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Similarity measures on generalized interval-valued trapezoidal intuitionistic fuzzy number

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Abstract
A Distinct novel method is introduced in this paper to find the similarity measures between two generalized interval-valued trapezoidal intuitionistic fuzzy numbers (GIVTIFNs). This notion is studied with the help of generalized trapezoidal intuitionistic fuzzy numbers (GTIFNs). Proper numerical illustrations are given to justify the proposed notion.

Keywords
Trapezoidal fuzzy number, Intuitionistic Trapezoidal fuzzy number, Interval – valued Trapezoidal fuzzy number, Interval – valued Trapezoidal Intuitionistic fuzzy number, Similarity Measure.

AMS Subject Classification:
03F55,03B52, 28E10

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Contents

1. Introduction

Fuzzy similarity measures are used to compare the distance between the different kinds of objects. Similarity is an amount that reflects the strength of relation between the data items. Several extensions of fuzzy Sets [12] have been proposed and developed by many researchers. In [10] Turksen introduced the interval valued fuzzy sets based on Normal forms. Atanassov [1] proposed several methods for measuring the degree of similarity between intuitionistic fuzzy sets. Chen and Chen [2] proposed a similarity measure on generalized trapezoidal fuzzy numbers. Chen [3] proposed a similarity measure for GIVFN using the geometric mean average operator to overcome the drawbacks of the similarity measure. Hung and Yang [6] proposed some methods to calculate the degree of similarity between intuitionistic fuzzy sets. Li e.tal in [7] gave a detailed survey on similarity measures between intuitionistic fuzzy sets. In [11] Xu proposed the similarity measures of interval –valued intuitionistic fuzzy sets fuzz risk analysis, and also an overview of distance and similarity measures of intuitionistic fuzzy sets. Stephen Dinagar and Fany Helena [8,9] proposed a similarity measures for trapezoidal intuitionistic fuzzy number based on valued ambiguity and also with centroid ranking. Based on the above we proposed two similarity measures for generalized interval – valued trapezoidal intuitionistic fuzzy numbers.

This paper is organized as follows, we gave the basic definitions of generalized trapezoidal fuzzy number, generalized interval – valued trapezoidal fuzzy number, generalized trapezoidal intuitionistic fuzzy number, generalized interval – valued trapezoidal intuitionistic fuzzy number in section 2. Section 3 we proposed a method to calculate the degree of similarity between generalized interval – valued trapezoidal intuitionistic fuzzy numbers and example are illustrated .In sections 4 we proposed an another degree of similarity for

GIVTIFNs based on Chen's method and some numerical examples are presented. In Section 5 Comparison for two proposed similarity measure are discussed. Finally the conclusion is given in section 6.

2. Preliminaries

Definition 2.1. *A generalized trapezoidal fuzzy number (GTFN)* $A = (a_1, a_2, a_3, a_4; w_A)$ *is a fuzzy set on R with the membership function given by*

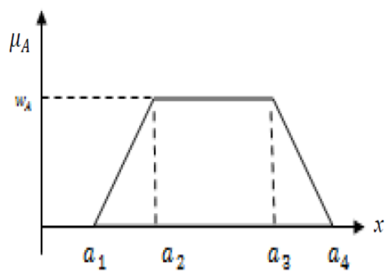

Figure 1

$$\mu_A(x) = \begin{cases} w_A \frac{x-a_1}{a_2-a_1}, & \text{if } x \in [a_1, a_2) \\ w_A, & \text{if } x \in [a_2, a_3] \\ w_A \frac{a_4-x}{a_4-a_3}, & \text{if } x \in (a_3, a_4] \\ 0, & \text{otherwise}. \end{cases}$$

Where $0 \leq w_A \leq 1$ *and* $a_1 \leq a_2 \leq a_3 \leq a_4$.

