Regular weakly quotient map and space

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Abstract
In this paper, we introduce the new regular weakly quotient map (briefly, rw-quotient map), strongly rw-quotient map and rw*-quotient map. Also, we investigate some important properties and consequences associated with usual quotient and bi, tri-quotient maps.

Keywords
rw-quotient map, strongly rw-quotient map, rw*-quotient map, rw-quotient space.

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1. Introduction

In the literature of Mathematics, quotient maps are generally called strong continuous maps or identification maps, because of strong conditions of continuity i.e. \( h^{-1}(V) \) is open in \( M \) if and only if \( V \) is open in \( N \) of a surjective function \( h : M \to N \) and their importance for the philosophy of gluing. Benchalli et al. introduced Regular weakly closed (briefly, rw-closed) sets in topological spaces \([10]\) and Regular weakly continuous maps, respective open map and closed map have been introduced and studied by M. Karpagadevi et al. \([7]\). Lellis Thivagar \([6]\) and Balamani et al. \([9]\) discussed generalized quotient maps and \( \psi^\alpha \)-quotient maps in topological space respectively. Bi-quotient, Tri-quotient maps studied by E. Michal \([2, 3]\).

We introduce such class of maps called rw-quotient map, strongly rw-quotient map, rw*-quotient map in topological space. Which have many rich consequences concern to usual quotient and bi, tri-quotient maps. Here we have given example and counter example for respective result.

2. Preliminaries

Throughout this paper \( M, N \) and \( R \) represent the topological spaces on which no separation axioms are assumed, unless otherwise mentioned. Map here mean function and for a subset \( F \) of topological space \( M \), the \( M \setminus F \) denotes the complement of \( F \) in \( M \). We recall the following definitions.

Definition 2.1. \([10]\) A subset \( F \) of a space \( M \) is said to be regular weakly closed (briefly, rw-closed) set, if \( \text{cl}(F) \subseteq U \) whenever \( F \subseteq U \) and \( U \) is regular semi open set in \( M \). Respectively called regular weakly open (briefly, rw-open) set, if \( M \setminus F \) is rw-closed set in \( M \). We denote the set of all rw-open sets in \( M \) by \( \text{RWO}(M) \).

Definition 2.2. A map \( h : M \to N \) is said to be rw-continuous map \([7]\), if \( h^{-1}(F) \) is rw-closed set of \( M \), for every closed set \( F \) of \( N \).

Definition 2.3. \([7]\) A map \( h : M \to N \) is said to be regular weakly closed (briefly, rw-closed) map, if the image of every closed set in \( M \) is rw-closed in \( N \). Respectively called regular weakly open (briefly, rw-open) map, if the image of every open set in \( M \) is rw-open in \( N \).

Definition 2.4. \((\text{We consider in this paper})\) A map \( h : M \to N \) is said to be rw*-closed map, if the image of every rw-closed set in \( M \) is rw-closed in \( N \). Respectively called rw*-open map, if the image of every rw-open set in \( M \) is rw-open in \( N \).

Definition 2.5. A rw-homeomorphism \( h : M \to N \) is a bijection and both rw-continuous and rw-open (or, both \( h \) and \( h^{-1} \) are rw-continuous).
Definition 3.6. A topological space $M$ is called $\eta_{rw}$-space [7], if every rw-closed set is closed.

### 3. Rw-Quotient Maps

We introduce here some class of maps rw-quotient map, strongly rw-quotient map, rw*-quotient map in topological space. Investigate some relations between them with usual quotient maps. Also we characterise few notions.

**Definition 3.1.** Let $M$ and $N$ be two topological spaces then a surjective map $h : M \rightarrow N$ is said to be rw-quotient map if $h$ is rw-continuous and $h^{-1}(V)$ is open in $M$ implies $V$ is rw-open set in $N$.

