Role of catastrophes, failures and repairs for a two server queueing systems

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Abstract
Consider a two server queueing system in which catastrophes, failures and repairs are employed. The two systems meet failures due to the occurrence of catastrophes and repaired. The expected queue size and second factorial moment are derived. In addition that some particular cases are discussed.

Keywords
Catastrophes, failures, repairs, Laplace Transform, expected queue size.

AMS Subject Classification
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1. Introduction

Queueing models have been analysed and applied in the evaluation of computer systems and communication networks. In case of non-existence of simulations, for analyzing a queueing model there is a need of tractable model for analysis and techniques.

Even though the literature on queueing theory is vast, very few work is identified regarding transient behavior of queueing systems. Transient solutions for queueing systems are generally assumed to be an intractable problem with minimum practical interest.

In queueing theory, the researchers have analysed the models by adopting the concepts like bulk size, priority, vacation, retrial, catastrophe and so on. The catastrophes arrive as unsupportive customers to the system and induced to remove all the customers in the system. If in a computer networks, job is infected with a virus, it transforms the virus to other systems. Because of this reason, virus infected computer networks may be modeled by queueing operations with catastrophes. In practice, due to the occurrence of catastrophes, there exists an impact of failure and repair of systems on the flow of tasks that have to be handled by the operators.

Based on the above stated concepts, in this paper, a queueing system is analysed with the role of catastrophes, failures and repairs. The jobs arrive to the system according to Poisson process. The two systems meet failures due to the occurrence of catastrophes and repaired. The expected queue size and second factorial moment are derived. In addition that some particular cases are discussed.

2. Review of Literature

Parthasarathy and Sudhesh (2005)[7] have obtained time dependent and stationary probabilities for a Markovian multiprocessor system with heterogeneous service and failure times. Jain and Kanethia (2006)[3] have studied application of catastrophes on the transient behavior of the finite Markovian queue with repairable single server. They obtained the system size in terms of Laplace Stieltjes transform. Kumar et al., (2007)[5] have discussed a transient solution using continued fractions for the system size in an M/M/1 queueing system with catastrophes, server failures and non-zero repair time. They investigated some key performance measures, namely,
throughput, loss probability and response time for the system. Further, they analysed the reliability and availability of the system. Kalidass et al., (2012) [4] have discussed a single server Markovian queueing system with a finite buffer with additional assumption of system with Poisson stream of negative arrivals called as catastrophes may occur with the server is idle or busy. Also they discussed the performance measures and the busy period of the system dependent on time and the under steady state conditions were derived. Ayyappan and Devipriya (2013)[1] have analysed a bulk service queue with catastrophe. The analytical solutions for mean and variance of the number of customers in the system, by using probability generating function technique and Rouche’s theorem, have been derived. Nompazy and Yechiali (2014) [6] have discussed a single server Markovian queue in which the system suffers disastrous failure, causing on present job to be lost. After the repair is over, the job moves to the servicing phase. The mean queue sizes, the mean waiting time and fraction of lost customers have been derived. Sherif I. Ammar (2014) [8] studied application of catastrophes, server failures and repairs on heterogeneous time dependent two processor system with Poisson arrival and exponential service and derived the expression for mean and variance of the system size distribution. Balasubramanian et al., (2015) [2] have studied a two servers queue with blocking and catastrophe. The expressions for the expected number of units in the system and lost probability of units are mathematically derived.

3. Model Description

Consider a queueing model with two heterogeneous servers who are called as computer system operators. The different jobs arrive to the systems and the arrival process is assumed to be Poisson process with mean arrival rate \( \lambda \). The two operators who are named as faster and slower, provide services with mean service rates \( \mu_1 \) and \( \mu_2 \) respectively under the condition that \( \mu_1 > \mu_2 \). Each job demands its service from any one of the operators. The systems may meet failure due to the occurrence of catastrophes and that to be repaired.

When the systems are idle or busy, catastrophes occur at the service stations according to Poisson process with mean rate \( \gamma \). At the moment of the occurrence of catastrophes in the system, all jobs are smashed, the two operators get idled. Both failed systems are repaired and it time is identical independently distributed with exponential distribution and mean rate \( \eta \). After repairing the system, the operators are ready for service at the arrival of a new job.

