Gaussian anti magic labeling in some special graphs

K. Thirusangu\(^1\)* and A. Selvaganapathy\(^2\)

**Abstract**

In this paper we examine the existence of Gaussian antimagic labeling for some special graphs such as Middle graph of path, Split graph of path, Shadow graph of path, Total path graph, Middle cycle graph, Ladder graph, Square comb, Splitting comb graph, Theta graph, Sum of two copies of Theta graph by the path of length two, Cubic graph and Path union of two copies of Cubic graph.

**Keywords**

Graph labeling, magic graphs, antimagic graphs, Gaussian antimagic graphs.

**AMS Subject Classification**

05C78.

\(^1\)Department of Mathematics, S.I.VE.T College, Gowrivakkam, Chennai-600073, India.

\(^2\)Department of Mathematics, SRM Arts and Science College, Chennai-603203, India.

*Corresponding author: \(^1\)kthirusangu@gmail.com; \(^2\)selva78.chennai@gmail.com

**Article History:** Received 01 January 2020; Accepted 12 February 2020

---

1. Introduction

The concept of graph labeling was introduced by Rosa [2] in 1967. Graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. In the intervening years various labeling of graphs such as graceful labeling, harmonious labeling, magic labeling, antimagic labeling, bimagic labeling, prime labeling, cordial labeling, total cordial labeling, k-graceful labeling and odd graceful labeling etc., have been studied in over 2300 papers[1].

The concept of Gaussian antimagic labeling was recently introduced in [3]. It is proved that graphs such as paths, cycles, Y-tree, comb, triangular snake and chordal graphs are Gaussian antimagic. In this paper we prove that Middle graph of path, Split graph of path, Shadow graph of path, Total path graph, Middle cycle graph, Ladder graph, Square comb, Splitting comb graph, Theta graph, Sum of two copies of Theta graph by the path of length two, Cubic graph and Path union of two copies of Cubic graph are Gaussian antimagic.

---

2. Preliminaries

In this section we give the basic notions relevant to this paper.

**Definition 2.1.** The middle graph of path G is the graph whose vertex set is \(V(G) \cup E(G)\) and in which two vertices are adjacent if and only if either they are adjacent edges of G (or) one is a vertex of G and other is an edge incident on it.

**Definition 2.2.** The middle graph of cycle \(C_m\) is the graph whose vertex set is \(V(G) \cup E(G)\) and in which two vertices are adjacent if and only if either they are adjacent edges of G (or) one is a vertex of G and other is an edge incident on it.

**Definition 2.3.** The Total graph \(T(P_n)\) of a graph S is the graph whose vertex set is \(V \cup E\) and two vertices are adjacent whenever they are either adjacent or incident in S.

**Definition 2.4.** For a graph G the split graph is obtained by adding to each vertex v a new vertex such that v’ is adjacent to every vertex that is adjacent to v in G. The resultant graph is denoted as spl(G).

**Definition 2.5.** The shadow graph \(D_2(G)\) of a connected graph G is constructed by taking two copies of G say G’ and G”. Join each vertex v’ in G’ to the neighbors of the corresponding vertex v” in G”.

**Definition 2.6.** A Comb is a caterpillar in which each vertex in the path is joined to exactly one pendant vertex. The square comb is a caterpillar in which each vertex in the path is joined to exactly two pendant vertices.
comb graph \((P_6 \odot K_2)\) is a graph on the same vertex set but in which two vertices are adjacent if and only if they are at distance at most 2 in \(G\).

**Definition 2.7.** The Ladder graph \(L_m\) is a planar undirected graph with \(2m\) vertices and \(3m - 2\) edges.

**Definition 2.8.** A block with two non-adjacent vertices of degree 3 and all other vertices of degree 2 is called a Theta graph.

**Definition 2.9.** Let \(G\) be the graph obtained by the path union of two copies \(S'_m\) and \(S''_m\) of Theta graph \(S_m\).

**Definition 2.10.** A cubic graph is a graph in which all vertices have degree three. Cubic graphs are also called trivalent graphs.

**Definition 2.11.** Let \(S_p\) be the graph obtained by taking the path union of two copies of cubic graphs with \(2m\) vertices \(S'\) and \(S''\) respectively.