Example 3.2. Let $M = \{a, b, c, d\}$ be topological space by $\eta = \{\emptyset, M, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ a topology on it, induce the $\text{RWO}(M) = \{\emptyset, M, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$ and $N = \{(p, q)\}$ with topology $\xi = \{(0, N), (p, q)\}$. Let the map $h : M \rightarrow N$ be $h(a) = p$, $h(b) = p$, $h(c) = q$, $h(d) = q$. Then obviously $h^{-1}(\emptyset) = \emptyset$, $h^{-1}(N) = M$, $h^{-1}(\{p\}) = \{a, b\}$ and $h^{-1}(\{(q)\}) = \{c, d\}$ are rw-open sets. Hence $h$ is rw-continuous. Also $h^{-1}(\emptyset) = \emptyset$, $h^{-1}(\{p\}) = \{a, b\}$ are all open set in $M$, implies obviously $\emptyset, N, \{p\}$ are rw-open set in $N$. Therefore $h$ is rw-quotient map.

Example 3.3. Let $h : (R, \eta_a) \rightarrow (R, \eta_b)$ and $\eta_a$ be usual topology on $R$, the map $h$ is defined by $h(x) = 5x$ is obviously (See 3.4) rw-quotient map.

**Theorem 3.4.** Every quotient map is rw-quotient map.

**Proof.** Let $h : M \rightarrow N$ be any quotient map, by definition $h$ is surjective. It is known that every continuous map is rw-continuous map, hence $h$ is rw-continuous. Since $h$ is a quotient map, $h^{-1}(V)$ is open in $M$, implies $V$ is open in $N$. As every open set is rw-open set, implies $V$ is rw-open set in $N$. Therefore $h$ is rw-quotient map.

Example 3.5. Converse of 3.4 need not hold. From example 3.2, $h$ is not continuous because $h^{-1}(\{q\}) = \{c, d\}$ which is not open in $M$. Hence $h$ is not a quotient map.

**Theorem 3.6.** [4] If $h : M \rightarrow N$ is rw-continuous and $h : N \rightarrow M$ is continuous, such that $h \circ k : N \rightarrow N$ is identity then $h$ is rw-quotient map.

**Proof.** Since $h \circ k = \text{Id}$ implies $h$ is bijective. $h$ is rw-continuous by hypothesis. For any $V \subset N$ with $h^{-1}(V)$ be open in $M$, continuity of $k$ gives the $k^{-1}(h^{-1}(V)) = (h \circ k)^{-1}(V) = (\text{Id})^{-1}(V) = V$ is open in $N$, implies $V$ is rw-open set in $N$. Therefore $h$ is rw-quotient map.

**Theorem 3.7.** $h$ become rw-quotient map, whenever $h : N \rightarrow M$ is surjective, continuous and open maps.

**Proof.** By theorem 3.4, $h$ is rw-quotient map, because every $h : N \rightarrow M$ is surjective, continuous and open maps are quotient map.

**Theorem 3.8.** $h$ become rw-quotient map, whenever $h : N \rightarrow M$ is surjective, continuous and closed map.

**Theorem 3.9.** $h$ become rw-quotient map, whenever $h : N \rightarrow M$ is surjective, continuous and rw-open map.

**Theorem 3.10.** $h$ become rw-quotient map, whenever $h : N \rightarrow M$ is surjective, rw-continuous and rw-open closed map.

**Proof.** Two conditions are obvious by the hypothesis. For last condition, any $V \subset N$ with $h^{-1}(V)$ be open in $M$, implies $M \setminus h^{-1}(V)$ is closed set in $M$. Since $h$ is rw-open map implies $h(M \setminus h^{-1}(V)) = N \setminus V$ is rw-closed in $N$. Therefore $V$ is rw-open set in $N$, $h$ is rw-quotient map.

**Definition 3.11.** Let $M$ and $N$ be two topological spaces, a surjective map $h : N \rightarrow M$ is said to be strongly rw-quotient map provided $V \subset N$ is open in $N$ if and only if $h^{-1}(V)$ is rw-open in $M$.

**Definition 3.12.** Let $M$ and $N$ be two topological spaces, a surjective map $h : N \rightarrow M$ is said to be rw*-quotient map if $h$ is rw-irresolute and $h^{-1}(V)$ is rw-open in $M$ implies $V$ is open set in $N$.

