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Intuitionistic fuzzy β^{**} generalized continuous functions

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Abstract
The aim of this paper is to introduce β^{**} generalized continuous functions and examine their characterizations and properties in intuitionistic fuzzy topological spaces.

Keywords
Intuitionistic fuzzy topology, intuitionistic fuzzy β^{**} generalized $T_{1/2}$ space, intuitionistic fuzzy β^{**} generalized continuous functions.

AMS Subject Classification
03F55, 54A40.

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1. Introduction

Zadeh [9] introduced the notion of a fuzzy set, which is the mapping from K to the unit interval and which it has exposed the applications in various area of studies. The concept of fuzzy set was welcomed due to uncertainty and its vagueness in which ordinary set could not deal. Later the generalizations of the consideration were enacted by several researches on a fuzzy set. Intuitionistic fuzzy sets and intuitionistic fuzzy topological spaces were introduced by Atanassov [1] and Coker [2] resp., which contain both the degrees of membership and non membership subject to the dependent that their summation does not surpassing one. Intuitionistic fuzzy β^{**}GC Sets which defined initially by Sudha and Jayanthi [6]. Here we define the concept of intuitionistic fuzzy β^{**} generalized continuous(cont.)functions and examine their characterizations and properties.

2. Preliminaries

Definition 2.1. *[1] A set E of the form $E = \{\langle k, \mu_E(k), \nu_E(k)\rangle : k \in K\}$ is an intuitionistic fuzzy set(IFSet) where the functions $\mu_E : K \to [0,1]$ and $\nu_E : K \to [0,1]$ denote the degree of membership $(\mu_E(k))$ and the degree of non-membership $(\nu_E(k))$ of $k \in K$ to the set E resp., and $0 \leq \mu_E(k) + \nu_E(k) \leq 1$ for each $k \in K$.*

Definition 2.2. *[1] Let C and D be IFSets where of $C = \{\langle k, \mu_C(k),$
$\nu_C(k)\rangle : k \in K\}$ and $D = \{\langle k, \mu_D(k), \nu_D(k)\rangle : k \in K\}$, then*

- $C \subseteq D$ iff $\mu_C(k) \leq \mu_D(k)$ and $\nu_C(k) \geq \nu_D(k) \ \forall \ k \in K$,
- C = D iff $C \subseteq D$and$C \supseteq D$,
- $C^c = \{\langle k, \nu_C(k), \mu_C(k) : k \in K\rangle\}$
- $C \cup D = \{\langle k, \mu_C(k) \vee \mu_D(k), \nu_C(k) \wedge \nu_D(k) : k \in K\rangle\}$
- $C \cap D$ =$\{\langle k, \mu_C(k) \wedge \mu_D(k), \nu_C(k) \vee \nu_D(k) : k \in K\rangle\}$

The IFSets $0_{\sim}$ = $\langle k, 0, 1\rangle$ and $1_{\sim}$ = $\langle k, 1, 0\rangle$ resp., the empty set and the whole set of K.

Definition 2.3. *[2] An intuitionistic fuzzy topology (IFT) on K is a collection τ of IFSets in K with the below axioms:*

- $0_{\sim}, 1_{\sim} \in \tau$

- $O_1 \cap O_2 \in \tau$ for any $O_1, O_2 \in \tau$
- $\cup O_i \in \tau$ for any family $\{O_i :\in I\} \subseteq \tau$

Here (K, τ) is called the intuitionistic fuzzy topological space (IFTSpace) and any IFSet in τ is an intuitionistic fuzzy open set in K. Its complement is called an intuitionistic fuzzy closed set (IFCSet) in K.