**Definition 2.12.** Gaussian antimagic labeling in a \((p,q)\) graph is a function \(f: V \rightarrow \{a + ib/a, b \in \mathbb{N}\} \leq a \leq b \leq q\) such that the induced function \(f^*: E \rightarrow \mathbb{N}\) defined by \(f^*(uv) = |f(u)|^2 + |f(v)|^2\) results all the edge labels are distinct. A graph which admits Gaussian antimagic labeling is called Gaussian antimagic graph.

**Theorem 3.1.** The middle graph of \(D_2(P_6)\) admits Gaussian antimagic labeling.

**Proof.** Let \(V = \{v_1, v_2, v_3, \ldots, v_{m+2}\}\) be the vertices and \(E = \{v_kv_{k+1}/1 \leq k \leq (m-3)/2, m \in \mathbb{N}\} \cup \{v_{k+1}v_{k+2}/0 \leq k \leq m-1, m \in \mathbb{N}\} \cup \{v_{m+1}v_{m+2}/1 \leq k \leq m-1, m \in \mathbb{N}\}\) be the edges of the middle graph of \(P_6\).

Define a function \(f: V \rightarrow \{a + ib/a, b \in \mathbb{N}\} \leq a \leq b \leq q\) such that \(f(v_i) = s + (s+1)i, 1 \leq s \leq n\).

Define the induced function \(f^*: E \rightarrow \mathbb{N}\) such that \(f^*(uv) = |f(u)|^2 + |f(v)|^2\).

The edge labels are obtained as follows:

- \(f^*(v_{k+1}v_{k+2}) = 16k^2 + 40k + 30, 0 \leq k \leq (m-3)/2, m \in \mathbb{N}\)
- \(f^*(v_{k+2}v_{k+3}) = 16k^2 + 40k + 30, 0 \leq k \leq (m-3)/2, m \in \mathbb{N}\)
- \(f^*(v_{m+1}v_{m+2}) = 4m^2 + 12m + 14, m \in \mathbb{N}\)
- \(f^*(v_{m+2}v_{m+3}) = 16k^2 + 40k + 30, 0 \leq k \leq (m-3)/2, m \in \mathbb{N}\)

Thus \(f^*(E) = \{30, 86, 174, \ldots, 4m^2 - 4m + 6, 18, 38, 66, \ldots, 4m^2 + 2\} \cup \{2m^2 + 6m + 10, 4m^2 + 12m + 14\}\) in which all the elements are distinct. Therefore, the middle graph of \(P_6\) admits Gaussian antimagic labeling.

**Theorem 3.2.** The Split graph of \(P_6\) admits Gaussian antimagic labeling.

**Proof.** Let \(V = \{v_1, v_2, v_3, \ldots, v_{2m}\}\) be the vertices and \(E = \{v_kv_{k+1}/1 \leq k \leq 2m-1, m \in \mathbb{N}\} \cup \{v_{k+1}v_{k+2}/0 \leq k \leq 2m-1, m \in \mathbb{N}\} \cup \{v_{m+k}v_{m+k+1}/1 \leq k \leq m-1, m \in \mathbb{N}\}\) be the edges of the Split graph of \(P_6\).

Define a function \(f: V \rightarrow \{a + ib/a, b \in \mathbb{N}, b = a + 1, 1 \leq a \leq n\} m \in \mathbb{N}\) such that \(f(v_i) = s + (s+1)i, 1 \leq s \leq n\).

Define the induced function \(f^*: E \rightarrow \mathbb{N}\) such that \(f^*(uv) = |f(u)|^2 + |f(v)|^2\).

The edge labels are obtained as follows:

- \(f^*(v_{k+1}v_{k+2}) = 16k^2 + 40k + 30, 0 \leq k \leq (m-3)/2, m \in \mathbb{N}\)
- \(f^*(v_{k+2}v_{k+3}) = 16k^2 + 40k + 30, 0 \leq k \leq (m-3)/2, m \in \mathbb{N}\)
- \(f^*(v_{m+1}v_{m+2}) = 4m^2 + 12m + 14, m \in \mathbb{N}\)
- \(f^*(v_{m+2}v_{m+3}) = 16k^2 + 40k + 30, 0 \leq k \leq (m-3)/2, m \in \mathbb{N}\)

Thus \(f^*(E) = \{30, 86, 174, \ldots, 4m^2 - 4m + 6, 18, 38, 66, \ldots, 4m^2 + 2\} \cup \{2m^2 + 6m + 10, 4m^2 + 12m + 14\}\) in which all the elements are distinct. Therefore, the middle graph of \(P_6\) admits Gaussian antimagic labeling.

