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A study on generalized quadrilateral fuzzy numbers

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Abstract

Fuzzy numbers are the crucial elements to represent the imprecise quantities. In this paper, defuzzification method for generalized quadrilateral fuzzy numbers (GQFN) and some arithmetic operations on GQFN's based on the defuzzification method have been proposed. Also some properties of defuzzification method are presented and relevant numerical examples are provided to prove the properties.

Keywords

Fuzzy number, Generalizedquadrilateral fuzzy number, Classical equivalent fuzzy mean.

AMS Subject Classification:

03E72.

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Contents

1. Introduction

The theory of fuzzy sets were introduced by Prof. Lotfi A Zadeh[10] in 1965. After that the concept of fuzzy numbers gains the enormous applications in various fields. Dubois and Prade[1,2] discussed the idea of fuzzy sets, fuzzy numbers and their arithmetic operations. In the fuzzy literature, some types of fuzzy numbers proposed depends upon their diagrammatical representations like triangular, trapezoidal, pentagonal fuzzy numbers etc. In the same way, Stephen Dinagar and Abirami[6] proposed the more generalized interval valued fuzzy numbers. Pathinathan and Santhoshkumar[4] introduced perfect pentagonal fuzzy numbers and quadrilateral fuzzy numbers as a generalized version of pentagonal fuzzy numbers. As from analytical geometry, quadrilateral has only four vertices, but those quadrilateral fuzzy numbers has five vertices. So it does not gives the exact shape of a quadrilateral. Stephen Dinagar and Christopar Raj[7] proposed the new concept of generalized quadrilateral fuzzy numbers. The interval arithmetic on fuzzy numbers and its arithmetic operations were proposed by Kauffmann and Gupta[3]. Also they studied some novel functions on fuzzy numbers. Rezvani[5] defuzzifies the triangular fuzzy numbersbased on the graded mean representation method. For trapezoidal fuzzy numbers, some linear and some nonlinear arithmetic operations were studied by Vahidi and Rezvani[9]. Stephen Dinagar and Latha[8] discussed the ranking function for type-2 triangular fuzzy numbers and its arithmetic operations.In this paper, we have introduced the defuzzification method and some arithmetic operations on GQFN's. The paper has been organised as follows: Firstly in section-2, we review the basic definitions and introduced the defuzzification method. In section-3, some arithmetic operations on generalizedquadrilateral fuzzy numbers are proposed. In section-4, few properties of proposed defuzzification method are discussed. In section-5, relevant numerical examples are given to prove the properties. Finally in section-6, conclusion is also included.

2. Preliminaries

Definition 2.1. Fuzzy Set

A fuzzy set A in X characterized by a membership function $\mu_A(x)$ which mapping from the elements of X to the interval [0,1]. The value of $\mu_A(x)$ is called the membership grade of x in A.

Definition 2.2. Fuzzy Number

A fuzzy set $\widetilde{A}$ defined on R is said to be a fuzzy number if

1. *$\widetilde{A}$ is normal*

2. $\widetilde{A}$ *is convex*

3. *Supp* $\widetilde{A}$ *is closed and bounded.*

Definition 2.3. Generalized Quadrilateral Fuzzy Number *A fuzzy number $\widetilde{A}$ is said to be a generalized quadrilateral fuzzy number (GQFN) represented by $\widetilde{A} = [a_1, a_2, a_3, a_4; l_1, l_2]$, where $a_1 \le a_2 \le a_3 \le a_4$, when its membership function is given as*
$$\mu_A(x) = \begin{cases} l_1 \frac{x-a_1}{a_2-a_1}, & \text{if } a_1 \le x \le a_2 \\ \frac{(x-a_2)l_2+(a_3-x)l_1}{a_3-a_2}, & \text{if } a_2 \le x \le a_3 \\ l_2 \frac{x-a_4}{a_3-a_4}, & \text{if } a_3 \le x \le a_4 \\ 0, & \text{otherwise.} \end{cases}$$
And its pictorial representation is given by,

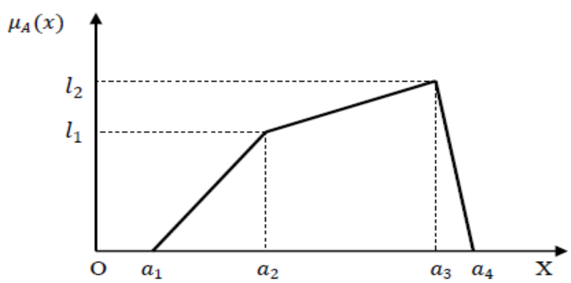

Figure 1

When $l_1 = l_2 = 1$, the generalized quadrilateral fuzzy number becomes a trapezoidal fuzzy number. When $a_1 = a_2$ or $a_2 = a_3$ or $a_3 = a_4$, then the generalized quadrilateral fuzzy number becomes a generalized triangular fuzzy number.