Definition 2.2. *A generalized interval – valued trapezoidal fuzzy number (GIVTFN)* $\tilde{A}$ *is an interval – valued fuzzy set on R, defined by* $\tilde{A} = \left[\tilde{A}^L, \tilde{A}^U\right]$. *Where the generalized trapezoidal fuzzy numbers are* $\tilde{A}^L = \left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{A}}^L\right)$ *and* $\tilde{A}^U = \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{A}}^U\right)$ *whose membership functions are*

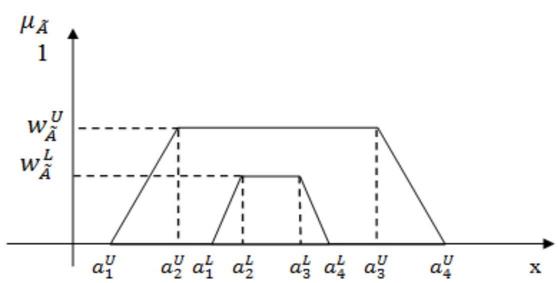

Figure 2

$$\mu_{\tilde{A}}^L(x) = \begin{cases} w_{\tilde{A}}^L \frac{x-a_1^L}{a_2^L-a_1^L}, & \text{if } x \in \left[a_1^L, a_2^L\right) \\ w_{\tilde{A}}^L, & \text{if } x \in \left[a_2^L, a_3^L\right] \\ w_{\tilde{A}}^L \frac{a_4^L-x}{a_4^L-a_3^L}, & \text{if } x \in \left(a_3^L, a_4^L\right] \\ 0, & \text{otherwise}. \end{cases}$$

$$\mu_{\tilde{A}}^U(x) = \begin{cases} w_{\tilde{A}}^U \frac{x-a_1^U}{a_2^U-a_1^U}, & \text{if } x \in \left[a_1^U, a_2^U\right) \\ w_{\tilde{A}}^U, & \text{if } x \in \left[a_2^U, a_3^U\right] \\ w_{\tilde{A}}^U \frac{a_4^U-x}{a_4^U-a_3^U}, & \text{if } x \in \left(a_3^U, a_4^U\right] \\ 0, & \text{otherwise}. \end{cases}$$

such that $a_1^L \leq a_2^L \leq a_3^L \leq a_4^L$, $a_1^U \leq a_2^U \leq a_3^U \leq a_4^U, 0 < w_{\tilde{A}}^L \leq w_{\tilde{A}}^U < 1$ *and* $\tilde{A}^L \subseteq \tilde{A}^U$.

Definition 2.3. *A generalized trapezoidal intuitionistic fuzzy number (GTIFN)* $\tilde{A}^i = [(a_1, a_2, a_3, a_4; w_{\tilde{A}^i})(b_1, b_2, b_3, b_4; u_{\tilde{A}^i})]$ *is an intuitionistic fuzzy set on R, with the membership function* $\mu_{\tilde{A}^i}$ *and the non – membership function* $\nu_{\tilde{A}^i}$ *defined as*

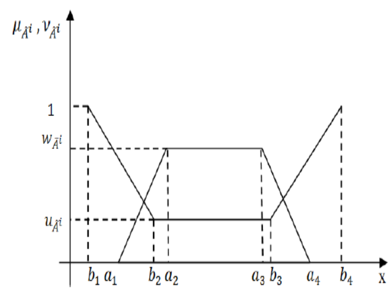

Figure 3

$$\mu_{\tilde{A}^i}(x) = \begin{cases} 0, & \text{if } x < a_1 \\ w_{\tilde{A}^i}\left(\frac{x-a_1}{a_2-a_1}\right), & \text{if } a_1 \leq x \leq a_2 \\ w_{\tilde{A}^i}, & \text{if } a_2 \leq x \leq a_3 \\ w_{\tilde{A}^i}\left(\frac{a_4-x}{a_4-a_3}\right), & \text{if } a_3 \leq x \leq a_4 \\ 0, & \text{if } x > a_4. \end{cases}$$

$$\nu_{\tilde{A}^i}(x) = \begin{cases} 1, & \text{if } x < b_1 \\ \frac{(b_2-x)+\mu_{\tilde{A}^i}(x-b_1)}{b_2-b_1}, & \text{if } b_1 \leq x \leq b_2 \\ u_{\tilde{A}^i}, & \text{if } b_2 \leq x \leq b_3 \\ \frac{(x-b_3)+\mu_{\tilde{A}^i}(b_4-x)}{b_4-b_3}, & \text{if } b_3 \leq x \leq b_4 \\ 1, & \text{if } x > b_4. \end{cases}$$

Where $0 < w_{\tilde{A}^i} \leq 1$, $0 \leq u_{\tilde{A}^i} \leq 1$ *and* $0 < w_{\tilde{A}^i} + u_{\tilde{A}^i} \leq 1$.