**Theorem 3.3.** The Shadow graph of \(D_2(P_6)\) admits Gaussian antimagic labeling.

**Proof.** Let \(V = \{v_1, v_2, v_3, \ldots, v_{2m}\}\) be the vertices and \(E = \{v_kv_{k+1}/1 \leq k \leq m-1, m \in \mathbb{N}\} \cup \{v_{k+1}v_{k+2}/0 \leq k \leq m-2, m \in \mathbb{N}\} \cup \{v_{m+k}v_{m+k+1}/1 \leq k \leq (m-2)/2, m \in \mathbb{N}\} \cup \{v_{m+k+1}v_{m+k+2}/1 \leq k \leq (m-1)/2, m \in \mathbb{N}\}\) be the edges of the Shadow graph of \(D_2(P_6)\).

Define a function \(f: V \rightarrow \{a + ib/a, b \in \mathbb{N}, b = a + 1, 1 \leq a \leq n\} m \in \mathbb{N}\) such that \(f(v_i) = s + (s+1)i, 1 \leq s \leq n\).

Define the induced function \(f^*: E \rightarrow \mathbb{N}\) such that \(f^*(uv) = |f(u)|^2 + |f(v)|^2\).

The edge labels are obtained as follows:

- \(f^*(v_{k+1}v_{k+2}) = 16k^2 + 40k + 30, 0 \leq k \leq (m-3)/2, m \in \mathbb{N}\)
- \(f^*(v_{k+2}v_{k+3}) = 16k^2 + 40k + 30, 0 \leq k \leq (m-3)/2, m \in \mathbb{N}\)
- \(f^*(v_{m+k}v_{m+k+1}) = 16k^2 + 40k + 30, 0 \leq k \leq (m-3)/2, m \in \mathbb{N}\)
- \(f^*(v_{m+k+1}v_{m+k+2}) = 16k^2 + 40k + 30, 0 \leq k \leq (m-3)/2, m \in \mathbb{N}\)

Thus \(f^*(E) = \{30, 86, 174, \ldots, 4m^2 - 4m + 6, 18, 38, 66, \ldots, 4m^2 + 2\} \cup \{2m^2 + 6m + 10, 4m^2 + 12m + 14\}\) in which all the elements are distinct. Therefore, the middle graph of \(P_6\) admits Gaussian antimagic labeling.
Theorem 3.4. The Total path graph admits Gaussian antimagic labeling.

Proof. Let \( V = \{v_1, v_2, v_3, \ldots, v_{2m-1}\} \) be the vertices and \( E = \{v_{2k}v_{k+1}/1 \leq k \leq m-1, m \in N\} \cup \{v_{2k+1}/m + k \leq 2m - 2, m \in N\} \cup \{v_{2m+k+1}/1 \leq m-1, m \in N\} \cup \{v_{m+k}v_{k+1}/1 \leq k \leq m-1, m \in N\} \) be the edges of the Total path graph.

Define a function \( f : V \rightarrow \{a + ib/a, b \in N, b = a + 1, 1 \leq a \leq n\} \)

\[ f(v_i) = s + (s + 1)i, 1 \leq s \leq n \]

Define the induced function \( f^* : E \rightarrow \{v_i \} \)

\[ f^*(uv) = |f(u)|^2 + |f(v)|^2 \]

The edge labels are obtained as follows:

\[ f^*(v_k, v_{k+1}) = 4k^2 + 8k + 6, 1 \leq k \leq m - 1, m \in N \]

\[ f^*(v_k, v_{k+1}) = 4k^2 + 8k + 6, m + 1 \leq k \leq 2m - 2, m \in N \]

\[ f^*(v_k, v_{m+k}) = 4k^2 + 4k + 2m^2 + 4mk + 2m + 2, 1 \leq k \leq m - 1, m \in N \]

\[ f^*(v_{m+k}v_{k+1}) = 4k^2 + 8k + 2m^2 + 4mk + 2m + 6, 1 \leq k \leq m - 1, m \in N \]

Thus \( f^*(E) = \{18, 38, 66, \ldots, 4m^2 + 2\}, \{4m^2 + 16m + 18, 4m^2 + 3m, \ldots, 16m^2 - 16 + 6\}, \{2m^2 + 6m + 10, 2m^2 + 10m + 26, \ldots, 10m^2 - 6m + 2\}, \{2m^2 + 6m + 10, 2m^2 + 10m + 26, \ldots, 10m^2 + 6m + 2\} \]

in which all the elements are distinct. Therefore, the Middle cycle graph admits Gaussian antimagic labeling.

