015) developed inventory model for non-instantaneous deteriorating items with price dependent demand under permissible delay in payments. Anchal et al. (2016) developed a partial backlogging inventory model for non-instantaneous deteriorating items with trade credit facility.

Adler and Nanda (1974) studied the impact of learning in optimal lot determination on a single product. Yelle (1979) provided a comprehensive review and survey of learning effect. Lapre et al. (2000) studied the impact of learning effect on waste material reduction. They derived a quality learning curve linking various types of learning to the evolution of factory's waste. Balkhi (2003) studied impact of learning on production lot size for deteriorating items. Sangal et al. (2016) developed a fuzzy inventory model for deteriorating items with learning effect. They assumed inventory is partially backlogged. Jawla and Singh (2016) considered an imperfect production process with preservation technology and developed a reverse logistic inventory model under learning effect.

The objective of this study is to develop an optimal inventory model for non-instantaneous deteriorating environment in Green Supply Chain. This study covers a large range of products and inventory that faces a challenge of how to be eco-friendly as well as make profit in a highly competitive environment when inventory is deteriorating and has shorter life cycle. We assume that customers are environment savvy and are more interested in products with low carbon emission. Further, we consider learning effect in various processes during manufacturing and remanufacturing.

2. Notations

Following parameters are used throughout the model:

η, θ: Deteriorating rate parameters for deterioration $(\eta+\theta t)$ where $\eta > \theta$

$CO_2(Q)$: Carbon emission Units in a cycle

T_{MR}: Total cycle time including single manufacturing and single remanufacturing cycle

δ: Carbon emission demand parameter

Z: Number of shipments

δ: Learning parameter

Manufacturing Parameters

p, q: Production rate parameters for production $(p+qt)$ where $p > q$

D_{SM}: Demand rate

C_{SM}: Set up cost parameter

C_{PDM}: Production cost parameter

C_{PM}: Procurement cost parameter

C_{DM}: Deterioration cost parameter

C_{HM}: Holding cost parameter

W_{MD}, W_{MLD}: Learning effect parameters for deterioration cost

W_{MH}, W_{MLH}: Learning effect parameters for holding cost

Remanufacturing Parameters

X_R: Reproduction rate

D_{SR}: Demand rate

C_{SR}: Set up cost parameter

C_{PDR}: Production cost parameter

C_{PR}: Procurement cost parameter

C_{DR}: Deterioration cost parameter

C_{HR}: Holding cost parameter

W_{RD}, W_{RLD}: Learning effect parameters for deterioration cost

W_{RH}, W_{RLH}: Learning effect parameters for holding cost

Retailer's Parameters

$S_B(t)$: Inventory level at time t in the range $0 \le t \le t_B$

u, v: Demand parameters for demand $(u+vt)$ where $u > v$

Q_B: Initial Quantity level during retailer's cycle

t_B: Time at which inventory reached zero

I_B: Maximum inventory level at time $t = 0$

C_{BO}: Ordering cost parameter

C_{BP}: Purchasing cost parameter

C_{BD}: Deterioration cost parameter

C_{BH}: Holding cost parameter

J_1: Number of retailers cycles in one manufacturing cycle

J_2: Number of retailers cycles in one remanufacturing cycle

W_{BD}, W_{BLD}: Learning effect parameters for deterioration cost

W_{BH}, W_{BLH} Learning effect parameters for holding cost

Collection Parameters

I_{CL}: Max Collection inventory

τ: Returned rate parameter

μ: Production rate parameter for collected inventory

3. Assumptions

The mathematical models in the analysis have the following assumptions:

- Initially stock level is zero for manufacturing and remanufacturing.
- Lead time is negligible.
- Single cycle of manufacturing is followed by single cycle of remanufacturing.
- Each manufacturing/remanufacturing cycle includes multiple retailer cycle.
- Deterioration is assumed to be linear and is given by (η + θt) where $\eta > \theta$.
- Learning effect takes place resulting in reduced holding and deterioration cost.
- Remanufactured products are as good as new products.
- Shortages are not allowed.
- Waste products are collected at rate $\tau(D_{SM}+D_{SR})$ and remanufactured at rate μ.
- Deterioration takes place on inventory after a fixed time interval.

4. Mathematical Modeling

In the development of this model, we consider single manufacturer and single retailer. For each cycle of manufacture, we assume corresponding cycle of remanufacture. Collection of used products happens at the retailer end throughout manufacturing and remanufacturing and it is transferred back to the manufacturer. There, the product is recycled and remanufactured to be as good as new product. The objective is to minimize overall cost.

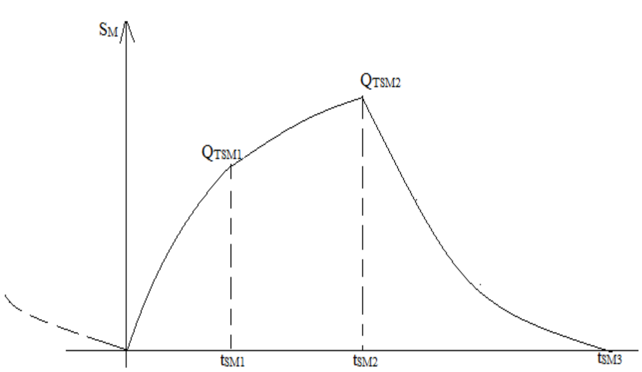

Figure 1

Manufacturing Cycle

We consider production rate is linear and is given by $p+qt$. t_{SM2} is the production time and t_{SM3} is the manufacture cycle time. Deterioration is assumed to be non-instantaneous and it starts at t_{SM1}. Supplier inventory level at time t considering demand as D_{SM} and deterioration rate as $\eta+\theta t$ is given by:

$$\frac{dS_{M1}(t)}{dt} = p+qt-D_{SM} \quad 0 \le t \le t_{SM1} \tag{1}$$

$$\frac{dS_{M2}(t)}{dt} = p+qt-D_{SM}-(\eta+\theta t)S_{M2}(t) \quad t_{SM1} \le t \le t_{SM2} \tag{2}$$

$$\frac{dS_{M3}(t)}{dt} = -D_{SM}-(\eta+\theta t)S_{M3}(t) \quad t_{SM2} \le t \le t_{SM3} \tag{3}$$

$$\text{At } t=0,\ S_{M1}(t)=0 \tag{4}$$

$$S_{M1}(t) = \frac{qt^2}{2}+pt-D_{Sm}.t \tag{5}$$

$$\text{At } t=t_{SM1},\ S_{M1}(t=t_{SM1}) = S_{M2} \quad (t=t_{SM1}) \tag{6}$$

$$\begin{aligned} S_{M2}(t) = e^{-\eta t-\theta\frac{t^2}{2}}\{pt+\frac{qt^2}{2}+\eta(\frac{pt^2}{2}+\frac{qt^3}{3}) \\ +\frac{\theta}{2}(\frac{pt^3}{3}+\frac{qt^4}{4})-D_{SM}(t+\frac{\eta t^2}{2}+\frac{\theta t^3}{6}) \\ +e^{\eta.t_{SM1}+\frac{\theta.t_{SM1}^2}{2}}(\frac{q.t_{SM1}^2}{2}+p.t_{SM1}-D_{SM}.t_{SM1}) \\ -(pt_{SM1}+\frac{qt_{SM1}^2}{2})-\eta(\frac{pt_{SM1}^2}{2}+\frac{qt_{SM1}^3}{3}) \\ -\frac{\theta}{2}(\frac{pt_{SM1}^3}{3}+\frac{qt_{SM1}^4}{4})+D_{SM}(t_{SM1}+\frac{\eta t_{SM1}^2}{2} \\ +\frac{\theta t_{SM1}^3}{6})\} \end{aligned} \tag{7}$$

$$\text{At } t=t_{SM3},\ S_{M3}(t)=0 \tag{8}$$

$$\begin{aligned} S_{M3}(t) = -e^{-\eta t-\theta\frac{t^2}{2}}D_{SM}\{(t+\frac{\eta t^2}{2}+\frac{\theta t^3}{6}) \\ +(t_{SM3}+\frac{\eta t_{SM3}^2}{2}+\frac{\theta t_{SM3}^3}{6})\} \end{aligned} \tag{9}$$

Following are the various costs for supplier:

$$\textbf{Set Up Cost: } (SC)_M = C_{SM} \tag{10}$$

Production Cost

$$\begin{aligned} (PC)_M &= C_{PDM}[\int_0^{t_{SM1}}(p+qt)dt+\int_{t_{SM1}}^{t_{SM2}}(p+qt)dt] \\ &= C_{PDM}(pt_{SM2}+\frac{qt_{SM2}^2}{2}) \end{aligned} \tag{11}$$

Procurement Cost

$$(PC_1)_M = C_{PM}[\int_0^{t_{SM1}}S_{M1}(t)dt+\int_{t_{SM1}}^{t_{SM2}}S_{M2}(t)dt] \tag{12}$$

$$
\begin{aligned}
&= C_{PM}\{(\frac{pt_{SM1}^2}{2}+\frac{qt_{SM1}^3}{6})-D_{SM}\frac{t_{SM1}^2}{2}\\
&\quad +p[(\frac{t_{SM2}^2}{2}-\frac{\eta t_{SM2}^3}{3}-\frac{\theta t_{SM2}^4}{8})\\
&\quad -(\frac{t_{SM1}^2}{2}-\frac{\eta t_{SM1}^3}{3}-\frac{\theta t_{SM1}^4}{8})]\\
&\quad +\frac{q}{2}[(\frac{t_{SM2}^3}{3}-\frac{\eta t_{SM2}^4}{4}-\frac{\theta t_{SM2}^5}{10})\\
&\quad -(\frac{t_{SM1}^3}{3}-\frac{\eta t_{SM1}^4}{4}-\frac{\theta t_{SM1}^5}{10})]\\
&\quad +\eta[(\frac{pt_{SM2}^3}{6}+\frac{qt_{SM2}^4}{12})-(\frac{pt_{SM1}^3}{6}+\frac{qt_{SM1}^4}{12})]\\
&\quad +\frac{\theta}{2}[(\frac{pt_{SM2}^4}{12}+\frac{qt_{SM2}^5}{20})-(\frac{pt_{SM1}^4}{12}+\frac{qt_{SM1}^5}{20})]\\
&\quad -D_{SM}[(\frac{t_{SM2}^2}{2}-\frac{\eta t_{SM2}^3}{6}-\frac{\theta t_{SM2}^4}{12})\\
&\quad -(\frac{t_{SM1}^2}{2}-\frac{\eta t_{SM1}^3}{6}-\frac{\theta t_{SM1}^4}{12})]\\
&\quad +(t_{SM2}-\frac{\eta t_{SM2}^2}{2}-\frac{\theta t_{SM2}^3}{6}\\
&\quad -t_{SM1}+\frac{\eta t_{SM1}^2}{2}+\frac{\theta t_{SM1}^3}{6})[e^{\eta.t_{SM1}+\frac{\theta.t_{SM1}^2}{2}}\\
&\quad (\frac{q.t_{SM1}^2}{2}+p.t_{SM1}-D_{SM}.t_{SM1})\\
&\quad -(pt_{SM1}+\frac{qt_{SM1}^2}{2})-\eta(\frac{pt_{SM1}^2}{2}+\frac{qt_{SM1}^3}{3})\\
&\quad -\frac{\theta}{2}(\frac{pt_{SM1}^3}{3}+\frac{qt_{SM1}^4}{4})+D_{SM}(t_{SM1}+\frac{\eta t_{SM1}^2}{2}\\
&\quad +\frac{\theta t_{SM1}^3}{6})]\} \qquad (13)
\end{aligned}
$$