**Theorem 3.13.** Every injective rw*-quotient map is rw*-open map.

**Proof.** Let $h : N \rightarrow M$ be any injective rw*-quotient map. For every rw-open set $V$ in $M$, implies $h^{-1}(h(V)) = V$ is rw-open set in $M$. Since $h$ is rw*-quotient map, implies $h(V)$ is open set as well rw-open in $N$.

**Theorem 3.14.** Every injective rw*-quotient map is rw*-closed map.

**Proof.** Let $h : N \rightarrow M$ be injective rw*-quotient map. For every rw-closed set $F$ in $M$, implies $h^{-1}(h(F)) = F$ is rw-open set in $M$. By Theorem 3.9, $h(M \setminus F) = N \setminus h(F)$ is rw-open set in $N$. Hence $h(F)$ is rw-closed set in $N$.

**Theorem 3.15.** Every strongly rw-quotient map is rw-quotient map.

**Proof.** Let $h : N \rightarrow M$ be strongly rw-quotient map, obviously the first two conditions hold. For $V \subset N$ with $h^{-1}(V)$ be an open set in $M$, also that become rw-open in $M$. Since $h$ is strongly rw-quotient map, implies $V$ is open set in $N$, therefore $V$ is rw-open set in $N$.

**Example 3.16.** Converse of 3.14 need not hold. Let $M = \{a, b, c, d\}$ be topological spaces with two topologies on it by, $\eta_1 = \{\emptyset, M, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ for which $\text{RWO}(M, \eta_1) = \{\emptyset, M, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$ and another topology $\eta_2 = \{\emptyset, M, \{a, b\}, \{c\}\}$ for which $\text{RWO}(M, \eta_2) = P(M) = \text{power set of } M$ and the map $f : (M, \eta_1) \rightarrow (M, \eta_2)$ be $h(x) = x$, $\forall x \in M$ then obviously $h$ is rw-quotient map. But not strongly rw-quotient map, because $h^{-1}(\{b\}) = \{b\}$ is rw-open sets in $(M, \eta_1)$ but $\{b\}$ is not open set in $(M, \eta_2)$. 

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Theorem 3.17. Every rw*-quotient map is strongly rw-quotient map.

Proof. Let $h : M \to N$ be strongly rw*-quotient map, two conditions are obvious. Because $V$ is any open set in $N$ is also rw-open set in $N$. Since $h$ is rw-irresolute implies $h^{-1}(V)$ is rw-open set in $M$. For $W \subset N$ with $h^{-1}(W)$ be an open set in $M$, implies $h^{-1}(W)$ is rw-open in $M$. Since $h$ is rw*-quotient map, implies $W$ is open set in $N$.

Theorem 3.18. Every rw*-quotient map is rw-quotient map.

Proof. It followed by theorem 3.15 and 3.17.

4. Consequences on Compositions

Theorem 4.1. Composition of two quotient maps is rw-quotient map.

Proof. We know composition of two quotient maps is quotient map [5; 3.29]. Hence by theorem 3.4 which is rw-quotient map.

Example 4.2. Converse of 5.1 need not hold. Let $M = \{a, b, c, d\}$ be topological space with topology considered is, $\eta = \{\emptyset, M, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $N = \{p\}$ be space with topology $\xi = \{\emptyset, N\}$ then the maps $h : M \to N$ by $h(x) = a, \forall x \in M$ and $k : M \to N$ by $k(x) = p, \forall x \in M$ obviously $k \circ h : M \to N$ become $k \circ h(x) = p$ which is rw-quotient map but $h$ is not quotient map.

Remark 4.3. Composition of quotient map with rw-quotient map need not be rw-quotient map.

Remark 4.4. Composition of rw-quotient map with quotient map need not be rw-quotient map.

Remark 4.5. Composition of two rw-quotient maps need not be rw-quotient map.

Theorem 4.6. If $N$ is $\eta_{rw}$ topological space, $h : M \to N$ is rw-quotient map and $k : N \to R$ is quotient map then $k \circ h$ is rw-quotient map.

Proof. Obviously $k \circ h$ is surjective. For every open set $U \subset R$, the $k \circ h)^{-1}(U) = h^{-1}(k^{-1}(U))$ is open in $M$. For every $W \subset R$ with $(k \circ h)^{-1}(W) = h^{-1}(k^{-1}(W))$ open in $M$. Since $h$ is rw-quotient map implies $k^{-1}(U)$ is open in $N$. Since $k$ is strongly rw-quotient map implies $U$ is open set in $R$. Hence $k \circ h$ is strongly rw-quotient map.