Theorem 3.5. The Middle cycle graph admits Gaussian antimagic labeling.

Proof. Let \( V = \{v_1, v_2, v_3, \ldots, v_{2m}\} \) be the vertices of the Middle cycle graph.

Define a function \( f : V \rightarrow \{a + ib/a, b \in N, b = a + 1, 1 \leq a \leq n\} \)

\[ f(v_i) = s + (s + 1)i, 1 \leq s \leq n \]

Define the induced function \( f^* : E \rightarrow \{v_i \} \)

\[ f^*(uv) = |f(u)|^2 + |f(v)|^2 \]

The edge labels are obtained as follows:

\[ f^*(v_k, v_{k+1}) = 4k^2 + 8k + 6, 1 \leq k \leq m - 1, m \in N \]

\[ f^*(v_k, v_{k+1}) = 4k^2 + 8k + 6, m + 1 \leq k \leq 2m - 2, m \in N \]

\[ f^*(v_k, v_{m+k}) = 4k^2 + 4k + 2m^2 + 4mk + 2m + 2, 1 \leq k \leq m - 1, m \in N \]

\[ f^*(v_{m+k}v_{k+1}) = 4k^2 + 8k + 2m^2 + 4mk + 2m + 6, 1 \leq k \leq m - 1, m \in N \]

Thus \( f^*(E) = \{18, 38, 66, \ldots, 4m^2 + 2\}, \{4m^2 + 16m + 18, 4m^2 + 3m, \ldots, 16m^2 - 16 + 6\}, \{2m^2 + 6m + 10, 2m^2 + 10m + 26, \ldots, 10m^2 + 6m + 2\} \]

in which all the elements are distinct. Therefore, the Middle cycle graph admits Gaussian antimagic labeling.

Theorem 3.6. The Ladder graph admits Gaussian antimagic labeling.

Proof. Let \( V = \{v_1, v_2, v_3, \ldots, v_{2m}\} \) be the vertices and \( E = \{v_{2k}v_{k+1}/1 \leq k \leq m - 1, m \in N\} \cup \{v_{2k+1}/m + k \leq 2m - 1, m \in N\} \cup \{v_{2m+k+1}/1 \leq k \leq m, m \in N\} \)

be the edges of the Ladder graph. Define a function \( f : V \rightarrow \{a + ib/a, binN, b = a + 1, 1 \leq a \leq n\} \)

\[ f(v_i) = s + (s + 1)i, 1 \leq s \leq n \]

Define the induced function \( f^* : E \rightarrow \{v_i \} \)

\[ f^*(uv) = |f(u)|^2 + |f(v)|^2 \]

The edge labels are obtained as follows:

\[ f^*(v_k, v_{k+1}) = 4k^2 + 8k + 6, 1 \leq k \leq m - 1, m \in N \]

\[ f^*(v_k, v_{k+1}) = 4k^2 + 8k + 6, m + 1 \leq k \leq 2m - 1, m \in N \]

\[ f^*(v_k, v_{m+k}) = 4k^2 + 4k + 2m^2 + 4mk + 2m + 2, 1 \leq k \leq m, m \in N \]

Thus \( f^*(E) = \{18, 38, 66, \ldots, 4m^2 + 2\}, \{4m^2 + 16m + 18, 4m^2 + 3m, \ldots, 16m^2 - 16 + 6\}, \{2m^2 + 6m + 10, 2m^2 + 10m + 26, \ldots, 10m^2 + 6m + 2\} \]

in which all the elements are distinct. Therefore, the Ladder graph admits Gaussian antimagic labeling.

Theorem 3.7. The Square comb graph admits Gaussian antimagic labeling.