Note 2.4. Classical Equivalent Fuzzy Mean(CEFM) *If F(R) is a set of generalized quadrilateral fuzzy numbers. The classical equivalent fuzzy mean M assigns a real number for each fuzzy number in F(R).*
For $\widetilde{A} = [a_1, a_2, a_3, a_4; l_1, l_2] \in F(R)$, then the classical equivalent fuzzy mean M is defined as
$$M\left(\widetilde{A}\right) = \frac{(a_1+a_2+a_3+a_4)(l_1+l_2)}{8}$$
Also we define orders on F(R) by,
$M(\widetilde{A}) \ge M(\widetilde{B})$ *if and only if* $\widetilde{A} \underset{M}{\geq} \widetilde{B}$,
$M(\widetilde{A}) \le M(\widetilde{B})$ *if and only if* $\widetilde{A} \underset{M}{\leq} \widetilde{B}$,
and $M(\widetilde{A}) = M(\widetilde{B})$ *if and only if* $\widetilde{A} \underset{M}{\equiv} \widetilde{B}$.
This method is the way to defuzzify the generalized quadrilateral fuzzy numbers. We called this method as a classical equivalent fuzzy mean only because of the properties in the section-4. Also it is known that based on the above said notion, the unique arithmetic operations on GQFN's have been proposed as follows.

3. Arithmetic Operations on Generalized Quadrilateral Fuzzy Numbers Based on CEFM

Let $\widetilde{A} = [a_1, a_2, a_3, a_4; l_1, l_2]$ and $\widetilde{B} = [b_1, b_2, b_3, b_4; m_1, m_2]$ be two GQFN's.

Take $\sigma_l = l_1 + l_2$ and $\sigma_m = m_1 + m_2$.

(i) Addition:
$$\widetilde{A} + \widetilde{B} = [\tfrac{2(a_1\sigma_l+b_1\sigma_m)}{\sigma_l+\sigma_m}, \tfrac{2(a_2\sigma_l+b_2\sigma_m)}{\sigma_l+\sigma_m}, \tfrac{2(a_3\sigma_l+b_3\sigma_m)}{\sigma_l+\sigma_m}, \tfrac{2(a_4\sigma_l+b_4\sigma_m)}{\sigma_l+\sigma_m}; \tfrac{l_1+m_1}{2}, \tfrac{l_2+m_2}{2}]$$

(ii) Subtraction:
$$\widetilde{A} - \widetilde{B} = [\tfrac{2(a_1\sigma_l-b_4\sigma_m)}{\sigma_l+\sigma_m}, \tfrac{2(a_2\sigma_l-b_3\sigma_m)}{\sigma_l+\sigma_m}, \tfrac{2(a_3\sigma_l-b_2\sigma_m)}{\sigma_l+\sigma_m}, \tfrac{2(a_4\sigma_l-b_1\sigma_m)}{\sigma_l+\sigma_m}; \tfrac{l_1+m_1}{2}, \tfrac{l_2+m_2}{2}]$$

(iii) Scalar multiplication:
If $k > 0$, $k\widetilde{A} = [ka_1, ka_2, ka_3, ka_4; l_1, l_2]$
If $k < 0$, $k\widetilde{A} = [ka_4, ka_3, ka_2, ka_1; l_1, l_2]$

(iv) Multiplication:
If $M(\widetilde{B}) > 0$,
$$\widetilde{A}.\widetilde{B} = [\tfrac{2a_1\sigma_l}{\sigma_l+\sigma_m}M\left(\widetilde{B}\right), \tfrac{2a_2\sigma_l}{\sigma_l+\sigma_m}M\left(\widetilde{B}\right), \tfrac{2a_3\sigma_l}{\sigma_l+\sigma_m}M\left(\widetilde{B}\right), \tfrac{2a_4\sigma_l}{\sigma_l+\sigma_m}M\left(\widetilde{B}\right); \tfrac{l_1+m_1}{2}, \tfrac{l_2+m_2}{2}]$$
If $M(\widetilde{B}) < 0$,
$$\widetilde{A}.\widetilde{B} = [\tfrac{2a_4\sigma_l}{\sigma_l+\sigma_m}M\left(\widetilde{B}\right), \tfrac{2a_3\sigma_l}{\sigma_l+\sigma_m}M\left(\widetilde{B}\right), \tfrac{2a_2\sigma_l}{\sigma_l+\sigma_m}M\left(\widetilde{B}\right), \tfrac{2a_1\sigma_l}{\sigma_l+\sigma_m}M\left(\widetilde{B}\right); \tfrac{l_1+m_1}{2}, \tfrac{l_2+m_2}{2}]$$