Example 4.7. Converse of 4.6 need not hold, refer and earn arguments from 4.2.

Theorem 4.8. If $N$ is $\eta_{rw}$ topological space, $h : M \to N$ is rw-quotient map and $k : N \to R$ is rw-quotient map then $k \circ h$ is rw-quotient map.

Proof. Similar arguments with essential changes in 4.6 can work.

Example 4.9. Converse of 4.8 need not hold, refer and earn arguments from 4.2.

Corollary 4.10. If $N$ is $\eta_{rw}$ topological space, $h : M \to N$ is rw-quotient map and $k : N \to R$ is rw-quotient map then $k \circ h$ is rw-quotient map.

Proof. It follow by combining the theorems 5.6 and 5.8 and (Converse does not hold).

Theorem 4.11. If $h : M \to N$ is a strong rw-quotient map and $k : N \to R$ is a quotient map then $k \circ h$ is strong rw-quotient map.

Proof. Here surjective of $k$ and $h$ gives $k \circ h$ is surjective and $k \circ h$ is rw-continuous map due to composition of continuous and rw-continuous. Lastly if $h^{-1}(k^{-1}(V))$ is open in $M$. Since $h$ is strong rw-quotient map and quotient of $k$ implies $V$ is open in $R$. Therefore $k \circ h$ is strong rw-quotient map.

Theorem 4.12. If $h : M \to N$ is open surjective, rw-irresolute and $k : N \to R$ is rw-quotient map then $k \circ h$ is rw-quotient map.

Proof. Obviously $k \circ h$ surjective. For every open set $U \subset R$, the $k \circ h)^{-1}(U) = h^{-1}(k^{-1}(U))$ is open in $M$. For every $W \subset R$ with $(k \circ h)^{-1}(W) = h^{-1}(k^{-1}(W))$ open in $M$. Since $h$ is open map $h^{-1}(k^{-1}(W)) = k^{-1}(W)$ is open in $N$. Given $k$ is rw-quotient map implies $W$ is rw-open set in $R$. Hence $k \circ h$ is rw-quotient map.

Theorem 4.13. If $h : M \to N$ is $\eta_{rw}$-open surjective and rw-irresolute and $k : N \to R$ is strongly rw-quotient map then $k \circ h$ is strongly rw-quotient map.

Proof. Obviously $k \circ h$ surjective and rw-continuous. For every $U \subset R$ with $(k \circ h)^{-1}(U) = h^{-1}(k^{-1}(U))$ be rw-open in $M$, since $h$ is $\eta_{rw}$-open map implies $h(h^{-1}(k^{-1}(U))) = k^{-1}(U)$ is open in $N$. Since $k$ is strongly rw-quotient map implies $U$ is open set in $R$. Hence $k \circ h$ is strongly rw-quotient map.

Theorem 4.14. If $h : M \to N$ is $\eta_{rw}$-open, surjective and rw-irresolute and $k : N \to R$ is rw-quotient map then $k \circ h$ is $\eta_{rw}$-quotient map.


Theorem 4.15. If $h : M \to N$ and $k : N \to R$ are $\eta_{rw}$-quotient maps then $k \circ h$ is $\eta_{rw}$-quotient map.

Proof. Obviously $k \circ h$ surjective and rw irresolute. For every $U \subset R$ with $(k \circ h)^{-1}(U) = h^{-1}(k^{-1}(U))$ be rw-open in $M$, since $h$ is rw-quotient map implies $k^{-1}(U)$ is open and rw-open in $N$. Since $k$ is $\eta_{rw}$-quotient map implies $U$ is open set in $R$. Hence $k \circ h$ is $\eta_{rw}$-quotient map.
5. Standard Comparisons and Applications

Theorem 5.1. If $h : M \to N$ is any map, where $M$ and $N$ are $\eta_{rw}$ topological spaces then following are equivalent.
i) $h$ is $rw^*$-quotient map.
ii) $h$ is strongly $rw$-quotient map.
iii) $h$ is $rw$-quotient map.