The important transient probabilities are stated as follows:

\( Q(t) \): is the probability of the systems under repair at time \( t \) and \( Q(0) = 0 \)

\( X(t) \): is the number of jobs in the system at time \( t \).

\( P_{01}(t) = P[X(t) = n], n = 1, 2, 3, \ldots \) is the probability of \( n \) jobs in the system at time \( t \).

\( P_{00}(t) = P[X(t) = 0] \), is the probability of the system is empty at time \( t \).

\( P_{0}(t) = P[X(t) = 1] \) is the probability of there is one job in the system and it is served by faster operator.

\( P_{01}(t) = P[X(t) = 1] \) is the probability of there is one job in the system and that is served by slower operator.

4. Transient state equations

Based on the above stated assumptions, the required Chapman-Kolmogorov forward equations are written as,

\[
\dot{Q}(t) = -\eta Q(t) + \gamma (1 - Q(t)) \tag{4.1}
\]

\[
P_{00}'(t) = -(\lambda + \gamma)P_{00}(t) + \mu_1 P_{10}(t) + \mu_2 P_{01}(t) + \eta Q(t) \tag{4.2}
\]

\[
P_{10}'(t) = -(\lambda + \mu_1 + \gamma)P_{10}(t) + \lambda P_{00}(t) + \mu_2 P_{11}(t) \tag{4.3}
\]

\[
P_{01}'(t) = -(\lambda + \mu_2 + \gamma)P_{01}(t) + \mu_1 P_{11}(t) \tag{4.4}
\]

\[
P_{11}'(t) = -(\lambda + \mu_1 + \mu_2 + \gamma)P_{11}(t) + \lambda \{P_{01}(t) + P_{10}(t)\} + (\mu_1 + \mu_2)P_{21}(t) \tag{4.5}
\]

\[
P_{n1}'(t) = -(\lambda + \mu_1 + \mu_2 + \gamma)P_{n1}(t) + \lambda P_{n-1,1}(t) + (\mu_1 + \mu_2)P_{n+1,1}(t) \tag{4.6}
\]

Now, define Laplace transforms of \( P_{ni}(t) \) and \( Q(t) \) respectively as

\[
P_{ni}(z) = \int_0^\infty e^{-zt} P_{ni}(t) dt \quad \text{and} \quad Q^*(z) = \int_0^\infty Q(t) dt \tag{4.7}
\]

Apply Laplace transforms defined in (4.7) to the equations from (4.1) to (4.6) which become

\[
(z + \eta + \gamma)Q^*(z) = \frac{\gamma}{z} \tag{4.8}
\]

\[
(z + \eta + \gamma)P_{00}(z) = \mu_1 P_{10}(z) + \mu_2 P_{01}(z) + \eta Q^*(z) \tag{4.9}
\]

\[
(z + \lambda + \mu_1 + \gamma)P_{10}(z) = \lambda P_{00}(z) + \mu_2 P_{11}(z) \tag{4.10}
\]

\[
(z + \lambda + \mu_2 + \gamma)P_{01}(z) = \mu_1 P_{11}(z) \tag{4.11}
\]

\[
(z + \lambda + \mu_1 + \mu_2 + \gamma)P_{11}(z) = \lambda (P_{01}(z) + P_{10}(z)) + (\mu_1 + \mu_2)P_{21}(z) \tag{4.12}
\]
The variance of the number of jobs in the queue is obtained by differentiation of the probability generating function given in (4.15) and (4.16). On differentiating the expression with the equations (4.9), (4.10) and (4.11) which provide the required probability generating function of Laplace transforms as,

\[ G_1(x,z) = \sum_{n=0}^{\infty} P_n^s(z)x^n \]  

(4.14)

Multiply the equations (4.5) and (4.6) by the proper powers of \( x \) and summing over \( n = 1, 2, 3, ..., \infty \). Adding the resultant expression with the equations (4.9), (4.10) and (4.11) which provide the required probability generating function of Laplace transforms as,

\[ \{z - \lambda x - (\mu_1 + \mu_2)x^{-1}\}G_1(x,z) = (\mu_1 + \mu_2) \]

(4.15)

\[ (1 - x^{-1})P^o_{01}(z) - \{z + \lambda (1-x) + \gamma\}P^o_{10}(z) + \eta Q^s(z) \]

\[ + \eta Q^s(z) \]