Deterioration Cost

$$
\begin{aligned}
(DC)_M &= C_{DM}[\int_{t_{SM1}}^{t_{SM2}}(\eta+\theta t).S_{M2}(t)dt\\
&\quad +\int_{t_{SM2}}^{t_{SM3}}(\eta+\theta t).S_{M3}(t)dt] \qquad (14)
\end{aligned}
$$

$$
\begin{aligned}
&= C_{DM}\{\frac{\eta p}{2}(t_{SM2}^2-t_{SM1}^2)+(\frac{\eta q}{2}+\theta p)(\frac{t_{SM2}^3}{3}-\frac{t_{SM1}^3}{3})\\
&\quad +\frac{\theta q}{8}(t_{SM2}^4-t_{SM1}^4)-D_{SM}[\frac{\eta}{2}(t_{SM2}^2-t_{SM1}^2)\\
&\quad +\frac{\theta}{3}(t_{SM2}^3-t_{SM1}^3)-(\eta(t_{SM3}-t_{SM2})\\
&\quad +\frac{\theta}{2}(t_{SM3}^2-t_{SM2}^2))(t_{SM3}+\frac{\eta}{2}t_{SM3}^2\\
&\quad +\frac{\theta}{6}t_{SM3}^3)-(\frac{\eta}{2}(t_{SM3}^2-t_{SM2}^2)+\frac{\theta}{3}(t_{SM3}^3-t_{SM2}^3))]\\
&\quad +(\eta t_{SM2}+\frac{\theta t_{SM2}^2}{2}-\eta t_{SM1}-\frac{\theta t_{SM1}^2}{2})\\
&\quad [e^{\eta.t_{SM1}+\frac{\theta.t_{SM1}^2}{2}}(\frac{q.t_{SM1}^2}{2}+p.t_{SM1}-D_{SM}.t_{SM1})\\
&\quad -(pt_{SM1}+\frac{qt_{SM1}^2}{2})-\eta(\frac{pt_{SM1}^2}{2}+\frac{qt_{SM1}^3}{3})\\
&\quad -\frac{\theta}{2}(\frac{pt_{SM1}^3}{3}+\frac{qt_{SM1}^4}{4})+D_{SM}(t_{SM1}\\
&\quad +\frac{\eta t_{SM1}^2}{2}+\frac{\theta t_{SM1}^3}{6})]\} \qquad (15)
\end{aligned}
$$

Holding Cost

$$
\begin{aligned}
(HC)_M &= C_{HM}[\int_0^{t_{SM1}}S_{M1}(t)dt+\int_{t_{SM1}}^{t_{SM2}}S_{M2}(t)dt\\
&\quad +\int_{t_{SM1}}^{t_{SM2}}S_{M3}(t)dt] \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
&= C_{HM}\{(\frac{pt_{SM1}^2}{2}+\frac{qt_{SM1}^3}{6})-D_{SM}\frac{t_{SM1}^2}{2}\\
&\quad +p[(\frac{t_{SM2}^2}{2}-\frac{\eta t_{SM2}^3}{3}-\frac{\theta t_{SM2}^4}{8})\\
&\quad -(\frac{t_{SM1}^2}{2}-\frac{\eta t_{SM1}^3}{3}-\frac{\theta t_{SM1}^4}{8})]\\
&\quad +\frac{q}{2}[(\frac{t_{SM2}^3}{3}-\frac{\eta t_{SM2}^4}{4}-\frac{\theta t_{SM2}^5}{10})\\
&\quad -(\frac{t_{SM1}^3}{3}-\frac{\eta t_{SM1}^4}{4}-\frac{\theta t_{SM1}^5}{10})]\\
&\quad +\eta[(\frac{pt_{SM2}^3}{6}+\frac{qt_{SM2}^4}{12})-(\frac{pt_{SM1}^3}{6}+\frac{qt_{SM1}^4}{12})]\\
&\quad +\frac{\theta}{2}[(\frac{pt_{SM2}^4}{12}+\frac{qt_{SM2}^5}{20})-(\frac{pt_{SM1}^4}{12}+\frac{qt_{SM1}^5}{20})]\\
&\quad -D_{SM}[(\frac{t_{SM2}^2}{2}-\frac{\eta t_{SM2}^3}{6}-\frac{\theta t_{SM2}^4}{12})\\
&\quad -(\frac{t_{SM1}^2}{2}-\frac{\eta t_{SM1}^3}{6}-\frac{\theta t_{SM1}^4}{12})]\\
&\quad +(t_{SM2}-\frac{\eta t_{SM2}^2}{2}-\frac{\theta t_{SM2}^3}{6}-t_{SM1}+\frac{\eta t_{SM1}^2}{2}+\frac{\theta t_{SM1}^3}{6})\\
&\quad [e^{\eta.t_{SM1}+\frac{\theta.t_{SM1}^2}{2}}(\frac{q.t_{SM1}^2}{2}+p.t_{SM1}-D_{SM}.t_{SM1})\\
&\quad -(pt_{SM1}+\frac{qt_{SM1}^2}{2})-\eta(\frac{pt_{SM1}^2}{2}+\frac{qt_{SM1}^3}{3})\\
&\quad -\frac{\theta}{2}(\frac{pt_{SM1}^3}{3}+\frac{qt_{SM1}^4}{4})\\
&\quad +D_{SM}(t_{SM1}+\frac{\eta t_{SM1}^2}{2}+\frac{\theta t_{SM1}^3}{6})]\\
&\quad +D_{SM}[((t_{SM3}-t_{SM2})-\frac{\eta}{2}(t_{SM3}^2-t_{SM2}^2)\\
&\quad -\frac{\theta}{6}(t_{SM3}^3-t_{SM2}^3))(t_{SM3}+\frac{\eta t_{SM3}^2}{2}+\frac{\theta t_{SM3}^3}{6})\\
&\quad -(\frac{(t_{SM3}^2-t_{SM2}^2)}{2}-\frac{\eta(t_{SM3}^3-t_{SM2}^3)}{6}\\
&\quad -\frac{\theta(t_{SM3}^4-t_{SM2}^4)}{12})]\} \qquad (17)
\end{aligned}
$$