(v) Division:
If $M(\widetilde{B}) > 0$,
$$\frac{\widetilde{A}}{\widetilde{B}} = [\tfrac{2a_1\sigma_l}{(\sigma_l+\sigma_m)M(\widetilde{B})}, \tfrac{2a_2\sigma_l}{(\sigma_l+\sigma_m)M(\widetilde{B})}, \tfrac{2a_3\sigma_l}{(\sigma_l+\sigma_m)M(\widetilde{B})}, \tfrac{2a_4\sigma_l}{(\sigma_l+\sigma_m)M(\widetilde{B})}; \tfrac{l_1+m_1}{2}, \tfrac{l_2+m_2}{2}]$$
If $M(\widetilde{B}) < 0$,
$$\frac{\widetilde{A}}{\widetilde{B}} = [\tfrac{2a_4\sigma_l}{(\sigma_l+\sigma_m)M(\widetilde{B})}, \tfrac{2a_3\sigma_l}{(\sigma_l+\sigma_m)M(\widetilde{B})}, \tfrac{2a_2\sigma_l}{(\sigma_l+\sigma_m)M(\widetilde{B})}, \tfrac{2a_1\sigma_l}{(\sigma_l+\sigma_m)M(\widetilde{B})}; \tfrac{l_1+m_1}{2}, \tfrac{l_2+m_2}{2}]$$

(vi) Square :
$$\widetilde{A}^2 = [a_1M\left(\widetilde{A}\right), a_2M\left(\widetilde{A}\right), a_3M\left(\widetilde{A}\right), a_4M\left(\widetilde{A}\right); l_1, l_2]$$

(vi) Square root :
If $M(\widetilde{A}) > 0$,
$$\sqrt{\widetilde{A}} = \left[\frac{a_1}{\sqrt{M(\widetilde{A})}}, \frac{a_2}{\sqrt{M(\widetilde{A})}}, \frac{a_3}{\sqrt{M(\widetilde{A})}}, \frac{a_4}{\sqrt{M(\widetilde{A})}}; l_1, l_2\right]$$

4. Properties of Classical Equivalent Fuzzy Mean

Proposition 4.1. *If $\widetilde{A}$ and $\widetilde{B}$ be any two GQFN's and $M\left(\widetilde{A}\right)$,$M\left(\widetilde{B}\right)$ be their respective classical equivalent fuzzy mean values. Then the fuzzy mean values of sum of the GQFN's $\widetilde{A}$ and $\widetilde{B}$ is equal to the sum of the fuzzy mean values of $\widetilde{A}$ and $\widetilde{B}$.*
i.e., $M\left(\widetilde{A}+\widetilde{B}\right) = M\left(\widetilde{A}\right) + M\left(\widetilde{B}\right)$

Proof. Let $\widetilde{A} = [a_1, a_2, a_3, a_4; l_1, l_2]$ and $\widetilde{B} = [b_1, b_2, b_3, b_4; m_1, m_2]$ be two GQFN's.
Then
$$M\left(\widetilde{A}\right) = \frac{(a_1+a_2+a_3+a_4)(l_1+l_2)}{8}$$
$$M\left(\widetilde{B}\right) = \frac{(b_1+b_2+b_3+b_4)(m_1+m_2)}{8}$$
Since,