Proof. Let \( V = \{v_1, v_2, v_3, \ldots, v_{2m}\} \) be the vertices and \( E = \{v_{2k}v_{k+1}/1 \leq k \leq m - 1, m \in N\} \cup \{v_{2k+1}/m + k \leq 2m - 1, m \in N\} \cup \{v_{2m+k+1}/1 \leq k \leq m, m \in N\} \cup \{v_{2k+1}v_{k+1}/0 \leq k \leq (m - 4)/2, m \in N\} \cup \{v_{2k+1}v_{k+1}/1 \leq k \leq (m - 2)/2, m \in N\} \)

be the edges of the Square comb graph. Define a function \( f : V \rightarrow \{a + ib/a, b \in N, b = a + 1, 1 \leq a \leq n\} \)

\[ f(v_i) = s + (s + 1)i, 1 \leq s \leq n \]

Define the induced function \( f^* : E \rightarrow \{v_i \} \)

\[ f^*(uv) = |f(u)|^2 + |f(v)|^2 \]

The edge labels are obtained as follows:

\[ f^*(v_k, v_{k+1}) = 4k^2 + 8k + 6, 1 \leq k \leq m - 1, m \in N \]

\[ f^*(v_k, v_{k+1}) = 4k^2 + 8k + 6, m + 1 \leq k \leq 2m - 1, m \in N \]

\[ f^*(v_k, v_{m+k}) = 4k^2 + 4k + 2m^2 + 4mk + 2m + 2, 1 \leq k \leq m, m \in N \]

\[ f^*(v_{m+k}v_{k+1}) = 4k^2 + 8k + 4mk + 2m^2 + 6m + 6, 1 \leq k \leq m, m \in N \]

\[ f^*(v_{m+k}v_{k+1}) = 4k^2 + 8k + 4mk + 2m^2 + 6m + 6, 1 \leq k \leq m - 1, m \in N \]

\[ f^*(v_{m+k}v_{k+1}) = 16k^2 + 40k + 30, 0 \leq k \leq (m - 4)/2, m \in N \]

Thus \( f^*(E) = \{18, 38, 66, \ldots, 4m^2 + 2\}, \{4m^2 + 16m + 18, 4m^2 + 3m, \ldots, 16m^2 - 16 + 6\}, \{2m^2 + 6m + 10, 2m^2 + 10m + 26, \ldots, 10m^2 + 6m + 2\} \)

in which all the elements are distinct. Therefore, the Ladder graph admits Gaussian antimagic labeling.
Theorem 3.8. The splitting comb graph admits Gaussian antimagic labeling.

Proof. Let $V = \{v_{1}, v_{2}, v_{3}, \ldots, v_{m}\}$ be the vertices and $E = \{v_{i}v_{i+1}/1 \leq k \leq m-1, m \in N\} \cup \{v_{i}v_{m+k}/1 \leq k \leq m, m \in N\} \cup \{v_{m+k}v_{2m+k}/1 \leq k \leq m, m \in N\} \cup \{v_{m+k}v_{m+k+1}/1 \leq k \leq m-1, m \in N\} \cup \{v_{m+k}v_{m+k+1}/1 \leq k \leq m, m \in N\} \cup \{v_{1}v_{m+k}v_{n+k+1}/1 \leq k \leq m-1, m \in N\}$ be the edges of the splitting comb graph. Define a function $f: V \rightarrow \{a + ib/a, b = a + 1, 1 \leq a \leq n\}$ such that $f(v_{i}) = s + (s+1)i, 1 \leq s \leq n$. Define the induced function $f^{*}: E \rightarrow N$ such that $f^{*}(uv) = |f(u)|^{2} + |f(v)|^{2}$.

The edge labels are obtained as follows:

- $f^{*}(v_{i}v_{i+1}) = 4k^{2} + 8k + 1, 1 \leq k \leq m-1, m \in N$
- $f^{*}(v_{i}v_{m+k}) = 4k^{2} + 8k + 8m^{2} + 8m + 2, 1 \leq k \leq m, m \in N$
- $f^{*}(v_{m+k}v_{2m+k}) = 4k^{2} + 8k + 12mk + 18m^{2} + 6m + 2, 1 \leq k \leq m, m \in N$
- $f^{*}(v_{i}v_{m+k}) = 4k^{2} + 8k + 4mk + 2m^{2} + 6m + 6, 1 \leq k \leq m-1, m \in N$
- $f^{*}(v_{m+k}v_{m+k+1}) = 4k^{2} + 8k + 4mk + 2m^{2} + 6m + 10m + 2, 1 \leq k \leq m-1, m \in N$