Definition 2.4. *A generalized interval – valued trapezoidal intuitionistic fuzzy number be* $\tilde{\tilde{A}}^i = [(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}^i}^L)$ $(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}^i}^U); (b_1^L, b_2^L, b_3^L, b_4^L; u_{\tilde{\tilde{A}}^i}^L)(b_1^U, b_2^U, b_3^U, b_4^U; u_{\tilde{\tilde{A}}^i}^U)]$ *Whose membership and non membership function are defined as*

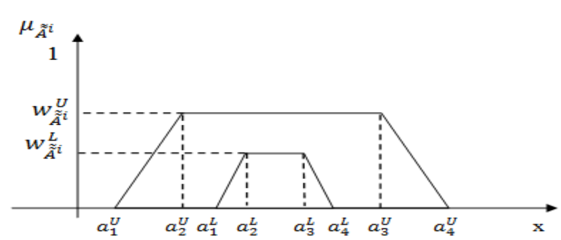

Figure 4

$$\mu^L_{\tilde{\tilde{A}}^i}(x)=\begin{cases}0, & if\ x<a^L_1\\ w^L_{\tilde{\tilde{A}}^i}\left(\frac{x-a^L_1}{a^L_2-a^L_1}\right), & if\ a^L_1\le x\le a^L_2\\ w^L_{\tilde{\tilde{A}}^i}, & if\ a^L_2\le x\le a^L_3\\ w^L_{\tilde{\tilde{A}}^i}\left(\frac{a^L_4-x}{a^L_4-a^L_3}\right), & if\ a^L_3\le x\le a^L_4\\ 0, & if\ x>a^L_4.\end{cases}$$

$$\mu^U_{\tilde{\tilde{A}}^i}(x)=\begin{cases}0, & if\ x<a^L_1\\ w^U_{\tilde{\tilde{A}}^i}\left(\frac{x-a^U_1}{a^U_2-a^U_1}\right), & if\ a^U_1\le x\le a^U_2\\ w^U_{\tilde{\tilde{A}}^i}, & if\ a^U_2\le x\le a^U_3\\ w^U_{\tilde{\tilde{A}}^i}\left(\frac{a^U_4-x}{a^U_4-a^U_3}\right), & if\ a^U_3\le x\le a^U_4\\ 0, & if\ x>a^U_4.\end{cases}$$

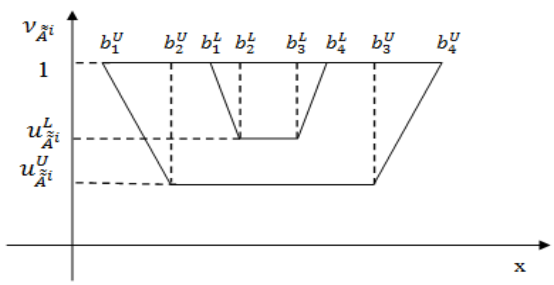

Figure 5

$$\nu^L_{\tilde{\tilde{A}}^i}(x)=\begin{cases}1, & if\ x<b^L_1\\ \frac{(b^L_2-x)+u^L_{\tilde{\tilde{A}}^i}(x-b^L_1)}{(b^L_2-b^L_1)}, & if\ b^L_1\le x\le b^L_2\\ u^L_{\tilde{\tilde{A}}^i}, & if\ b^L_2\le x\le b^L_3\\ \frac{(x-b^L_3)+\ u^L_{\tilde{\tilde{A}}^i}(b^L_4-x)}{b^L_4-b^L_3}, & if\ b^L_3\le x\le b^L_4\\ 1, & if\ x>b^L_4.\end{cases}$$

$$\nu^U_{\tilde{\tilde{A}}^i}(x)=\begin{cases}1, & if\ x<b^U_1\\ \frac{(b^U_2-x)+u^U_{\tilde{\tilde{A}}^i}(x-b^U_1)}{(b^U_2-b^U_1)}, & if\ b^U_1\le x\le b^U_2\\ u^U_{\tilde{\tilde{A}}^i}, & if\ b^U_2\le x\le b^U_3.\\ \frac{(x-b^U_3)+u^U_{\tilde{\tilde{A}}^i}(b^U_4-x)}{b^U_4-\ b^U_3}, & if\ b^U_3\le x\le b^U_4\\ 1, & if\ x>b^U_4\end{cases}$$