$\widetilde{A}+\widetilde{B}=[\frac{2(a_1\sigma_l+b_1\sigma_m)}{\sigma_l+\sigma_m},\frac{2(a_2\sigma_l+b_2\sigma_m)}{\sigma_l+\sigma_m},\frac{2(a_3\sigma_l+b_3\sigma_m)}{\sigma_l+\sigma_m},$
$\frac{2(a_4\sigma_l+b_4\sigma_m)}{\sigma_l+\sigma_m};\frac{l_1+m_1}{2},\frac{l_2+m_2}{2}]$

$$M\left(\widetilde{A}+\widetilde{B}\right)=\frac{2[(a_1+a_2+a_3+a_4)\sigma_l+(b_1+b_2+b_3+b_4)\sigma_m]\left[\frac{(l_1+l_2+m_1+m_2)}{2}\right]}{(\sigma_l+\sigma_m)8}$$
$$=\frac{2[(a_1+a_2+a_3+a_4)\sigma_l+(b_1+b_2+b_3+b_4)\sigma_m]\left[\frac{(\sigma_l+\sigma_m)}{2}\right]}{(\sigma_l+\sigma_m)\,8}$$
$$=\frac{[(a_1+a_2+a_3+a_4)\sigma_l+(b_1+b_2+b_3+b_4)\sigma_m]}{8}$$
$$=\frac{(a_1+a_2+a_3+a_4)\sigma_l}{8}+\frac{(b_1+b_2+b_3+b_4)\sigma_m}{8}$$
$$M\left(\widetilde{A}+\widetilde{B}\right)=M\left(\widetilde{A}\right)+M\left(\widetilde{B}\right)$$

Hence proved. □

Proposition 4.2. *If $\widetilde{A}$ and $\widetilde{B}$ be any two GQFN's and $M\left(\widetilde{A}\right)$, $M\left(\widetilde{B}\right)$ be their respective classical equivalent fuzzy mean values. Then the fuzzy mean values of subtraction of the GQFN's $\widetilde{A}$ and $\widetilde{B}$ is equal to the subtraction of the fuzzy mean values of $\widetilde{A}$ and $\widetilde{B}$.*
i.e., $M\left(\widetilde{A}-\widetilde{B}\right)=M\left(\widetilde{A}\right)-M\left(\widetilde{B}\right)$

Proof. Similar to the proof of proposition-4.1. □

Proposition 4.3. *If $\widetilde{A}$ and $\widetilde{B}$ be any two GQFN's and $M\left(\widetilde{A}\right)$, $M\left(\widetilde{B}\right)$ be their respective classical equivalent fuzzy mean values. Then the fuzzy mean values of product of the GQFN's $\widetilde{A}$ and $\widetilde{B}$ is equal to the product of the fuzzy mean values of $\widetilde{A}$ and $\widetilde{B}$.*
i.e., $M\left(\widetilde{A}.\widetilde{B}\right)=M\left(\widetilde{A}\right).M\left(\widetilde{B}\right)$

Proof. Let $\widetilde{A}=[a_1,a_2,a_3,a_4;l_1,l_2]$ and $\widetilde{B}=[b_1,b_2,b_3,b_4;m_1,m_2]$ be two GQFN's.
Then
$$M\left(\widetilde{A}\right)=\frac{(a_1+a_2+a_3+a_4)(l_1+l_2)}{8}$$
$$M\left(\widetilde{B}\right)=\frac{(b_1+b_2+b_3+b_4)(m_1+m_2)}{8}$$

Case (i):
If $M(\widetilde{B})>0$,
$\widetilde{A}.\widetilde{B}=[\frac{2a_1\sigma_l}{\sigma_l+\sigma_m}M\left(\widetilde{B}\right),\frac{2a_2\sigma_l}{\sigma_l+\sigma_m}M\left(\widetilde{B}\right),\frac{2a_3\sigma_l}{\sigma_l+\sigma_m}M\left(\widetilde{B}\right),$
$\frac{2a_4\sigma_l}{\sigma_l+\sigma_m}M\left(\widetilde{B}\right);\frac{l_1+m_1}{2},\frac{l_2+m_2}{2}]$

$$M\left(\widetilde{A}.\widetilde{B}\right)=\frac{2\left[(a_1+a_2+a_3+a_4)\sigma_lM\left(\widetilde{B}\right)\right]\left[\frac{(l_1+l_2+m_1+m_2)}{2}\right]}{(\sigma_l+\sigma_m)\,8}$$
$$=\frac{2\left[(a_1+a_2+a_3+a_4)\sigma_lM\left(\widetilde{B}\right)\right]\left[\frac{(\sigma_l+\sigma_m)}{2}\right]}{(\sigma_l+\sigma_m)\,8}$$
$$=\frac{\left[(a_1+a_2+a_3+a_4)\sigma_lM\left(\widetilde{B}\right)\right]}{8}$$
$$M\left(\widetilde{A}.\widetilde{B}\right)=M\left(\widetilde{A}\right).M\left(\widetilde{B}\right)$$

Hence proved.