Total average cost for manufacturer is given by:

$$
(TAC)_{SM}=\frac{1}{t_{SM3}}(SC+PC+PC_1+DC+HC)_M \qquad (18)
$$

Remanufacturing cycle

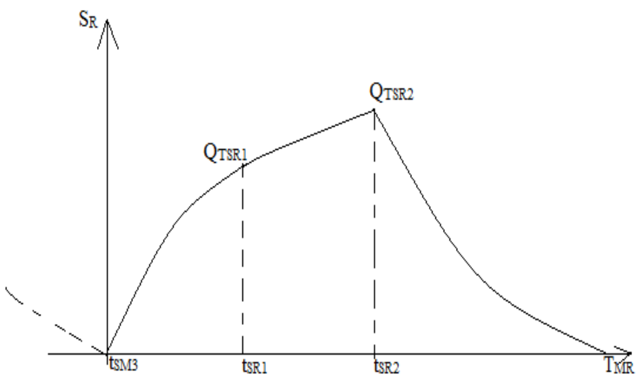

Figure 2

Assuming production rate as X_R and demand is D_{SR}during remanufacturing, inventory level at time t considering linear deterioration rate $\eta+\theta t$ and total cycle time T_{MR} is given by:

$$\frac{dS_{R1}(t)}{dt}=\chi_R-D_{SR} \quad t_{SM3}\le t\le t_{SR1} \tag{19}$$

$$\frac{dS_{R2}(t)}{dt}=\chi_R-D_{SR}-(\eta+\theta t).S_{R2}(t) \quad t_{SR1}\le t\le t_{SR2} \tag{20}$$

$$\frac{dS_{R3}(t)}{dt}=-D_{SR}-(\eta+\theta t).S_{R3}(t) \quad t_{SR2}\le t\le T_{MR} \tag{21}$$

$$att=t_{SM3};S_{R1}(t)=0 \tag{22}$$

$$S_{R1}(t)=(\chi_R-D_{SR})(t-t_{SM3}) \tag{23}$$

$$att=t_{SR1};S_{R1}(t)=S_{R2}(t) \tag{24}$$

$$S_{R1}(t)=(\chi_R-D_{SR})(t-t_{SM3}) \tag{25}$$

$$att=t_{SR1};S_{R1}(t)=S_{R2}(t) \tag{26}$$

$$S_{R2}(t)=e^{-\eta t-\frac{\theta t^2}{2}}(\chi_R-D_{SR})\{(t+\frac{\eta t^2}{2}+\frac{\theta t^3}{6}) + [e^{\eta t_{SR1}+\frac{\theta t_{SR1}^2}{2}}(t_{SR1}-t_{SM3}) - (t_{SR1}+\frac{\eta_{SR1}^2}{2}+\frac{\theta t_{SR1}^3}{6})]\} \tag{27}$$

$$att=T_{MR};S_{R3}(t)=0 \tag{28}$$

$$S_{R3}(t)=-D_{SR}.e^{-\eta t-\frac{\theta t^2}{2}}\{(T_{MR}+\frac{\eta T_{MR}^2}{2}+\frac{\theta T_{MR}^3}{6}) - (t+\frac{\eta t^2}{2}+\frac{\theta t^3}{6})\} \tag{29}$$

Following are the various costs during remanufacturing:

Set up Cost: $(SC)_R=C_{SR}$ (30)

Production Cost

$$(PC)_R=C_{PDR}[\int_{t_{SM3}}^{t_{SR1}}\chi_R dt+\int_{t_{SR1}}^{t_{SR2}}\chi_R dt] = C_{PDR}.\chi_R.(t_{SR2}-t_{SM3}) \tag{31}$$