5. Queueing performances:

In this section, the mean and variance of the number of jobs in the system are derived. On differentiating the expression of the probability generating function given in (4.15) twice, the expected queue size and second factorial moment are obtained respectively as,

\[ G'_1(1,z) = (z + \gamma)^{-1}[(\mu_1 + \mu_2)(P^o_{01}(z) + P^o_{10}(z)) + (\mu_1 + \mu_2 - \lambda)(P^o_{00}(z) - \eta(z + \gamma)^{-1}Q^s(z))] \]

(5.1)

and

\[ G''_1(1,z) = 2(z + \gamma)^{-2}[(\mu_1 + \mu_2)[\eta Q^s(z) - (z + \gamma)(P^o_{10}(z) + P^o_{00}(z)) + P^o_{01}(z) + \lambda P^o_{10}(z) - \eta(z + \gamma)^{-1}Q^s(z))] + (\mu_1 + \mu_2 - \lambda)(P^o_{10}(z) + P^o_{00}(z)) - \eta(z + \gamma)^{-1}Q^s(z))] \]

(5.2)

The variance of the number of jobs in the queue is obtained by using the relation

\[ V(1,z) = G''_1(1,z) + G'_1(1,z) - (G'_1(1,z))^2 \]

6. Particular case:

Case (i) : No catastrophes is allowed.

Apply \( \gamma = 0 \Rightarrow \eta = 0 \) in equations (5.1) and (5.2) which yield

\[ G'_1(1,z) = z^{-1}[(\mu_1 + \mu_2)(P^o_{01}(z) + P^o_{10}(z)) + (\mu_1 + \mu_2 - \lambda)P^o_{00}(z)] \]

(6.1)

and

\[ G''_1(1,z) = 2z^{-2}[(\mu_1 + \mu_2)\{-z(P^o_{01}(z) + P^o_{10}(z) + P^o_{00}(z))\} - (\mu_1 + \mu_2 - \lambda)\{\mu_1 + \mu_2\}P^o_{01}(z) + \lambda P^o_{10}(z) + (\mu_1 + \mu_2 - \lambda)\{P^o_{10}(z) + P^o_{00}(z)\}] \]

(6.2)

Case (ii) : Absence of second operator. Here we use \( \mu_2 = 0 \), then the equations (5.1) and (5.2) becomes

\[ G'_1(1,z) = (z + \gamma)^{-1}\left[\mu_1(P^o_{01}(z) + P^o_{10}(z)) + (\mu_1 - \gamma)P^o_{00}(z) - \gamma(z + \gamma)^{-1}Q^s(z)\right] \]

(6.3)

and

\[ G''_1(1,z) = 2(z + \gamma)^{-2}\left[(\mu_1)(\eta Q^s(z) - (z + \gamma)(P^o_{10}(z)) + P^o_{01}(z) + \lambda P^o_{00}(z) - (\mu_1 - \gamma)\{P^o_{10}(z) + P^o_{00}(z)\}) - (\mu_1 - \gamma)(P^o_{01}(z) + \lambda P^o_{10}(z)) + (\mu_1 - \gamma)(P^o_{10}(z) - \eta(z + \gamma)^{-1}Q^s(z))]\right] \]

(6.4)

These results coincide with the results of Krishnakumar et al, [2007][5], and Kalidass et al,[2012][4].

Case (iii) : Absence of catastrophes and second operator. The equations (5.2) and (5.3) are reduced after applying \( \mu_2 = 0 \) and \( \gamma = \eta = 0 \)

\[ G'_1(1,z) = z^{-1}[\mu_1(P^o_{01}(z) + P^o_{10}(z)) + (\mu_1 - \lambda)P^o_{00}(z)] \]

(6.5)

and

\[ G''_1(1,z) = 2z^{-2}\left[-z[\mu_1(P^o_{01}(z) + P^o_{10}(z)) + P^o_{00}(z)] - (\mu_1 - \lambda)\{\mu_1(P^o_{01}(z) + \lambda P^o_{10}(z)) + (\mu_1 - \lambda)(P^o_{10}(z) + P^o_{00}(z))]\right] \]

(6.6)

7. Conclusion

In this paper, a two server queueing systems with Poisson arrival is considered. The two systems meet failures due to the occurrence of catastrophes and subjected to repair. The expected queue size and second factorial moment have been derived. Particular cases are discussed under certain conditions.

References


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