Case (ii):
If $M(\widetilde{B})<0$,
$\widetilde{A}.\widetilde{B}=[\frac{2a_4\sigma_l}{\sigma_l+\sigma_m}M\left(\widetilde{B}\right),\frac{2a_3\sigma_l}{\sigma_l+\sigma_m}M\left(\widetilde{B}\right),\frac{2a_2\sigma_l}{\sigma_l+\sigma_m}M\left(\widetilde{B}\right),$
$\frac{2a_1\sigma_l}{\sigma_l+\sigma_m}M\left(\widetilde{B}\right);\frac{l_1+m_1}{2},\frac{l_2+m_2}{2}]$

$$M\left(\widetilde{A}.\widetilde{B}\right)=\frac{2\left[(a_4+a_3+a_2+a_1)\sigma_lM\left(\widetilde{B}\right)\right]\left[\frac{(l_1+l_2+m_1+m_2)}{2}\right]}{(\sigma_l+\sigma_m)8}$$
$$=\frac{2\left[(a_4+a_3+a_2+a_1)\sigma_lM\left(\widetilde{B}\right)\right]\left[\frac{(\sigma_l+\sigma_m)}{2}\right]}{(\sigma_l+\sigma_m)\,8}$$
$$=\frac{\left[(a_1+a_2+a_3+a_4)\sigma_lM\left(\widetilde{B}\right)\right]}{8}$$
$$M\left(\widetilde{A}.\widetilde{B}\right)=M\left(\widetilde{A}\right).M\left(\widetilde{B}\right)$$

Hence proved. □

Proposition 4.4. *If $\widetilde{A}$ and $\widetilde{B}$ be any two GQFN's and $M\left(\widetilde{A}\right)$, $M\left(\widetilde{B}\right)$ be their respective classical equivalent fuzzy mean values. Then the fuzzy mean values of division of the GQFN's $\widetilde{A}$ and $\widetilde{B}$ is equal to the division of the fuzzy mean values of $\widetilde{A}$ and $\widetilde{B}$.*
i.e., $M\left(\frac{\widetilde{A}}{\widetilde{B}}\right)=\frac{M\left(\widetilde{A}\right)}{M\left(\widetilde{B}\right)}$

Proof. Similar to the proof of proposition-4.3. □

Proposition 4.5. *If $\widetilde{A}$ be any GQFN and $M\left(\widetilde{A}\right)$ be the classical equivalent fuzzy mean of $\widetilde{A}$. Then*

(i) $M\left(k\widetilde{A}\right)=kM\left(\widetilde{A}\right)$

(ii) $M\left(\widetilde{A}^2\right)=\left(M\left(\widetilde{A}\right)\right)^2$

(iii) $M\left(\sqrt{\widetilde{A}}\right)=\sqrt{M\left(\widetilde{A}\right)}$

Proof. Let $\widetilde{A}=[a_1,a_2,a_3,a_4;l_1,l_2]$
Then
$$M\left(\widetilde{A}\right)=\frac{(a_1+a_2+a_3+a_4)(l_1+l_2)}{8}$$

(i) Case-(1)
If $k>0, k\widetilde{A}=[ka_1,ka_2,ka_3,ka_4;l_1,l_2]$
$$M\left(k\widetilde{A}\right)=\frac{(ka_1+ka_2+ka_3+ka_4)(l_1+l_2)}{8}$$
$$=\frac{k(a_1+a_2+a_3+a_4)(l_1+l_2)}{8}$$
$$M\left(k\widetilde{A}\right)=kM\left(\widetilde{A}\right)$$

Case-2
If $k<0, k\widetilde{A}=[ka_4,ka_3,ka_2,ka_1;l_1,l_2]$
$$M\left(k\widetilde{A}\right)=\frac{(ka_4+ka_3+ka_2+ka_1)(l_1+l_2)}{8}$$
$$=\frac{k(a_1+a_2+a_3+a_4)(l_1+l_2)}{8}$$
$$M\left(k\widetilde{A}\right)=kM\left(\widetilde{A}\right)$$

Hence proved.

(ii) $\widetilde{A}^2=\left[a_1M\left(\widetilde{A}\right),a_2M\left(\widetilde{A}\right),a_3M\left(\widetilde{A}\right),a_4M\left(\widetilde{A}\right);l_1,l_2\right]$
$$M\left(\widetilde{A}^2\right)=\frac{\left(a_1M\left(\widetilde{A}\right)+a_2M\left(\widetilde{A}\right)+a_3M\left(\widetilde{A}\right)+a_4M\left(\widetilde{A}\right)\right)(l_1+l_2)}{8}$$
$$=\frac{(a_1+a_2+a_3+a_4)M\left(\widetilde{A}\right)(l_1+l_2)}{8}$$
$$=M\left(\widetilde{A}\right)M\left(\widetilde{A}\right)$$
$$M\left(\widetilde{A}^2\right)=\left(M\left(\widetilde{A}\right)\right)^2$$

(iii) Similar to the proof of (ii).