Proof. (i)⇒(ii) Surjective is obvious and since every $rw$-irresolute map is $rw$-continuous map. Third condition is trivial by the definition of $rw^*$-quotient map.
(ii)⇒(iii) Obvious by the similar arguments in above.
(iii)⇒(i) Surjective is obvious and $h$ is $rw$-irresolute, because every continuous map is $rw$-irresolute because $N$ is $\eta_{rw}$. Let $h^{-1}(V)$ be $rw$-open set in $M$, implies $h^{-1}(V)$ is open set in $M$. The $rw$-quotient map of $h$ implies $V$ is $rw$-open set and open set in $N$.

Theorem 5.2. following are equivalent for surjective, $rw$-continuous map $h : (M, \eta) \to (N, \xi)$
i) $h$ is strongly $rw$-quotient map.
ii) For any $k : (N, \xi) \to (R, \tau)$ then $k$ is continuous if and only if $k\circ h$ is $rw$-continuous.
iii) For fixed topology $\eta$ on $M$ then $\xi$ is the maximal topology for $h$ to be $rw$-continuous.

Proof. (i)⇒(ii) For every open set $U \subset R$, the $(k\circ h)^{-1}(U) = h^{-1}(k^{-1}(U))$ is $rw$-open set in $(M, \eta)$ because of $k$ is continuous and $h$ is $rw$-continuous. Therefore $k\circ h$ is $rw$-continuous. Conversely, for every open set $U$ of $(R, \tau)$, the $(k\circ h)^{-1}(U) = h^{-1}(k^{-1}(U))$ is $rw$-open set in $(M, \eta)$. Since $h$ is strongly $rw$-quotient map gives $k^{-1}(U)$ is open in $(N, \xi)$. Therefore $k$ is continuous.
(ii)⇒(iii) Obvious by the similar arguments in above.
(iii)⇒(i) Surjective is obvious and $h$ is $rw$-irresolute, because every continuous map is $rw$-irresolute because $N$ is $\eta_{rw}$. Let $h^{-1}(V)$ be $rw$-open set in $M$, implies $h^{-1}(V)$ is open set in $M$. The $rw$-quotient map of $h$ implies $V$ is $rw$-open set and open set in $N$.

Theorem 5.3. If $h : M \to N$ is any map, where $M$ and $N$ are $\eta_{rw}$ topological spaces then following are equivalent.
i) $h$ is $rw^*$-quotient map.
ii) $h$ is strongly $rw$-quotient map.
iii) $h$ is $rw$-quotient map.

Proof. (i)⇒(ii) Surjective is obvious and since every $rw$-irresolute map is $rw$-continuous map. Third condition is trivial by the definition of $rw^*$-quotient map.
(ii)⇒(iii) Obvious by the similar arguments in above.
(iii)⇒(i) Surjective is obvious and $h$ is $rw$-irresolute, because every continuous map is $rw$-irresolute because $N$ is $\eta_{rw}$. Let $h^{-1}(V)$ be $rw$-open set in $M$, implies $h^{-1}(V)$ is open set in $M$. The $rw$-quotient map of $h$ implies $V$ is $rw$-open set and open set in $N$.

Theorem 5.4. If $h : (M, \eta) \to (N, \xi)$ is surjective, $rw$-continuous map then following are equivalent.
i) $h$ is strongly $rw$-quotient map.
ii) For any $k : (N, \xi) \to (R, \tau)$ then $k$ is continuous if and only if $k\circ h$ is $rw$-continuous.
iii) For fixed topology $\eta$ on $M$ then $\xi$ is the maximal topology for $h$ to be $rw$-continuous.

