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Replacement timing for a one-shot system with minimal repair

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Abstract
A system that is available only once is called a one-shot system. To ensure its high availability, an inspection is performed regularly because its failure is detected only through it. In this paper, we discuss management policies in replacement timings of a single-unit one-shot system whose failures are removed by minimal repairs. We compare the two replacement policies that manages the number of failures and the number of periodic inspections. Our objective is to compare the two management policies of replacement timings by formulating the incurred cost rate, which is cost per unit time, and the availability.

Keywords
One-shot system, Storage system, Periodic replacement, IFR, Cost rate.

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Contents

1. Introduction

A system that can be used only once such as a missile or an air-bag is called one-shot system [1] or dormant system as it spends almost all its life in repository. Since one-shot system is mainly used for some critical missions including combat and life-saving, it is imperatively important to maintain its high availability.

Systems deteriorate with time even they are in storage, and it is difficult to judge whether a one-shot system has fault or not by a simple visual check. Thus, detailed inspections are performed to detect failures. We want to inspect a system as frequently as possible to ensure a high reliability, but the frequent inspection may incur an expensive cost which may be unacceptable for users. Thus, we should optimize inspection schedules.

Many researchers have studied the optimal inspection schedules [2]-[8]. Although they considered that a system is replaced upon a failure detection, many real systems such as military missiles are minimally repaired and resume their operation after the repair. Minimal repair recovers the failed system without disturbing the distribution or hazard rate of the repaired system [9]. To take this situation into consideration, we proposed maintenance models of one-shot system with minimal repair [10] [11]. When an IFR system is minimally repaired upon failures, it should be replaced at certain time to renew the system. We assumed that a system is replaced at the detection of nth failure.

However, not all real one-shot systems are replaced upon a failure detection. Periodic replacement has rather advantage in the preparation for replacement work due to the fixed and predetermined time. Unfortunately, the comparison between two management polices of replacement tinging, a policy based on the number of failures and a periodic policy had not been conducted for a one-shot system. For a system whose failure is detected immediately without inspection and is repaired minimally, Park [12] showed that number-of-failure-based policy has advantage in cost when failures occur following

Weibull distributions. In this paper, we compare the two policies for a one-shot system. We call the two management policies the number-of-failure-based (NOFB) policy and the periodic policy. We assume periodic inspection. The periodic policy replaces the system at mth inspection, whereas the NOFB policy does it at nth detection of failure. We define the optimal solution as the inspection interval and the number of failures until replacement or the number of inspections until replacement that minimizes the expected cost rate, which is incurred cost per unit time, ensuring a certain mean availability. Optimal cost rates in two policies given by the optimal solutions are compared.

Formulating the cost rate of the periodic policy is not easy due to the vast number of cases that will happen. To deal with the policy easily, we propose an approximation method and estimate its accuracy. This approximation is to be shown useful to know the rough area of optimal solution.

Notations used in the paper and model assumption are described in section 2 and 3, respectively. In section 4, we introduce the cost rate and the availability of two policies. Approximation method for periodic policy is proposed there. In section 5, we show the comparison results of two policies and the error of the approximation. Monte Carlo method is used to compare the two policies. Lastly, we conclude this paper.

2. Notations

We use the following notations in this paper.

n: number of failures until a replacement in the NOFB policy (the system is replaced at nth failure detection)

m: number of inspections until a replacement in the periodic policy (the system is replaced at mth inspection)

T: inspection interval

C_I: inspection cost per one time

C_R: minimal repair cost

C_P: replacement cost, $C_P > C_R$

$F^{(l)}(x)$: a failure distribution function of the system after the $(l-1)$th minimal repair, where $l = 1$ means the distribution function of new system and we write simply $F(x)$

$\bar{F}^{(l)}(x)$: a reliability function of the system after the $(l-1)$th minimal repair, $1 - F^{(l)}(x)$, and $\bar{F}(x)$ when $l = 1$