Definition 2.4. *[8] The IFSets E and D are q-coincident(E_qD) iff there exists an element $k \in K$ s.t $\mu_A(k) > \nu_B(k)$ or $\nu_A(k) < \mu_B(k)$*

Definition 2.5. *[3] An intuitionistic fuzzy point(IFPoint), written as $r_{(\alpha,\beta)}(k)$ is an IFSet of K given by*

$$r_{(\alpha,\beta)}(k) = \begin{cases} (\alpha,\beta) & \text{if } k = p, \\ (0,1) & \text{otherwise} \end{cases}$$

Definition 2.6. *[6] An IFSet E of an IFTSpace $(K, \tau)$is an intuitionistic fuzzy IFβ^{**} generalized closed set (IFβ^{**}GCSet) if $cl(int(cl(A)))\cap int(cl(int(A)))\subseteq O$ whenever $A \subseteq O$ and O is an IFOSet in (K, τ). its complement is an intuitionistic fuzzy β^{**} generalized open set (IFβ^{**}GOSet) in K.*

Definition 2.7. *[4] Let $s : (K, \tau) \to (P, \sigma)$. Then s is said to be an intuitionistic fuzzy cont. function if $s^{-1}(D) \in IFO(K)\ \forall\ D \in \sigma$*

3. Intuitionistic fuzzy β^{**} generalized continuous functions

Here we have defined intuitionistic fuzzy β^{**} generalized cont.functions and examine the existence of their properties. Also we have established the relation between the newly introduced functions and the already existing functions.

Definition 3.1. *A function $s : (K, \tau) \to (P, \sigma)$ is said to be an intuitionistic fuzzy β^{**} generalized continuous (IFβ^{**}G cont.) function if $s^{-1}(D)$ is an IFβ^{**}GC Set in $(K, \tau)\forall$ IFCSet D of (P, σ).*

Example 3.2. *Let $K = \{c, d\}$ and $P = \{e, f\}$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ IFTs on K and P resp., where $G_1 = \langle k, (0.5_c, 0.4_d), (0.5_c, 0.6_d)\rangle$, $G_2 = \langle k, (0.8_c, 0.6_d), (0.2_c, 0.4_d)\rangle$, $G_3 = \langle p, (0.3_e, 0.4_f), (0.5_e, 0.6_f)\rangle$. Define a function $s : (K, \tau) \to (P, \sigma)$ by $s(c) = e$ and $s(d) = f$. Here s is an IFβ^{**}G cont. function.*

Proposition 3.3. *Every IF cont.function [4] is an IFβ^{**}G cont.function but the reverse is not true in general.*

Proof. Let $s : (K, \tau) \to (P, \sigma)$ be an IF cont.function. Let D be an IFCSet in P. Then $s^{-1}(D)$ is an IFCSet in K and $s^{-1}(D)$ is an IFβ^{**}C Set in K. Finally s is an IFβ^{**}G cont.function. □

Example 3.4. *In Eg. 3.2, s is not an IF cont.function.*

Proposition 3.5. *Every IFSemi cont. function [5] is an IFβ^{**}G cont.function but the reverse is not true in general.*

Proof. Let $s : (K, \tau) \to (P, \sigma)$ be an IFSemi cont. function. Let D be an IFCSet in P. Then $s^{-1}(D)$ is an IFSCSet in K and $s^{-1}(D)$ is an IFβ^{**}GCSet in K. Finally s is an IFβ^{**}G. cont.function. □

Example 3.6. *In Eg. 3.2, s is an IFβ^{**}G cont.function but not an IFSemi cont.function.*

Proposition 3.7. *Every IFPrecont.function [5] is an IFβ^{**}G cont.function but the reverse is not true in general.*

Proof. Let $s : (K, \tau) \to (P, \sigma)$ be an IFPre cont.function. Let D be an IFCSet in P. Then $s^{-1}(D)$ an IFPCSet in K and $s^{-1}(D)$ is an IFβ^{**}GCSet in K. Finally s is an IFβ^{**}G. cont.function. □

Example 3.8. *Let $K = \{c, d\}$, $P = \{e, f\}$, and $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ and IFTs on K and P resp., where $G_1 = \langle k, (0.5_c, 0.6_d), (0.5_c, 0.4_d)\rangle$, $G_2 = \langle k, (0.4_c, 0.3_d), (0.6_c, 0.7_d)\rangle$, $G_3 = \langle p, (0.6_e, 0.6_f), (0.4_e, 0.4_f)\rangle$, Define a function $s : (K, \tau) \to (P, \sigma)$ by $s(c) = e$ and $s(d) = f$. Then s is an IFβ^{**}G cont.function but not an IFPre cont.function*