□

5. Numerical Examples

Let $\widetilde{A}=[4,8,9,11;0.6,0.4] \Rightarrow M\left(\widetilde{A}\right)=4$

$\widetilde{B}=[5,7,12,16;0.7,0.8] \Rightarrow M\left(\widetilde{B}\right)=7.5$

(i) Addition:

$\widetilde{A}+\widetilde{B}=[9.2,\ 14.8,\ 21.6,28;0.65,\ 0.6]$

$M\left(\widetilde{A}+\widetilde{B}\right)=\frac{(73.6)(1.25)}{8}=11.5$

$M\left(\widetilde{A}\right)+M\left(\widetilde{B}\right)=4+7.5=11.5$

$\therefore M\left(\widetilde{A}+\widetilde{B}\right)=M\left(\widetilde{A}\right)+M\left(\widetilde{B}\right)$

(ii) Subtraction:

$\widetilde{A}-\widetilde{B}=[-16,-8,-1.2,2.8;0.65,0.6]$

$M\left(\widetilde{A}-\widetilde{B}\right)=\frac{(-22.4)(1.25)}{8}=-3.5$

$M\left(\widetilde{A}\right)-M\left(\widetilde{B}\right)=4-7.5=-3.5$

$\therefore M\left(\widetilde{A}-\widetilde{B}\right)=M\left(\widetilde{A}\right)-M\left(\widetilde{B}\right)$

(iii) Scalar multiplication:

$3\widetilde{A}=[12,24,27,33;0.6,0.4]$

$M\left(3\widetilde{A}\right)=\frac{(96)(1)}{8}=12$

$kM\left(\widetilde{A}\right)=3\times 4=12$

$\therefore M\left(k\widetilde{A}\right)=kM\left(\widetilde{A}\right)$

(iv) Multiplication:

$\widetilde{A}.\widetilde{B}=[24,48,54,66;0.65,0.6]$

$M\left(\widetilde{A}.\widetilde{B}\right)=\frac{(192)(1.25)}{8}=30$

$M\left(\widetilde{A}\right).M\left(\widetilde{B}\right)=4\times 7.5=30$

$\therefore M\left(\widetilde{A}.\widetilde{B}\right)=M\left(\widetilde{A}\right).M\left(\widetilde{B}\right)$

(v) Division:

$\frac{\widetilde{A}}{\widetilde{B}}=[0.427,0.853,0.96,1.173;0.65,0.6]$

$M\left(\frac{\widetilde{A}}{\widetilde{B}}\right)=\frac{(3.413)(1.25)}{8}=0.533$

$\frac{M\left(\widetilde{A}\right)}{M\left(\widetilde{B}\right)}=\frac{4}{7.5}=0.533$

$\therefore M\left(\frac{\widetilde{A}}{\widetilde{B}}\right)=\frac{M\left(\widetilde{A}\right)}{M\left(\widetilde{B}\right)}$

(vi) Square:

$\widetilde{A}^2=[16,32,36,44;0.6,0.4]$

$M\left(\widetilde{A}^2\right)=\frac{(128)(1)}{8}=16$

$\left(M\left(\widetilde{A}\right)\right)^2=(4)^2=16$

$\therefore M\left(\widetilde{A}^2\right)=\left(M\left(\widetilde{A}\right)\right)^2$

(vi) Square root:

$\sqrt{\widetilde{A}}=[2,4,4.5,5.5;0.6,0.4]$

$M\left(\sqrt{\widetilde{A}}\right)=\frac{(16)(1)}{8}=2$

$\sqrt{M\left(\widetilde{A}\right)}=\sqrt{4}=2$

$\therefore M\left(\sqrt{\widetilde{A}}\right)=\sqrt{M\left(\widetilde{A}\right)}$

6. Conclusion

In this paper, the concept of generalized quadrilateral fuzzy numbers are reviewed. The new method of defuzzification, some arithmetic operations on GQFN's and some properties are proposed. Finally the properties are proved by using relevant numerical examples. By using these notions we may discuss the inventory models by GQFN's in the future.

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