Where $0\le\mu^L_{\tilde{\tilde{A}}^i}\le\mu^U_{\tilde{\tilde{A}}^i}\le 1, 0\le\nu^L_{\tilde{\tilde{A}}^i}\le\nu^U_{\tilde{\tilde{A}}^i}\le$ *1,0 ;* $w^L_{\tilde{\tilde{A}}^i}\le w^U_{\tilde{\tilde{A}}^i}<1$ *and* $0<w_{\tilde{\tilde{A}}^i}\le 1, 0\le u_{\tilde{\tilde{A}}^i}\le 1$ *and* $0<w_{\tilde{\tilde{A}}^i}+u_{\tilde{\tilde{A}}^i}\le 1.$

Proposition 2.5. *The mapping* $\psi: GTIFN(R)\to GIVTFN_*(R)$ *defined as*
$\psi[(a_1,a_2,a_3,a_4;w_{\widetilde{A}^i})(b_1,b_2,b_3,b_4;u_{\widetilde{A}^i})]=[(a^L_1,a^L_2,a^L_3,a^L_4;w^L_{\widetilde{A}})(a^U_1,a^U_2,a^U_3,a^U_4;w^U_{\widetilde{A}})]$
Where $a^L_i=a_i, i\in\{1,2,3,4\}$
$a^U_i=a_i, i\in\{1,2,3,4\}$
$w^L_{\widetilde{A}}=w_{\widetilde{A}^i}$
and $w^U_{\widetilde{A}}=1-u_{\widetilde{A}^i}$ *is a bijection*

Proof. It is immediate and $\psi^{-1}: GIVTFN_*(R)\to GTIFN(R)$ is defined as $\psi^{-1}[(a^L_1,a^L_2,a^L_3,a^L_4;w^L_{\widetilde{A}})(a^U_1,a^U_2,a^U_3,a^U_4;w^U_{\widetilde{A}})]=[(a_1,a_2,a_3,a_4;w_{\widetilde{A}^i})(b_1,b_2,b_3,b_4;u_{\widetilde{A}^i})]$
Where $a_i=a^L_i, i\in\{1,2,3,4\}$
$b_i=a^U_i, i\in\{1,2,3,4\}$
$w_{\widetilde{A}^i}=w^L_{\widetilde{A}}$
and $u_{\widetilde{A}^i}=1-w^U_{\widetilde{A}}$ is the inverse of ψ. □

Proposition 2.6. *The main properties of a similarity measure on GTFNs.* $S: GTFN(R)\times GTFN(R)\to[0,1]$
Sometimes considered as axiomatic requirements, are the following condition
$(S_1)S(A,B)=1$ *if and only if* $A=B$.
$(S_2)S(A,B)=S(B,A)$. *For every* $A,B\in GTFN(R)$
$(S_3)A\subseteq B\subseteq C$ *then* $S(A,C)\le S(A,B)$ *and* $S(A,C)\le S(B,C)$
(S_4) *If* $A=(a_1,a_2,a_3,a_4)$ *and* $B=(b_1,b_2,b_3,b_4)$ *then* $S(A,B)=1-|A-B|$
The above properties can be extended to similarity measure on GTIFNs and GIVTFNs in an obvious way. We denoted by $(\widetilde{S}^i_k), k\in\{1,2,3,4\}$ *the corresponding properties in the case of GTIFNS and by* $(\widetilde{S}_k), k\in\{1,2,3,4\}$ *the corresponding properties in the case of GIVTFNs.*