Procurement Cost:

$$(PC_1)_R=C_{PR}[\int_{t_{SM3}}^{t_{SR1}}S_{R1}(t)dt+\int_{t_{SR1}}^{t_{SR2}}S_{R2}(t)dt] \tag{32}$$

$$= C_{PR}(\chi_R-D_{SR})\{(\frac{t_{SR1}^2}{2}-t_{SR1}t_{SM3}-\frac{t_{SM3}^2}{2}) + ((\frac{t_{SR2}^2}{2}-\frac{\eta t_{SR2}^3}{6}-\frac{\theta t_{SR2}^4}{12}) - (\frac{t_{SR1}^2}{2}-\frac{\eta t_{SR1}^3}{6}-\frac{\theta t_{SR1}^4}{12})) + ((1+\eta t_{SR1}+\frac{\theta t_{SR1}^2}{2})(t_{SR1}-t_{SM3}) - (t_{SR1}\frac{\eta t_{SR1}^2}{2}+\frac{\theta t_{SR1}^3}{6}))((t_{SR2}-\frac{\eta t_{SR2}^2}{2}-\frac{\theta t_{SR2}^3}{6}) - (t_{SR1}-\frac{\eta t_{SR1}^2}{2}-\frac{\theta t_{SR1}^3}{6}))\} \tag{33}$$

Deterioration Cost

$$(DC)_R=C_{DR}[\int_{t_{SR1}}^{t_{SR2}}(\eta+\theta t).S_{R2}(t)dt + \int_{t_{SR2}}^{T_{MR}}(\eta+\theta t).S_{R3}(t)dt] \tag{34}$$

$$= C_{DR}\{(\chi_R-D_{SR})((\frac{\eta(t_{SR2}^2-t_{SR1}^2)}{2}+\frac{\theta(t_{SR2}^3-t_{SR1}^3)}{3}) + (\eta(t_{SR2}-t_{SR1})+\frac{\theta(t_{SR2}^2-t_{SR1}^2)}{2}) ((1+\eta t_{SR1}+\frac{\theta t_{SR1}^2}{2})(t_{SR1}-t_{SM3}) - (t_{SR1}+\frac{\eta t_{SR1}^2}{2}+\frac{\theta t_{SR1}^3}{6})))+D_{SR}[(\eta(T_{MR}-t_{SR2}) + \frac{\theta(T_{MR}^2-t_{SR2}^2)}{2})(T_{MR}+\frac{\eta T_{MR}^2}{2}+\frac{\theta T_{MR}^3}{6}) (\frac{\eta(T_{MR}^2-t_{SR2}^2)}{2}+\frac{\theta(T_{MR}^3-t_{SR2}^3)}{3})]\} \tag{35}$$

Holding Cost

$$(HC)_R=C_{HR}[\int_{t_{SM3}}^{t_{SR1}}S_{R1}(t)dt+\int_{t_{SR1}}^{t_{SR2}}S_{R2}(t)dt + \int_{t_{SR2}}^{T_{MR}}S_{R3}(t)dt] \tag{36}$$

$$= C_{HR}\{(\chi_R-D_{SR})[(\frac{t_{SR1}^2}{2}-t_{SR1}t_{SM3}-\frac{t_{SM3}^2}{2}) + ((\frac{t_{SR2}^2}{2}-\frac{\eta t_{SR2}^3}{6}-\frac{\theta t_{SR2}^4}{12}) - (\frac{t_{SR1}^2}{2}-\frac{\eta t_{SR1}^3}{6}-\frac{\theta t_{SR1}^4}{12})) + ((1+\eta t_{SR1}+\frac{\theta t_{SR1}^2}{2})(t_{SR1}-t_{SM3}) - (t_{SR1}+\frac{\eta t_{SR1}^2}{2}+\frac{\theta t_{SR1}^3}{6}))((t_{SR2}-\frac{\eta t_{SR2}^2}{2}$$

$$-\frac{\theta t_{SR2}^3}{6}) - (t_{SR1} - \frac{\eta t_{SR1}^2}{2} - \frac{\theta t_{SR1}^3}{6}))]$$
$$+ D_{SR}[\frac{(T_{MR}^2 - t_{SR2}^2)}{2} - \frac{\eta(T_{MR}^3 - t_{SR2}^3)}{6}$$
$$-\frac{\theta(T_{MR}^4 - t_{SR2}^4)}{12}] + D_{SR}[(T_{MR} - t_{SR2})$$
$$-\frac{\eta(T_{MR}^2 - t_{SR2}^2)}{2} - \frac{\theta(T_{MR}^3 - t_{SR2}^3)}{6})$$
$$(T_{MR} + \frac{\eta T_{MR}^2}{2} + \frac{\theta T_{MR}^3}{6})]\} \quad (37)$$

Total average cost for remanufacturing is given by:

$$(TAC)_{SR} = \frac{1}{(T_{MR} - t_{SM3})}[SC + PC + PC_1 + DC + HC]_R \quad (38)$$