Proposition 3.9. *Every IFα cont.function [5] is an IFβ^{**}G cont.function but the reverse is not true in general.*

Proof. Let $s : (K, \tau) \to (P, \sigma)$ be an IFα cont.function. Let D be an IFCSet in P. Then $s^{-1}(D)$ is an IFαCSet in K. Since every IFαCSet is an IFβ^{**}GC Set [5], $s^{-1}(D)$) is an IFβGCSet in K. Finally s is an IFβ^{**}G cont.function. □

Example 3.10. *In Eg. 3.2, s is an IFβ^{**}G cont.function but not an IFα cont.function*

Proposition 3.11. *Let $s : (K, \tau) \to (P, \sigma)$ be a function and $s^{-1}(E)$ be an IFRCSet in K $\forall$ IFCSet E in P. Then s is an IFβ^{**}G cont.function but reverse is not true in general.*

Proof. Let E be an IFCSet in P and $s^{-1}(E)$ be an IFRCSet in K. Since every IFRCSet is an IFβ^{**}GCSet [6], $s^{-1}(E)$ is an IFβ^{**}GCSet in K. Finally s is an IFβ^{**}G cont. function. □

Example 3.12. *In Eg 3.2, s is an IFβ^{**}G cont.function but not a function as in proposition 3.11.*

Proposition 3.13. *Every IFβ cont.function [5] is an IFβ^{**}G cont.function but the reverse is not true in general.*

Proof. Let s: (K, τ) →(P,σ) be an IFβ cont.function. Let D be an IFCSet in P. Then $s^{-1}(D)$ is an IFβCSet in K. Since every IFβCSet [8], $s^{-1}(D)$ is an IFβ^{**}GCSet in K.Finally s is an IFβ^{**}G cont. function. □

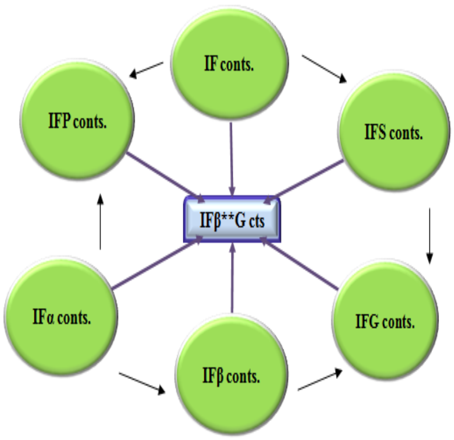
Example 3.14. *Let $K = \{c, d\}$ and Let $P = \{e, f\}$ Then $\tau = \{0_\sim,\ G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ be IFTs on K and P resp., where $G_1 = \langle k, (0.5_c, 0.6_d), (0.5_c, 0.4_d)\rangle$, $G_2 = \langle k, (0.6_c, 0.7_d), (0.4_c, 0.3_d)\rangle$, $G_3 = \langle p, (0.3_e, 0.2_f), (0.7_e, 0.8_f)\rangle$, Define a function s : (K, τ) →(P,σ)by s(c) = e and s(d) = s. Then s is an IFβ^{**}G cont.function but not an IFβ cont. function.*

Proposition 3.15. *Every IFG cont.function [7] is an IFβ^{**}G cont.function but thereverse is not true in general.*

Proof. Let s : (K, τ) →(P,σ) be an IFG cont.function. Let D be an IFCSet in P. Then $s^-1(D)$ is an IFGCSet in K. Since every IFGCSet is an IFβ^{**}GCSet [6], $s^{-1}(D)$ is an IFβ^{**}GCSet in K. Thus s is an IFβ^{**}G cont.function. □

Example 3.16. *In eg 3.2, s is an IFβ^{**}G cont.function and not an IFG cont.function,since G_3^c is an IFCSet in P, but $s^{-1}(G_3^c)$is not an IFGCSet in K,as $cl(s^{-1}(G_3^c)) = 1_\sim \nsubseteq G_2$,whereas $g^{-1}(G_3^c) \subseteq G_2$*

The interrelation of IF cont.function is depicted below.