3. Similarity Measures of Generalized Interval valued Triangular Fuzzy Number[5]

Farhadinia and Ban proposed a method to compute a similarity measure between two fixed point. Calculating the similarity between two GIVTFNs.
Let $p,q\in[0,1].P+q=1,\widetilde{A},\widetilde{B}\in GIVTFNs.$
$\widetilde{A}=\left[\widetilde{A}^L,\widetilde{A}^U\right]=[(a_1,a_2,a_3;w_{\widetilde{A}})_L(a_1,a_2,a_3;w_{\widetilde{A}})_U];$
$\widetilde{B}=\left[\widetilde{B}^L,\widetilde{B}^U\right]=[(b_1,b_2,b_3;w_{\widetilde{B}})_L(b_1,b_2,b_3;w_{\widetilde{B}})_U]$
$S_F\left(\widetilde{A},\widetilde{B}\right)=(S_F(\widetilde{A}^L,\widetilde{B}^L))^p.(S_F(\widetilde{A}^U,\widetilde{B}^U))^q$
Where,
$S_F(\widetilde{A}^L,\widetilde{B}^L)=[1-\frac{\sum_{i=1}^3|a^L_i-b^L_i|}{3}]\times\frac{\min(P_e(\widetilde{A}^L),P_e(\widetilde{B}^L))+min(w^L_{\widetilde{A}},w^L_{\widetilde{B}})}{\max(P_e(\widetilde{A}^L),P_e(\widetilde{B}^L))+max(w^L_{\widetilde{A}},w^L_{\widetilde{B}})}$
And

$$S_F\left(\widetilde{A}^U,\widetilde{B}^U\right)=\left[1-\frac{\sum_{i=1}^3|a^U_i-b^U_i|}{3}\right]\times\frac{\min\left(P_e\left(\widetilde{A}^U\right),P_e\left(\widetilde{B}^U\right)\right)+min\left(w^U_{\widetilde{A}},w^U_{\widetilde{B}}\right)}{\max\left(P_e\left(\widetilde{A}^U\right),P_e\left(\widetilde{B}^U\right)\right)+max\left(w^U_{\widetilde{A}},w^U_{\widetilde{B}}\right)}$$

Based on the above we proposed a similarity measure for generalized interval valued trapezoidal intuitionistic fuzzy number. The following algorithm is proposed.

3.1 Proposed Algorithm and Numerical Examples for GIVTIFNs

We give algorithms to calculate the similarity measures between two generalized interval – valued trapezoidal intuitionistic fuzzy numbers. GIVTIFN is a combination of interval – valued fuzzy number and intuitionistic fuzzy number. For intuitionistic interval value fuzzy number there will be two upper and lower numbers one is membership function and another non membership function.

$p,q \in [0,1]$. Where $P+q=1, \tilde{\tilde{A}}^i, \tilde{\tilde{B}}^i \in GIVTIFNs$, such that for all real numbers

$$\begin{aligned}\tilde{\tilde{A}}^i &= \left[\tilde{\tilde{A}}^{i^L}(\mu,\nu), \tilde{\tilde{A}}^{i^U}(\mu,\nu)\right] \\ &= [[\left(a_1,a_2,a_3,a_4;w_{\tilde{\tilde{A}}^i}\right)_L \left(a_1,a_2,a_3,a_4;w_{\tilde{\tilde{A}}^i}\right)_U]; \\ &[(b_1,b_2,b_3,b_4;u_{\tilde{\tilde{A}}^i})_L(b_1,b_2,b_3,b_4;u_{\tilde{\tilde{A}}^i})_U]]\end{aligned}$$

$$\begin{aligned}\tilde{\tilde{B}}^i &= \left[\tilde{\tilde{B}}^{i^L}(\mu,\nu), \tilde{\tilde{B}}^{i^U}(\mu,\nu)\right] \\ &= [[\left(c_1,c_2,c_3,c_4;w_{\tilde{\tilde{B}}^i}\right)_L \left(c_1,c_2,c_3,c_4;w_{\tilde{\tilde{B}}^i}\right)_U]; \\ &[\left(d_1,d_2,d_3,d_4;u_{\tilde{\tilde{B}}^i}\right)_L \left(d_1,d_2,d_3,d_4;u_{\tilde{\tilde{B}}^i}\right)_U]]\end{aligned}$$