Collection Cycle

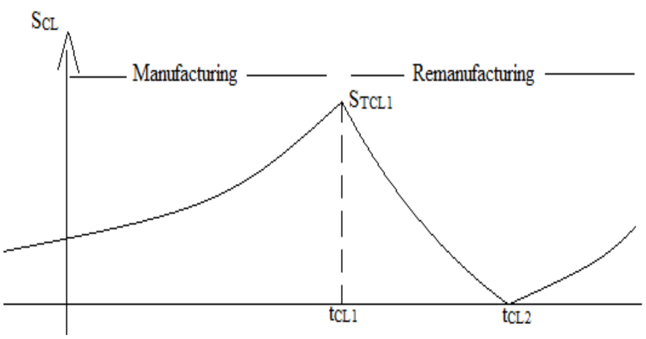

Figure 3

Ability to recycle product is a win-win situation for all. It allows customer to return the product after using it while retailer has an opportunity to retain customers. Also, it is beneficial for the environment. We assume that retailer collects the used product and transport it back to the manufacturer who then remanufactures it. Assuming τ is the returned rate during manufacturing and remanufacturing and μ is the production rate for collected inventory, collection inventory level at time t is given by:

$$\frac{dS_{CL1}(t)}{dt} = \tau\mu(D_{SR} + D_{SM}) - \chi_R \quad t_{CL1} \le t \le t_{CL2} \quad (39)$$

$$att = t_{CL1}; \; S_{CL1} = I_{CL} \quad (40)$$

$$S_{CL1} = [\mu(D_{SR} + D_{SM}) - \chi_R](t - t_{CL1}) + I_{CL} \quad (41)$$

$$\frac{dS_{CL2}(t)}{dt} = \tau\mu(D_{SR} + D_{SM}) \quad t_{CL2} \le t \le (T_{MR} + t_{CL1}) \quad (42)$$

$$att = t_{CL2}; \quad S_{CL2}(t) = 0 \quad (43)$$

$$S_{CL2}(t) = [\mu(D_{SR} + D_{SM})](t - t_{CL2}) \quad (44)$$

$$att = t_{CL2}; S_{CL1} = 0 \text{ and } att = T_{MR} + t_{CL1}; \; S_{CL2}(t) = I_{CL} \quad (45)$$

$$\chi_R = \tau\mu(D_{SR} + D_{SM})(\frac{I_{CL}}{\tau\mu(D_{SR} + D_{SM})T_{MR} - I_{CL}} + 1) \quad (46)$$

Retailer's Inventory

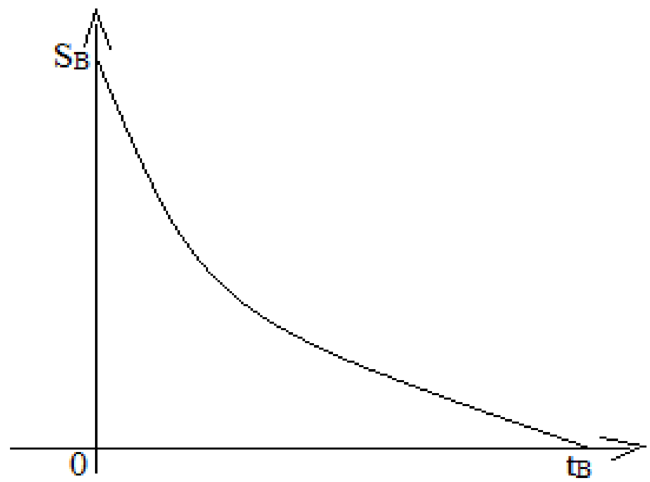

Figure 4

We consider customer is environment savvy and is interested in buying products with lower carbon emission. Considering demand to be linear at retailer's end and taking carbon emission effect into account, demand of the product is given by $u + vt - \delta CO_2(Q)$. Thus the inventory level considering deterioration rate as $\eta + \theta t$ and cycle time as t_B is given by:

$$\frac{dS_B(t)}{dt} = u + vt - \delta.CO_2(Q) - (\eta + \theta t).S_B(t)$$
$$0 \le t \le t_B \quad (47)$$

$$att = t_B; \; S_B(t) = 0 \quad (48)$$

$$S_B(t) = e^{-\eta t - \frac{\theta t^2}{2}}\{[(ut_B + \frac{v + \eta(u - \delta.CO_2(Q))}{2}t_B^2$$
$$+ [v\eta + \frac{\theta(u - \delta.CO_2(Q))}{2}]\frac{t_B^3}{3} + \theta v\frac{t_B^4}{8})$$
$$-(u - \delta.CO_2(Q))t + \frac{v + \eta(u - \delta.CO_2(Q))}{2}t^2$$
$$+ [v\eta + \frac{\theta(u - \delta.CO_2(Q))}{2}]\frac{t^3}{3} + \theta v\frac{t^4}{8})]\} \quad (49)$$

Following are the various costs for retailer:

Ordering Cost: $(OC)_B = C_{BO}$ (50)

Purchasing Cost

$$(PC)_B = C_{BP}\{(u - \delta.CO_2(Q))t_B$$
$$+ \frac{v + \eta(u - \delta.CO_2(Q))}{2}t_B^2$$
$$+ (\frac{\theta(v + \eta(u - \delta.CO_2(Q)))}{2}$$

$$+v\eta)\frac{t_B^3}{3}+\frac{\theta v}{8}t_B^4\} \quad (51)$$

Deterioration Cost

$$(DC)_B = C_{BD}\int_0^{t_B}(\eta+\theta t).S_B(t)dt \quad (52)$$

$$= C_{BD}\{(\eta t_B+\frac{\theta t_B^2}{2})((u-\delta.CO_2(Q))t_B + \frac{v+\eta(u-\delta.CO_2(Q))}{2}t_B^2 + (\frac{\theta(u-\delta.CO_2(Q))}{2}+v\eta)\frac{t_B^3}{3}+\frac{\theta v}{8}t_B^4)) - \eta[\frac{(u-\delta.CO_2(Q))t_B^2}{2}+\frac{v}{6}t_B^3] - \theta(\frac{(u-\delta.CO_2(Q))t_B^3}{3}+\frac{v}{8}t_B^4) + (\eta t_B+\frac{\theta t_B^2}{2})((u-\delta.CO_2(Q))t_B + \frac{v+\eta(u-\delta.CO_2(Q))}{2}t_B^2 + (v\eta+\frac{\theta(u-\delta.CO_2(Q))}{2})\frac{t_B^3}{3} + \frac{\theta v}{8}t_B^4)\} \quad (53)$$