$\mu^{(l)}$: mean time between the $(l-1)$th minimal repair and the next failure

$H(t)$: cumulative hazard rate function of the system

$h(t)$: hazard rate function of the system

η: scale parameter of a Weibull distribution

β: shape parameter of a Weibull distribution

$C_f(n,T)$: expected cost rate for $(0,\infty)$ in the NOFB policy

C_f^*: optimal cost rate in the NOFB policy

$C_p(m,T)$: expected cost rate for $(0,\infty)$ in the periodic policy

C_p^*: optimal cost rate in the periodic policy

$A_f(n,T)$: mean availability for $(0,\infty)$ in the NOFB policy

$A_p(m,T)$: mean availability for $(0,\infty)$ in the periodic policy

α: required mean availability for the system

3. Model Description

Our model is described as follows:

(1) A one-shot system consists of one unit and its hazard rate strictly increases with time.

(2) The system is inspected at every T.

(3) A failure is detected perfectly at the next inspection of the occurrence and is repaired minimally. Hazard rate of the system does not change by the repair.

(4) In the NOFB policy, a system is replaced and becomes "as good as new" when nth failure is detected. In periodic policy, the system is replaced and renewed at every mT. The system is replaced without inspection at mT in the periodic policy.

(5) All times needed for inspection, minimal repair, and replacement are negligible.

Figs 1 and 2 show the processes of the system under the NOFB policy and the periodic one, respectively.

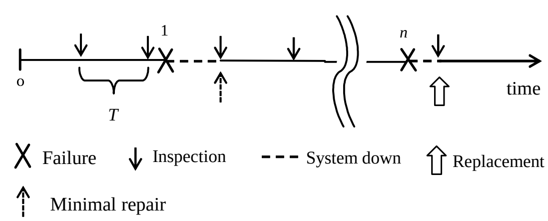

Figure 1. Process of the system under the NOFB policy.

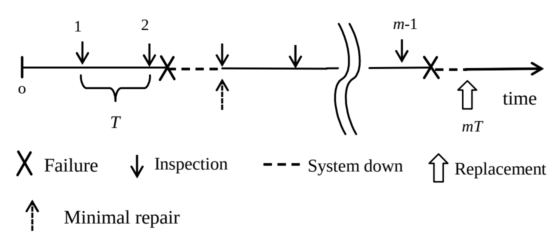

Figure 2. Process of the system under the periodic policy.

4. Analysis

In this section, we introduce the expected cost rate and the mean availability for two policies and compare the optimal cost rates analytically for special cases. In the periodic policy, an approximation method is proposed to simplify the calculation.

4.1 NOFB Policy

In this policy, we already derived its expected cost rate and the mean availability as shown in (1) and (2) [10].

$$C_f(n,T) = \frac{C_I}{T} + \frac{(n-1)C_R + C_P}{T\sum_{l=1}^{n}\sum_{k=0}^{\infty}\bar{F}^{(l)}(kT)}, \tag{4.1}$$

$$A_f(n,T) = \frac{\sum_{l=1}^{n}\mu^{(l)}}{T\sum_{l=1}^{n}\sum_{k=0}^{\infty}\bar{F}^{(l)}(kT)} \tag{4.2}$$

where

$$\bar{F}^{(l)}(x) = \int_0^{\infty}\bar{F}(x+y)\frac{[H(y)]^{l-2}}{(l-2)!}h(y)dy \tag{4.3}$$

and $\mu^{(l)} = \int_0^{\infty}\bar{F}^{(l)}(x)dx$.

4.2 Periodic Policy

In this policy, to formulate the cost rate and the availability is not easy. This is because we must handle myriad number of scenarios as m increases. For example, let us consider the case of $m=2$. The number of scenarios to be considered is four; the system does not fail until $2T$, the system firstly fails between T and $2T$, the system fails between 0 and T and does not fails after T, and the system fails twice from 0 to T and from T and $2T$. Apparently, this number accords with $2m$ and calculating the probability of each case becomes complicated, requiring multiple integrals, as m increases. Thus, we propose an approximation method.

If a failure were detected as soon as it occurs, the occurrences of failures would follow non-homogeneous Poisson process (NHPP). We assume that the time without downtime follows NHPP. Figure 3 gives a schematic concept of the assumption.