Thus $f^{*}(E) = \{18, 38, 66, \ldots, 4m^{2} + 2, 8m^{2} + 12m + 10, 8m^{2} + 30m + 26, \ldots, 2m^{2} + 8m + 2, 18m^{2} + 18m + 10, 18m^{2} + 30m + 26, \ldots, 34m^{2} + 10m + 2, 10m^{2} + 18m + 10, 10m^{2} + 30m + 26, \ldots, 26m^{2} + 10m^{2} + 2m^{2} + 10m + 18, 2m^{2} + 14m + 38, \ldots, 10m^{2} - 10m^{2} + 6m + 18, 2m^{2} + 14m + 38, \ldots, 10m^{2} - 2m^{2} + 2, 10m^{2} + 2m^{2} + 6m + 2, 10m^{2} + 2m^{2} + 6m + 38, \ldots, 10m^{2} - 2m^{2} + 2\}$ in which all the elements are distinct. Therefore, the splitting comb graph admits Gaussian antimagic labeling.

Theorem 3.9. The Theta graph $G_{1}$ admits Gaussian antimagic labeling.

Proof. Let $V = \{v_{1}, v_{2}, \ldots, v_{m}\}$ be the vertices and $E = \{v_{i}v_{i+1}/1 \leq k \leq m-1, m \in N\} \cup \{v_{i}v_{m+k}/1 \leq k \leq m, m \in N\} \cup \{v_{m+k}v_{2m+k}/1 \leq k \leq m, m \in N\} \cup \{v_{m+k}v_{m+k+1}/1 \leq k \leq m-1, m \in N\}$ be the edges of the Theta graph $G_{1}$. Define a function $f: V \rightarrow \{a + ib/a, b = a + 1, 1 \leq a \leq n\}$ such that $f(v_{i}) = s + (s+1)i, 1 \leq s \leq n$. Define the induced function $f^{*}: E \rightarrow N$ such that $f^{*}(uv) = |f(u)|^{2} + |f(v)|^{2}$.

The edge labels are obtained as follows:

- $f^{*}(v_{i}v_{i+1}) = 4k^{2} + 8k + 6, 1 \leq k \leq 5$
- $f^{*}(v_{i}v_{i+1}) = 90$
- $f^{*}(v_{1}v_{7}) = 118$
- $f^{*}(v_{1}v_{4}) = 154$

Thus $f^{*}(E) = \{18, 38, 66, \ldots, 146, 90, 118, 154\}$ in which all the elements are distinct. Therefore, the Theta graph $G_{1}$ admits Gaussian antimagic labeling.

Theorem 3.10. The sum of two copies of Theta graph by the path of length two admits Gaussian antimagic labeling.

Proof. Let $V = \{v_{1}, v_{2}, \ldots, v_{m}\}$ be the vertices and $E = \{v_{i}v_{i+k+1}/1 \leq k \leq 5, m \in N\} \cup \{v_{i}v_{i+k}\} \cup \{v_{i}v_{i+k+1}/8 \leq k \leq 12, m \in N\} \cup \{v_{i}v_{i+k}\}$ be the edges of the sum of two copies of Theta graph by the path of length two. Define a function $f: V \rightarrow \{a + ib/a, b = a + 1, 1 \leq a \leq n\}$ such that $f(v_{i}) = s + (s+1)i, 1 \leq s \leq n$. Define the induced function $f^{*}: E \rightarrow N$ such that $f^{*}(uv) = |f(u)|^{2} + |f(v)|^{2}$.

The edge labels are obtained as follows:

- $f^{*}(v_{i}v_{i+1}) = 4k^{2} + 8k + 6, 1 \leq k \leq 5$
- $f^{*}(v_{i}v_{i+1}) = 90$
- $f^{*}(v_{1}v_{7}) = 118$
- $f^{*}(v_{1}v_{4}) = 154$

Thus $f^{*}(E) = \{18, 38, 66, \ldots, 146, 90, 118, 154\}$ in which all the elements are distinct. Therefore, the sum of two copies of Theta graph by the path of length two admits Gaussian antimagic labeling.

Theorem 3.11. The cubic graph with $2m$ vertices admits Gaussian antimagic labeling.

Proof. Let $V = \{v_{1}, v_{2}, v_{3}, \ldots, v_{2m}\}$ be the vertices and $E = \{v_{i}v_{i+k+1}/1 \leq k \leq m-1, m \in N\} \cup \{v_{i}v_{i+1}\} \cup \{v_{i}v_{i+k}/1 \leq k \leq m, m \in N\} \cup \{v_{i}v_{i+k+1}/1 \leq k \leq m-1, m \in N\}$ be the edges of the cubic graph with $2m$ vertices. Define a function $f: V \rightarrow \{a + ib/a, b = a + 1, 1 \leq a \leq n\}$ such that $f(v_{i}) = s + (s+1)i, 1 \leq s \leq n$. Define the induced function $f^{*}: E \rightarrow N$ such that $f^{*}(uv) = |f(u)|^{2} + |f(v)|^{2}$.