Step 1

Calculate

$$P_e\left(\tilde{\tilde{A}}^{i^L}\right) = e^{(}\sqrt{\left(\frac{a_1^L+b_1^L}{2}-\frac{a_2^L+b_2^L}{2}\right)^2+\left(\frac{w^L_{\tilde{\tilde{A}}^i}+u^L_{\tilde{\tilde{A}}^i}}{2}\right)^2}+\left(\frac{a_3^L+b_3^L}{2}-\frac{a_2^L+b_2^L}{2}\right)$$
$$+\sqrt{\left(\frac{a_3^L+b_3^L}{2}-\frac{a_4^L+b_4^L}{2}\right)^2+\left(\frac{w^L_{\tilde{\tilde{A}}^i}+u^L_{\tilde{\tilde{A}}^i}}{2}\right)^2}+\left(\frac{a_4^L+b_4^L}{2}-\frac{a_1^L+b_1^L}{2}\right))$$

$$P_e\left(\tilde{\tilde{B}}^{i^L}\right) = e^{(}\sqrt{\left(\frac{c_1^L+d_1^L}{2}-\frac{c_2^L+d_2^L}{2}\right)^2+\left(\frac{w^L_{\tilde{\tilde{B}}^i}+u^L_{\tilde{\tilde{B}}^i}}{2}\right)^2}+\left(\frac{c_3^L+d_3^L}{2}-\frac{c_2^L+d_2^L}{2}\right)$$
$$+\sqrt{\left(\frac{c_3^L+d_3^L}{2}-\frac{c_4^L+d_4^L}{2}\right)^2+\left(\frac{w^L_{\tilde{\tilde{B}}^i}+u^L_{\tilde{\tilde{B}}^i}}{2}\right)^2}+\left(\frac{c_4^L+d_4^L}{2}-\frac{c_1^L+d_1^L}{2}\right))$$

$$P_e\left(\tilde{\tilde{A}}^{i^U}\right) = e^{(}\sqrt{\left(\frac{a_1^U+b_1^U}{2}-\frac{a_2^U+b_2^U}{2}\right)^2+\left(\frac{w^U_{\tilde{\tilde{A}}^i}+u^U_{\tilde{\tilde{A}}^i}}{2}\right)^2}+\left(\frac{a_3^U+b_3^U}{2}-\frac{a_2^U+b_2^U}{2}\right)$$
$$+\sqrt{\left(\frac{a_3^U+b_3^U}{2}-\frac{a_4^U+b_4^U}{2}\right)^2+\left(\frac{w^U_{\tilde{\tilde{A}}^i}+u^U_{\tilde{\tilde{A}}^i}}{2}\right)^2}+\left(\frac{a_4^U+b_4^U}{2}-\frac{a_1^U+b_1^U}{2}\right))$$

$$P_e\left(\tilde{\tilde{B}}^{i^U}\right) = e^{(}\sqrt{\left(\frac{c_1^U+d_1^U}{2}-\frac{c_2^U+d_2^U}{2}\right)^2+\left(\frac{w^U_{\tilde{\tilde{B}}^i}+u^U_{\tilde{\tilde{B}}^i}}{2}\right)^2}+\left(\frac{c_3^U+d_3^U}{2}-\frac{c_2^U+d_2^U}{2}\right)$$
$$+\sqrt{\left(\frac{c_3^U+d_3^U}{2}-\frac{c_4^U+d_4^U}{2}\right)^2+\left(\frac{w^U_{\tilde{\tilde{B}}^i}+u^U_{\tilde{\tilde{B}}^i}}{2}\right)^2}+\left(\frac{c_4^U+d_4^U}{2}-\frac{c_1^U+d_1^U}{2}\right))$$

Step 2

Calculate

$$S_F(\tilde{\tilde{A}}^{i^L}, \tilde{\tilde{B}}^{i^L}) = \left[1-\frac{\sum_{i=1}^4\left|\frac{a_i^L+b_i^L}{2}-\frac{c_i^L+d_i^L}{2}\right|}{4}\right]\times$$
$$\frac{\min\left(P_e\left(\tilde{\tilde{A}}^{i^L}\right),P_e\left(\tilde{\tilde{B}}^{i^L}\right)\right)+min\left(\left(\frac{w^L_{\tilde{\tilde{A}}^i}+u^L_{\tilde{\tilde{A}}^i}}{2}\right),\left(\frac{w^L_{\tilde{\tilde{B}}^i}+u^L_{\tilde{\tilde{B}}^i}}{2}\right)\right)}{\max\left(P_e\left(\tilde{\tilde{A}}^{i^L}\right),P_e\left(\tilde{\tilde{B}}^{i^L}\right)\right)+max\left(\left(\frac{w^L_{\tilde{\tilde{A}}^i}+u^L_{\tilde{\tilde{A}}^i}}{2}\right),\left(\frac{w^L_{\tilde{\tilde{B}}^i}+u^L_{\tilde{\tilde{B}}^i}}{2}\right)\right)}$$