Proposition 3.17. *A function $s:(K,\tau)\to(P,\sigma)$ be an IFβ^{**}G cont.function iff the inverse image of each IFOSet in P is an IFβ^{**}GOSet in K.*

Proof. **Necessity :** Let E be an IFOSet in P. Then E^c is an IFCSet in P. Since s is an IFβ^{**}G cont.function, $s^{-1}(E^c)$ is an IFβ^{**}GCSet in K. Since $s^{-1}(E^c) = (s^{-1}(E))^c, s^{-1}(E)$ is an IFβ^{**}GOSet in K.
Sufficiency : Let E be an IFCSet in P. Then E^c is an IFOSet in P. By assumption, $s^{-1}(E^c)$ is an IFβ^{**}GOSet in K. Since $s^{-1}(E^c) = (s^{-1}(E))^c, s^{-1}(E)$ is an IFβ^{**}GCSet in K. Hence s is an IFβ^{**}G cont.function. □

Proposition 3.18. *If $s:(K,\tau)\to(P,\sigma)$ is an IFβ^{**}G cont.function, then for each IFPoint $r(\alpha,\beta)$ of K and each $E\in\sigma$ s.t $s(r(\alpha,\beta))\in E$, there is an IFβ^{**}GOSet D of K s.t $r(\alpha,\beta)\in D$ and $s(D)\subseteq E$.*

Proof. Let $r(\alpha,\beta)$ be an IFPoint of K and $E\in\sigma$ s.t $s(r(\alpha,\beta))\in E$. Then $r(\alpha,\beta)\in s^{-1}(E)$. Put $D = s^{-1}(E)$. By assumption, D is anIFβ^{**}GOSet in K s.t $r(\alpha,\beta)\in D$ and $s(D) = s(s^{-1}(E))\subseteq E$. □

Proposition 3.19. *If $s:(K,\tau)\to(P,\sigma)$ is an IFβ^{**}GOSet cont.function, then for each IFPoint $r(\alpha,\beta)$ of K and each $E\in\sigma$ s.t $s(r(\alpha,\beta))_qE$, there is an IFβ^{**}GOS D of K s.t $r(\alpha,\beta)_qD$ and $s(D)\subseteq E$.*

Proof. Let $r(\alpha,\beta)$ be an IFPoint of K and $E\in\sigma$ s.t $s(r(\alpha,\beta))_qE$. Then $r(\alpha,\beta)_qs^{-1}(E)$. Put $D = s^{-1}(E)$. Then by assumption, D is an IFβ^{**}GOSet in K s.t $r(\alpha,\beta)_qD$ and $s(D) = s(s^{-1}(E))\subseteq E$. □

Proposition 3.20. *Let $s:(K,\tau)\to(P,\sigma)$ be an IFβ^{**}G cont.function, then s is an IFpre cont.function if K is an IFuzzy$\beta^{**}pT_{1/2}$ Space*

Proof. Let D be an IFCSet in P. Then $s^{-1}(D)$ is an IFβ^{**}GCSet in K, by assumption. Since K is an IFuzzyβ^{**}p$T_{1/2}$ space, $s^1(D)$ is an IFPCSet in K. Hence s is an IF pre cont.function. □

Proposition 3.21. *Let $s:(K,\tau)\to(P,\sigma)$ be an IFβ^{**}G cont.function, then s is an IFG cont.function if K is an IFuzzy$\beta_g^{**}T_{1/2}$ Space*

Proof. Let D be an IFCSet in P. Then $s^{-1}(D)$ is an IFβ^{**}GCSet in K, by assumption. Since K is an IFuzzyβ^{**}g $T_{1/2}$ space, $s^{-1}(D)$ is an IFGCSet in K. Hence s is an IFG cont.function. □

Proposition 3.22. *Let $s:(K,\tau)\to(P,\sigma)$ be an IFβ^{**}G cont.function and $t:(P,\sigma)\to(Q,\delta)$ is an IF cont.function then $t\circ s:(K,\tau)\to(Q,\delta)$ is an IFβ^{**}G cont.function.*