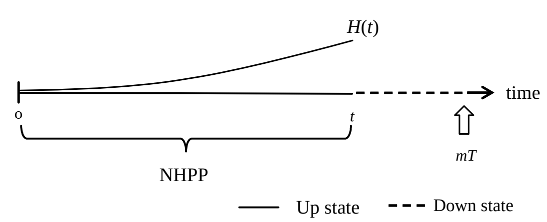

Figure 3. A schematic concept of assumption that satisfies NHPP during uptime.

Moreover, we assume that a duration between a failure and its detection is constantly $T/2$. This approximation is reasonable especially when T is small and has been known as good approximation [13]. This is because the failure distribution between sequential inspections can be regarded as linear when T is much small; namely it approaches the uniform distribution with pdf $\{F(kT) - F((k-1)T)\}/T$ between $(k-1)$th and kth inspection for a positive integer k. Using the two assumptions explained above, we determine the uptime until replacement time t, the time excluding downtime, by the solution of the following equation.

$$t + TH(t)/2 = mT \tag{4.4}$$

Since the l.h.s of (4) increases from 0 to infinity with t, (4) has unique solution. Let t^* be the solution. Then the expected number of failures until replacement is expressed as $H(t^*)$. For the failure occurred between time $(m-1)T$ and time mT, not a minimal repair but a replacement is taken. Thus, the expected number of minimal repairs is less than the expected number of failures and let us express $H(t^*-T)$ as the number of minimal repairs. The resulting cost rate is

$$C_p(m,T) = \frac{(m-1)C_I + H(t^*-T)C_R + C_P}{mT} \tag{4.5}$$

Under these assumptions, the mean availability is

$$A_p(m,T) = 1 - \frac{T/2\cdot H(t^*)}{mT} = 1 - \frac{H(t^*)}{2m} \tag{4.6}$$

4.3 Comparison for Special Cases

In this section, we compare the two policies in special cases analytically. Suppose that failures of the system follow a Weibull distribution with parameters η and β, namely $H(t) = (t/\eta)^{\beta}$.

4.3.1 $\beta = 1$ case

This case means that the system has constant failure rate, $1/\eta$ In the NOFB policy, replacements is never performed. The mean availability is

$$\int_0^T \bar{F}(t)dt = \eta\left(1 - e^{-T/\eta}\right)/T \tag{4.7}$$

and the optimal inspection interval, T^*, is the unique solution of the equation setting (7) equals α. The optimal cost rate is

$$C_f^* = C_f(\infty, T^*) = \frac{C_I}{T^*} + \frac{\alpha}{\eta}C_R \tag{4.8}$$

In the periodic policy, the analytical cost rate is derived as

$$C_p(m,T^*) = \frac{(m-1)C_I + (m-1)\left(1-e^{-T^*/\eta}\right)C_R + C_P}{mT^*} \tag{4.9}$$

where T^* is the same with in (8). Its difference with respect to m is

$$C_P(m+1,T^*) - C_p(m,T^*) = -\frac{C_p - C_I - \left(1-e^{-T^*/\eta}\right)C_R}{m(m+1)T^*} \tag{4.10}$$

Thus, if $C_P > C_I + \{1 - exp(-T^*/\eta)\}C_R = C_I + \alpha T^* C_R/\eta$, the optimal number of m is infinity. The resulting cost rate is

$$C_p^* = C_p(\infty, T^*) = \frac{C_I}{T^*} + \frac{\alpha}{\eta} C_R \tag{4.11}$$

which is exactly same as (8). Otherwise, $C_p^* = C_p(1, T^*) = C_P/T^*$ and the ratio of the two optimal cost rates is

$$\frac{C_f^*}{C_p^*} = \frac{\eta C_I + \alpha C_R T^*}{\eta C_P} > 1. \tag{4.12}$$

Thus, the periodic policy is better. This is because the policy does not need inspection at replacement.