Proof. (i)⇒(ii) For every open set $U \subset R$, the $(k\circ h)^{-1}(U) = h^{-1}(k^{-1}(U))$ is $rw$-open set in $(M, \eta)$ because of $k$ is continuous and $h$ is $rw$-continuous. Therefore $k\circ h$ is $rw$-continuous. Conversely, for every open set $U$ of $(R, \tau)$, the $(k\circ h)^{-1}(U) = h^{-1}(k^{-1}(U))$ is $rw$-open set in $(M, \eta)$. Since $h$ is strongly $rw$-quotient map gives $k^{-1}(U)$ is open in $(N, \xi)$. Therefore $k$ is continuous.
(ii)⇒(iii) Obvious by the similar arguments in above.
(iii)⇒(i) Surjective is obvious and $h$ is $rw$-irresolute, because every continuous map is $rw$-irresolute because $N$ is $\eta_{rw}$. Let $B = \eta \setminus \{U_0\}$, which induces topology $\xi^* = \xi \setminus \{0\}$ on $N$, which contains $U_0$, also $\xi^* \supseteq \xi$. But $h : (M, \eta) \to (N, \xi^*)$ also $rw$-continuous. Which contradicts to (iii), therefore $h$ is strongly $rw$-quotient map.

Theorem 5.5. [4] If $h : M \to N$ is $rw$-continuous and $k : N \to M$ is continuous, such that $h \circ k : N \to N$ is identity with $N$ is $\eta_{rw}$- topological space then $h$ is $rw^*$-quotient map.

Proof. Arguments in theorem 3.6 and $N$ is $\eta_{rw}$-space can give the proof.

Theorem 5.6. [4] If $h : M \to N$ is a strong $rw$-quotient map and $k : M \to R$ is a map that is constant on each set $h^{-1}(y)$ for $y \in N$, then
i) $k$ induces a map $l : N \to R$ such that $l \circ h = k$.
ii) The induced map $l$ is continuous iff $k$ is $rw$-continuous.
iii) The induced map $l$ is quotient iff $k$ strong $rw$-quotient map.

Proof. i) Since $k$ is constant on $h^{-1}(y)$ for $y \in N$, the set $k(h^{-1}(y))$ is a one point set in $R$. By considering $l(y)$ denote this point, then which is clear that $l$ is well-defined on $N$ and can see as each $x \in M$, $l(h(x)) = k(x)$.
ii) If $l$ is continuous and $h$ is $rw$-continuous implies $l \circ h = k$ is $rw$-continuous. On other hand let $U$ be any open set in $R$ then $k^{-1}(U)$ is an $rw$-open set due to $k$ is $rw$-continuous. But $k^{-1}(U) = h^{-1}(l^{-1}(U))$ is $rw$-open in $M$. Since $h$ is a strong $rw$-quotient map, $l^{-1}(U)$ is an open set. Hence $l$ is continuous.
iii) If $l$ is quotient map and $h$ is $rw$-quotient map by theorem 4.11, $l \circ h = k$ is strong $rw$-quotient map. On other hand, since $l \circ h = k$ surjective implies $l$ is surjective and $l$ is continuous by above result (ii). If $l^{-1}(U)$ is open in $N$ and $rw$-continuity of $h$ implies $h^{-1}(l^{-1}(U)) = k^{-1}(U)$ is $rw$-open in $M$. Since $k$
is strong rw-quotient map implies $U$ is open in $R$. Hence $l$ is quotient map. 

The following theorems 5.5 to 5.13 followed obviously from the results of [3] i.e. every bi-quotient map is quotient map and from [3, 8] implies that every tri-quotient maps are quotient maps Hence by 3.4 both class of maps become $h$ srw-quotient maps.

**Theorem 5.7.** Arbitrary product map of bi quotient maps are rw-quotient.

**Theorem 5.8.** If $N$ is Hausdorff space and any map $h : M \rightarrow N$ is surjective continuous then $h$ and $h \times \text{Id}_R$ are rw-quotient maps for every space $R$.

**Theorem 5.9.** If $N$ is Regular space, map $h : M \rightarrow N$ is surjective continuous map and $h \times k$ is quotient maps for every quotient maps $k$ then $h$ rw-quotient maps.

**Theorem 5.10.** If maps $h : M \rightarrow N$ and $k : R \rightarrow S$ are quotient maps and $M$ and $N \times S$ are Hausdorff $k$-spaces then $h \times k$ is rw-quotient map.

**Theorem 5.11.** Any surjective contiguous map $h$ from $M$ to a hausdorff space $N$ is rw-quotient map whenever $h$ is compact covering, and $N$ is locally compact.