Holding Cost

$$(HC)_B = C_{BH}\int_0^{t_B}S_B(t)dt \quad (54)$$

$$= C_{BH}\{(\frac{(u-\delta.CO_2(Q))t_B^2}{2} + \frac{v+\eta(u-\delta.CO_2(Q))}{6}t_B^3 + (v\eta+\frac{\theta(u-\delta.CO_2(Q))}{2})\frac{t_B^4}{12} + \frac{\theta v}{40}t_B^5) - \eta(\frac{(u-\delta.CO_2(Q))t_B^3}{3}+\frac{vt_B^4}{8}) - \theta(\frac{(u-\delta.CO_2(Q))t_B^4}{8}+\frac{vt_B^5}{20}) + (t_B-\frac{\eta t_B^2}{2}-\frac{\theta t_B^3}{6})(u-\delta.CO_2(Q))t_B + \frac{v+\eta(u-\delta.CO_2(Q))}{2}t_B^2 + (v\eta+\frac{\theta(u-\delta.CO_2(Q))}{2})\frac{t_B^3}{3} + \frac{\theta v}{8}t_B^4)\} \quad (55)$$

Total average cost for buyer during one cycle is given by:

$$(TAC)_B = \frac{1}{t_B}[OC+PC+DC+HC]B \quad (56)$$

Assuming J_1 buyer cycle during manufacturing, total average cost during manufacturing is given by:

$$TAC_{SBM} = \frac{1}{t_{M3}}(SC+PC+PC_1+DC+HC)_{SM} + \frac{J1}{t_{M3}}[OC+PC+DC+HC]B \quad (57)$$

And partial derivatives equations are given by:

$$\frac{\partial(TAC)_{SBM}(t_{SM2},t_{SM3})}{\partial t_{SM2}} = 0 \text{ and } \frac{\partial(TAC)_{SBM}(t_{SM2},t_{SM3})}{\partial t_{SM2}} = 0 \quad (58)$$

Provided

$$\frac{\partial^2(TAC)_{SBM}(t_{SM2},t_{SM3})}{\partial t_{SM2}^2}\frac{\partial^2(TAC)_{SBM}(t_{SM2},t_{SM3})}{\partial t_{SM3}^2} - \frac{\partial^2(TAC)_{SBM}(t_{SM2},t_{SM3})}{\partial t_{SM2}\partial t_{SM3}} > 0 \quad (59)$$

where

$$\frac{\partial^2(TAC)_{SBM}(t_{SM2},t_{SM3})}{\partial t_{SM2}^2} > 0 \quad (60)$$

During remanufacturing, assuming J2 cycle for buyer, total average cost is given by:

$$TAC_{SBR} = \frac{1}{T_{MR}-t_{SM3}}(SC+PC+PC_1+DC+HC)_{SR} + \frac{J2}{T_{MR}-t_{SM3}}[OC+PC+DC+HC]B \quad (61)$$

And partial derivatives equations are given by:

$$\frac{\partial(TAC)_{SBR}(t_{SR2},T_{MR})}{\partial t_{SR2}} = 0 \text{ and } \frac{\partial(TAC)_{SBR}(t_{SR2},T_{MR})}{\partial T_{MR}} = 0 \quad (62)$$

Provided

$$\frac{\partial^2(TAC)_{SBR}(t_{SR2},T_{MR})}{\partial t_{SR2}^2}\frac{\partial^2(TAC)_{SBR}(t_{SR2},T_{MR})}{\partial T_{MR}^2} - \frac{\partial^2(TAC)_{SBR}(t_{SR2},T_{MR})}{\partial t_{SR2}\partial T_{MR}} > 0 \quad (63)$$

where

$$\frac{\partial^2(TAC)_{SBR}(t_{SR2},T_{MR})}{\partial t_{SR2}^2} > 0 \quad (64)$$

5. Numerical Observations

Taking appropriate values for various input parameters and using Mathematica, we determine optimal cost: $t_{SM1} = 1$,

$J1 = 20$, $\delta = 0.2$, $CO_2(Q) = 0.3$, $p = 5$, $q = 0.1$, $D_{SM} = 5$, $C_{SM} = 100$, $C_{PDM} = 100$, $C_{PM} = 100$, $C_{DM} = 10$, $C_{HM} = 100$, $u = 2$, $v = 0.2$, $\eta = 0.05$, $\theta = 0.04$, $C_{BO} = 5$, $C_{BP} = 5$, $C_{BD} = 5$, $C_{BH} = 5$, $t_{SR1} = 5$, $J2 = 20$, $D_{SR} = 4$, $C_{SR} = 100$, $C_{PDR} = 90$, $C_{PR} = 100$, $C_{DR} = 20$, $I_{CL} = 3$, $\tau = 0.5$, $\mu = 0.8$.