4.3.2 $\beta = \infty$ case

In this case, a failure occurs exactly at constant time $1/\eta$ and the system does not work again after the failure unless it is replaced. Thus, the optimal cost rate in two policies are $C_f^* = \alpha(C_P + C_I)/\eta$ and $C_p^* = \alpha C_P/\eta$. The ratio is $C_f^*/C_p^* = 1 + C_I/C_p > 1$. The periodic policy is also better in this case.

5. Numerical Experiments

First, we show the accuracy of the approximation method for the periodic policy. Then, the comparison of optimal cost rates is shown.

5.1 Approximation Errors

We set parameters $\alpha = 0.9, H(t) = (t/3000)^{2.0}, C_I = 10, C_R = 40$ and $C_P = 400$ and compare the optimal cost rate by the approximation and by Monte Carlo simulation with 10 million samples. The results are shown in Table 1.

Table 1. Optimal solutions

Method	m	T	C_p^*
Approximation	21	325	0.1100
Monte Carlo	21	320	0.1114

We can see that the optimal value of m is same and the error of the optimal cost rate is -1.4%. Table 2 shows the detailed errors for fixed m. The error of the number of minimal repairs decreases with m. This is because the optimal T decreases with m and the approximation becomes more accurate. Although the accuracy of the approximation depends on the parameters, it is useful to find the rough space of optimal solution.

Table 2. Optimal solutions and errors

m	Optimal solution by the approximation		Errors (%)	
	T	C_p^*	Number of minimal repairs	Optimal cost rate
6	608	0.1321	-8.50	-0.61
8	527	0.1227	-6.09	-0.59
10	471	0.1174	-4.62	-0.55
12	430	0.1141	-3.70	-0.51
14	398	0.1121	-3.02	-0.47
16	372	0.1110	-2.54	-0.43
18	351	0.1103	-2.13	-0.39
20	333	0.1100	-1.83	-0.36
22	317	0.1101	-1.58	-0.33
24	304	0.1102	-1.36	-0.30
26	292	0.1107	-1.18	-0.27
28	281	0.1113	-1.05	-0.25

5.2 Comparison of Two Policies

We show the results of comparison of the two management policies, where we use Monte Carlo simulation with 10 million samples for the periodic policy.

Figures 4 and 5 show the optimal cost rates for fixed number of n and m as the qualitative comparison, where $\alpha = 0.9$, $H(t) = (t/3000)^{2.0}$, $C_I = 10$, $C_R = 40$ and $C_P = 400$. Each graph is convex.

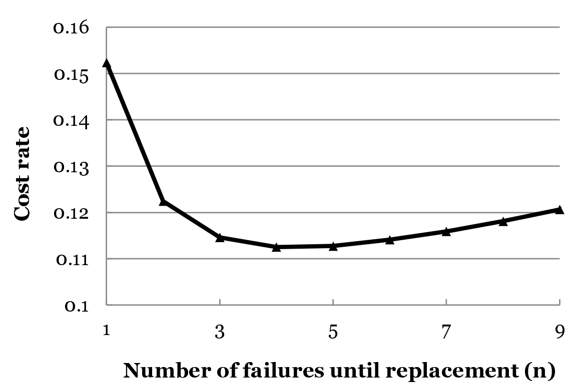

Figure 4. Minimal cost rates for fixed numbers of failures until replacement (n) in the NOFB policy.

Let the hazard rate follow Weibull distributions with scale parameter 3000. We observe how the ratio of the optimal cost rates of the two policies changes depending on shape parameter. Fig. 6 and 7 show results of the cases of $C_P = 400$ and $C_P = 80$ when $C_I = 10$ and $C_I = 2$, where $\alpha = 0.9$ and $C_R = 40$.