**Theorem 5.12.** The $h : M \rightarrow N$ is continuous surjective map become rw-quotient map whenever either $h$ is perfect or $h$ is compact-covering and $Y$ is locally compact and Hausdorff.

**Theorem 5.13.** If $M$ regular sieve-complete space, $N$ is paracompact space then every inductively perfect map $h : M \rightarrow N$ is rw-quotient.

**Theorem 5.14.** If $M$ completely metrizable, $N$ is paracompact space then following gives $h : M \rightarrow N$ is rw-quotient.

- i) Either $N$ is metrizable or $N$ is completely metrizable.
- ii) $Y$ is a countably bi-$k$-space.

**Theorem 5.15.** Product map $h \times k : M \times R \rightarrow N \times S$ is rw-quotient map whenever $h : M \rightarrow N$ and $k : R \rightarrow S$ are tri-quotient.

**Theorem 5.16.** Suppose $h : M \rightarrow N$ and $k : N \rightarrow R$ are continuous maps then

- i) If $h$ and $k$ are tri-quotient, or bi-quotient, or quotient then $k \circ h$ is rw-quotient map.
- ii) If $k \circ h$ is tri-quotient, or bi-quotient, or quotient, or rw-quotient, or strong rw-quotient map then $k$ is rw-quotient map.

**Proof.** i) Composition of two tri-quotients, or bi quotients, or quotients are respectively tri-quotients, or bi-quotients, or quotients. Hence by Theorem 3.4 $k \circ h$ is rw-quotient map. 

ii) Refer [2](Theorem 7.1 and 3.6) or Arguments are easy. 

**Theorem 5.17.** A map $h : M \rightarrow N$ is harmonious then $h$ is rw-quotient map.

**Proof.** Arguments in [8; Proposition 3.8] says harmonious implies tri-quotient and [3, 8] gives the proof.

**Corollary 5.18.** Let $M$ be a topological space and $N$ be a set. Let $h : M \rightarrow N$ be any surjective map construct a set $Q_{\eta_{rw}} = \{V \subset N|h^{-1}(V) \in RWO(M)\}$ then obviously the collection $Q_{\eta_{rw}}$ is not necessarily topology on $N$ (Because union of two rw-open sets need not rw-open) but some time (If $M$ with topology $\eta_{rw}$ then clear that $Q_{\eta_{rw}}$ become topology on $N$) this become topology. With this topology on $N$, $h : M \rightarrow N$ become rw-quotient map and quotient map. This topology $Q_{\eta_{rw}}$ we are calling rw-quotient topology and $(N, Q_{\eta_{rw}})$ is called rw-quotient topological space.

With which we can see weak sense of gluing or identifying or joining in standard constructions like Cone, cylinder, Sphere, Mobius strip, Torus, Klein bottle and suspension etc.[1].

For standard example If $\sim$ is any equivalence relations on $M$ with $\eta_{rw}$ then canonical projection $\sigma : M \rightarrow M/ \sim$ by,

$\sigma(x) = [x]$ is clear rw-quotient map, $M/ \sim$ is rw-quotient topology space under rw-quotient topology $Q_{\eta_{rw}}$.

**Theorem 5.19.** If $M$ with topology $\eta_{rw}$, $h : M \rightarrow N$ is rw quotient map and $M^* = \{h^{-1}(\{y\}) : y \in N\}$ then $N$ is rw-homeomorphic to $M^*$.

**Proof.** $M^* = Q_{\eta_{rw}}$ topological space define the map $k : M^* \rightarrow N$, by $k([x]) = h(x)$ became rw-homeomorphism.

The rw-quotient map and rw-quotient space are finding applications in gluing in some weak sense. Discussed map and space are weak sense of gluing or identifications and weakly glued space. As seen every gluing is rw-gluing but not converse. Anyone can see the characterization notion that, If $M$ is $\eta_{rw}$ given any space $N$ and map from $M$ to $N$ then “the map is rw-quotient map if and only if it is quotient map”. Hence gluing and weak gluing are same when $M$ (involved space) is a $\eta_{rw}$ space.

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**References**


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