Proof. Let D be an IFCSet in Q. Then $t^{-1}(D)$ is an IFCSet in P, by assumption. Since s is an IFβ^{**}G cont.function, $s^{-1}(t^{-1}(D))$ is an IFβ^{**}GCSet in K. Hence tos is an IFβ^{**}G cont.function as $(tos)^{-1}(D) = s^{-1}(t^{-1}(D))$. □

Proposition 3.23. *A function $s:(K,\tau)\to(P,\sigma)$ be an IFβ^{**}G cont.function if $cl(int(cl(s^{-1}(E))))\subseteq s^{-1}(cl(E))$ for every IFSet E in P.*

Proof. Let E be an IFCSet in P. By assumption, $cl(int(cl(s^{-1}(E))))\subseteq s^{-1}(cl(E)) = s^{-1}(E)$ And $s^{-1}(E)$ is an IFαCSet in K. Then $s^{-1}(E)$ is an IFβ^{**}GCSet in K and s is an IFβ^{**}G cont.function. □

Proposition 3.24. *Let $s:(K,\tau)\to(P,\sigma)$ be a function. Then the following conditions are equivalent if K and P are IFuzzy$\beta_p^{**}T_{1/2}$ space:*

(a) s is an IFβ^{**}G cont.function

(b) $s^{-1}(D)$ is an IFβ^{**}GOSet in K for each IFOSet D in P

(c) for each IFPoint $r(\alpha,\beta)$ in K and $\forall$ IFOSet D in P s. t $s(r(\alpha,\beta))\in D$, there is an IFβ^{**}GOSet E in K s.t $r(\alpha,\beta)\in E$ and $s(E)\subseteq D$.

Proof. (a) ⇔ (b): is clearly known from the Proposition 3.17.
(b) ⇒ (c): Let D be any IFOSet in K and P and let $r(\alpha,\beta)\in K$. Given $s(r(\alpha,\beta))\in D$. By assumption $s^{-1}(D)$ is an IFβ^{**}GOSet in K. Take E = $s^{-1}(D)$. Then $r(\alpha,\beta)\in s^{-1}(D) = E$ and $r(\alpha,\beta)\in E$ and $s(E) = s(s^{-1}(D))\subseteq D$.
(c) ⇒ (a): Let E be an IFCSet in P. Then $D = E^c$ is an IFOSet in P. Let $r(\alpha,\beta)\in K$ and $s(r(\alpha,\beta))\in D$. Then

IFβ^{**}GOSet, say F exists in K s.t $r(\alpha,\beta) \in F$ and $s(F) \subseteq D$. Therefore $r(\alpha,\beta) \in F \subseteq s^{-1}(s(F)) \subseteq s^{-1}(D)$ and hence $s^{-1}(D)$ is an IFβ^{**}GOSet in K [6]. Finally s is an IFβ^{**}G cont.function. □

Proposition 3.25. *Let $s : (K,\tau) \to (P,\sigma)$ be a function that satisfies $s^{-1}(int(B)) = int(cl(s^{-1}(int(B)))\forall$ IFSet D in P. Then s is an IFβ^{**}G cont.function.*

Proof. Let $D \subseteq P$ be an IFOSet.Then by assumption $s^{-1}(D) = int(cl(s^{-1}(D)))$. It gives $s^{-1}(D)$ is an IFROSet in K and it is an IFβ^{**}GOSet in K. Finally s is an IFβ^{**}G cont.function. □

Proposition 3.26. *Let $s : (K,\tau) \to (P,\sigma)$ be an IFβ^{**}G cont.function and $m : (P,\sigma) \to (Q,\delta)$ is an IFG cont.function and P is an IFuzzy$T_{1/2}$space, then $m \circ s : (K,\tau) \to (Q,\delta)$ is an IFβ^{**}G cont.function.*

Proof. Let D be an IFCSet in Q. Then $t^{-1}(D)$ is an IFGCSet in P, by assumption. Since P is an IFuzzy$T_{1/2}$ space, $t^{-1}(D)$ is an IFCSet in P. Therefore $s^{-1}(t^{-1}(D))$ is an IFβ^{**}GCSet in K, by assumption. Finally $m \circ s$ is an IFβ^{**}G cont.function. □

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