Total average cost during manufacturing: $(TAC)_{SBM} = 499.497$ units where $t_{M2} = 1.95917$, $t_{M3} = 3.04094$

Total average cost during remanufacturing: $(TAC)_{SBR} = 385.608$, $T_{SR2} = 6.67564$, $T_{MR} = 7.76931$

Total cost across manufacturing and remanufacturing is $(TAC)_{MR}$ = 885.105

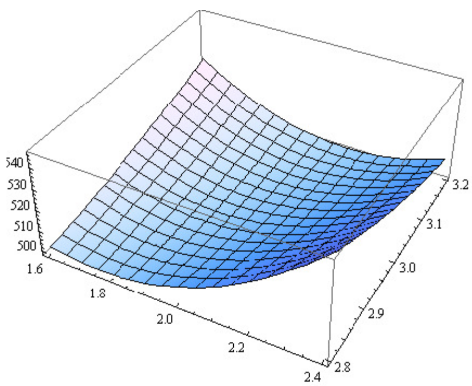

Figure 5

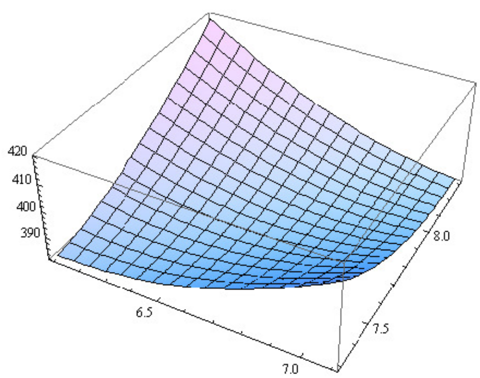

Figure 6

6. Learning Effect

We assume that holding cost and deterioration cost undergoes learning effect during manufacturing and remanufacturing. There are some parts of cost that doesn't have any learning effect, so it is assumed that every cost has one part which remains constant while another part that has learning effect.

Following are various costs during learning:

Learning Deterioration Cost and Holding Cost during manufacturing:

$$(LDC)_M = (W_{MD} + \frac{W_{MLD}}{Z^\gamma})[\int_{t_{SM1}}^{t_{SM2}} (\eta + \theta t).S_{M2}(t)dt + \int_{t_{SM2}}^{t_{SM3}} (\eta + \theta t).S_{M3}(t)dt] \quad (65)$$

$$(LHC)_M = (W_{MH} + \frac{W_{MLH}}{Z^\gamma})[\int_0^{t_{SM1}} S_{M1}(t)dt + \int_{t_{SM1}}^{t_{SM2}} S_{M2}(t)dt + \int_{t_{SM1}}^{t_{SM2}} S_{M3}(t)dt] \quad (66)$$

Learning Deterioration Cost and Holding Cost during remanufacturing:

$$(DC)_R = (W_{RD} + \frac{W_{RLD}}{Z^\gamma})[\int_{t_{SR1}}^{t_{SR2}} (\eta + \theta t).S_{R2}(t)dt + \int_{t_{SR2}}^{T_{MR}} (\eta + \theta t).S_{R3}(t)dt] \quad (67)$$

$$(LHC)_R = (W_{RH} + \frac{W_{RLH}}{Z^\gamma})[\int_{t_{SM3}}^{t_{SR1}} S_{R1}(t)dt + \int_{t_{SR1}}^{t_{SR2}} S_{R2}(t)dt + \int_{t_{SR2}}^{T_{MR}} S_{R3}(t)dt] \quad (68)$$

Learning Deterioration and Holding Cost for retailer:

$$(LDC)_B = (W_{BD} + \frac{W_{BLD}}{Z^\gamma}) \int_0^{t_B} (\eta + \theta t).S_B(t)dt \quad (69)$$

$$(LHC)_B = (W_{BH} + \frac{W_{BLH}}{Z^\gamma}) \int_0^{t_B} S_B(t)dt \quad (70)$$

Taking appropriate values for various parameters and finding the cost for different values of Z: $W_{MH} = 60$, $W_{MLH} = 40$, $W_{MD} = 6$, $W_{MLD} = 4$, $W_{RH} = 60$, $W_{RLH} = 40$, $W_{RD} = 12$, $W_{RLD} = 8$, $W_{BD} = 3$, $W_{BLD} = 2$, $W_{BH} = 3$, $W_{BLH} = 2$, $\mu = 0.2$.

Z	t_{SM2}	t_{SM3}	t_{SR2}	T_{MR}	$(TAC)_{MR}$
1	1.95917	3.04094	6.67564	7.76931	885.105
2	1.97797	3.11485	6.67151	7.8302	877.143
3	1.98648	3.15427	6.66918	7.86448	872.74
4	1.99163	3.18073	6.66755	7.88816	869.728
5	1.99518	3.20044	6.66631	7.90613	867.458

As expected, moving to next cycles result in decrease of total cost. Also, the cost change reduces as we move further.

7. Conclusion

In this study, we develop an optimal inventory model for non-instantaneous deteriorating product in a green supply chain. Used products are collected and transferred back to manufacture where they are remanufactured. We consider single cycle of manufacturing followed by single cycle of remanufacturing. We consider that customer is environment conscious and prefer products with lower carbon emission. Hence, demand is inversely dependent on carbon emission. Optimal cost is determined taking various parameters and constraints into account. Further, the effect of learning is studied on the overall cost.

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