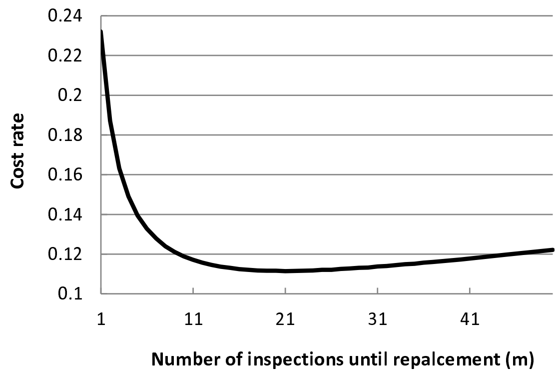

Figure 5. Minimal cost rates for fixed numbers of inspection until replacement (m) in the periodic policy.

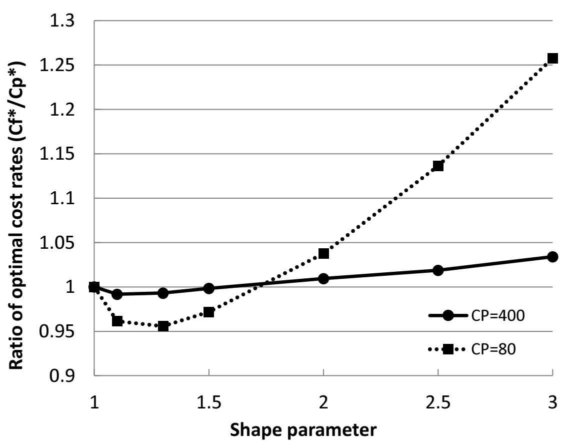

Figure 6. Ratios of optimal cost rates of the two policies with $C_I = 10$.

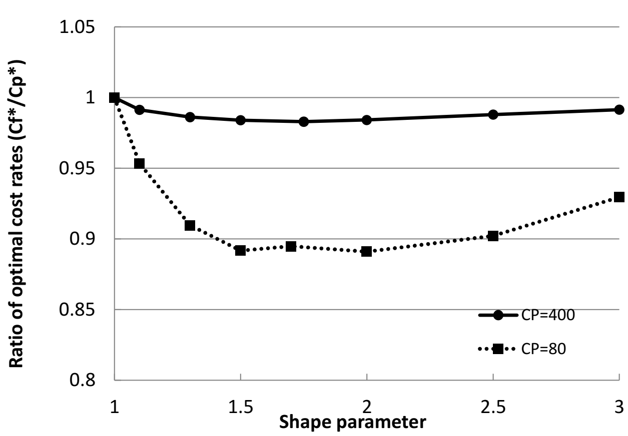

Figure 7. Ratios of optimal cost rates of the two policies with $C_I = 2$.

We can see that the ratio is less than 1 when shape parameter is relatively small in fig. 6. It means that the NOFB policy has advantage. When the shape parameter is large, however, the periodic policy becomes better. Fig. 7 shows the case that the inspection cost is less expensive than that of fig. 6. Advantage of the NOBF policy is observed in wider range. Note that the NOBF policy gets worse than the periodic policy when b increases infinitely according to Sec. IV, C. As we mentioned in Sec. I, Park [12] compared the two policies for a normal system, whose failure is detected immediately without inspections. He concluded that the ratio was always less than 1 in the condition. His condition is almost same with this study if we assume $C_I = 0$ and $T = 0$. Thus, we can conclude that the periodic policy is better when the inspection cost decreases for wide range.

6. Conclusion

We have surveyed two management policies for replacement timings of one-shot system; the NOBF policy and the periodic policy. To formulate the cost rate of the periodic policy, we have proposed an approximation method and checked its accuracy.

By comparing the two polices numerically when failures of a one-shot system follow Weibull distribution, we have found that the NOBF policy is better when shape parameter and the inspection cost is relatively small. This result follows the previous study [12] that showed the NOBF policy is always better for a normal system whose failure is detected immediately without inspection and so makes sense, because the previous one can be regarded as a special case that inspection cost and inspection interval are zero in our one-shot system.

However, the NOBF policy has disadvantage of uncertain replacement time. The time to replacement is random variable so the policy will get worse if preparation time for replacement required. Thus, the discussion in the paper is limited to the ideal condition that such the lead time is negligible. We have to consider quantitatively this uncertainty to determine which policy should be applied for a one-shot system.

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