The edge labels are obtained as follows:

- $f^{*}(v_{i}v_{i+1}) = 4k^{2} + 8k + 6, 1 \leq k \leq 5$
- $f^{*}(v_{i}v_{i+1}) = 90$
- $f^{*}(v_{i}v_{i+1}) = 118$
- $f^{*}(v_{i}v_{i+1}) = 154$

Thus $f^{*}(E) = \{18, 38, 66, \ldots, 146, 90, 118, 154\}$ in which all the elements are distinct. Therefore, the cubic graph with $2m$ vertices admits Gaussian antimagic labeling.
Theorem 3.12. The Path union of two copies of cubic graph with 4m vertices admits Gaussian antimagic labeling.

Proof. Let $V = \{v_1, v_2, v_3, \ldots, v_{4m}\}$ be the vertices and $E = \{v_kv_{k+1} / 1 \leq k \leq m - 1, m \in N\} \cup \{v_{4m}\}$ be the edges of the Path union of two copies of cubic graph with 4m vertices.

Define a function $f : V \to \{a + ib \mid a, b \in N, b = a + 1, 1 \leq a \leq n\}$ such that $f(v_i) = s + (s + 1)i, 1 \leq s \leq n$.

Define the induced function $f* : E \to N$ such that

$$f*(uv) = |f(u)|^2 + |f(v)|^2.$$

The edge labels are obtained as follows:

- $f*(v_1, v_2) = 4k^2 + 8k + 6, 1 \leq k \leq m - 1, m \in N$
- $f*(v_1, v_{4m}) = 2m^2 + 2m + 6, m \in N$
- $f*(vk, vm + k) = 4k^2 + 4k + 2m^2 + 4mk + 2m + 2, 1 \leq k \leq m, m \in N$
- $f*(v_{4m+1}, v_{m+1}) = 2m^2 + 2m + 6, m \in N$
- $f*(v_{4m+2}, v_{m+1}) = 4k^2 + 8k + 16m^2 + 16mk + 16m + 6, 1 \leq k \leq m - 1, m \in N$
- $f*(v_{4m+2}, v_{m+2}) = 26m^2 + 18m + 6, m \in N$
- $f*(v_{4m+3}, v_{m+1}) = 4k^2 + 4k + 26m^2 + 20mk + 10m + 2, 1 \leq k \leq m, m \in N$
- $f*(v_{4m+3}, v_{m+2}) = 4k^2 + 8k + 36m^2 + 24mk + 24m + 6, 1 \leq k \leq m - 1, m \in N$
- $f*(v_{4m+4}, v_{m+1}) = 26m^2 + 22m + 6, m \in N$

Thus $f*(E) = 18, 38, 66, \ldots, 4m^2 + 2, 2m^2 + 2m + 6, 2m^2 + 6m + 10, 2m^2 + 6m + 26, \ldots, 10m^2 + 6m + 2, 4m^2 + 16m + 18, 4m^2 + 24m + 38, \ldots, 16m^2 + 2, 10m + 10m + 6, 40m^2 + 12m + 2, 16m^2 + 32m + 18, 16m^2 + 48m + 38, \ldots, 36m^2 + 2, 26m^2 + 18m + 6, 26m^2 + 30m + 10, 26m^2 + 50m + 26, \ldots, 50m^2 + 14m + 2, 36m^2 + 48m + 18, 36m^2 + 72m + 38, \ldots, 64m^2 + 2, 6m^2 + 22m + 6$ in which all the elements are distinct. Therefore, the Path union of two copies of cubic graph with 4m vertices admits Gaussian antimagic labeling.

4. Conclusion

In this paper we have shown that the graphs such as Middle graph of path, Split graph of path, Shadow graph of path, Total path graph, Middle cycle graph, Ladder graph, Square comb, Splitting comb graph, Theta graph, Sum of two copies of Theta graph by the path of length two, Cubic graph and Path union of two copies of Cubic graph are Gaussian antimagic graphs.

References