and

$$S_F(\tilde{\tilde{A}}^{i^U}, \tilde{\tilde{B}}^{i^U}) = \left[1-\frac{\sum_{i=1}^4\left|\frac{a_i^U+b_i^U}{2}-\frac{c_i^U+d_i^U}{2}\right|}{4}\right]\times$$
$$\frac{\min\left(P_e\left(\tilde{\tilde{A}}^{i^U}\right),P_e\left(\tilde{\tilde{B}}^{i^U}\right)\right)+min\left(\left(\frac{w^U_{\tilde{\tilde{A}}^i}+u^U_{\tilde{\tilde{A}}^i}}{2}\right),\left(\frac{w^U_{\tilde{\tilde{B}}^i}+u^U_{\tilde{\tilde{B}}^i}}{2}\right)\right)}{\max\left(P_e\left(\tilde{\tilde{A}}^{i^U}\right),P_e\left(\tilde{\tilde{B}}^{i^U}\right)\right)+max\left(\left(\frac{w^U_{\tilde{\tilde{A}}^i}+u^U_{\tilde{\tilde{A}}^i}}{2}\right),\left(\frac{w^U_{\tilde{\tilde{B}}^i}+u^U_{\tilde{\tilde{B}}^i}}{2}\right)\right)}$$

Step 3

Calculate the similarity measure between generalized interval valued trapezoidal intuitionistic fuzzy numbers.$\tilde{\tilde{A}}^i$ and $\tilde{\tilde{B}}^i$ as

$$S_F\left(\tilde{\tilde{A}}^i,\tilde{\tilde{B}}^i\right) = \left(S_F\left(\tilde{\tilde{A}}^{i^L},\tilde{\tilde{B}}^{i^L}\right)\right)^p \cdot \left(S_F\left(\tilde{\tilde{A}}^{i^U},\tilde{\tilde{B}}^{i^U}\right)\right)^q$$

Example 3.1. *Let* $\tilde{\tilde{A}}^i, \tilde{\tilde{B}}^i \in GIVTIFN(R)$

$$\begin{aligned}\tilde{\tilde{A}}^i &= [\tilde{\tilde{A}}^{i^L}, \tilde{\tilde{A}}^{i^U}] \\ &= ([(0.3,0.46,0.5,0.7;0.4)(0.34,0.48,0.56,0.74;0.5)]; \\ &\quad [(0.2,0.34,0.6,0.8;0.1)(0.28,0.39,0.62,0.9;0.3)])\end{aligned}$$

$$\begin{aligned}\tilde{\tilde{B}}^i &= \left[\tilde{\tilde{B}}^{i^L}, \tilde{\tilde{B}}^{i^U}\right] \\ &= [(0.2,0.4,0.49,0.52;0.3)(0.3,0.52,0.68,0.74;0.6)]; \\ &\quad [(0.1,0.3,0.5,0.6;0.2)(0.23,0.41,0.7,0.8;0.4)]\end{aligned}$$

Using step 1 and 2 of algorithm , we get

$$S_F\left(\tilde{\tilde{A}}^{i^L},\tilde{\tilde{B}}^{i^L}\right) = 0.7999$$

$$S_F\left(\tilde{\tilde{A}}^{i^U},\tilde{\tilde{B}}^{i^U}\right) = 0.7883$$

taking $p=q=\frac{1}{2}$ *in step 3 i.e.*

$$S_F\left(\tilde{\tilde{A}}^i,\tilde{\tilde{B}}^i\right) = \left(S_F\left(\tilde{\tilde{A}}^{i^L},\tilde{\tilde{B}}^{i^L}\right)\right)^p \cdot \left(S_F\left(\tilde{\tilde{A}}^{i^U},\tilde{\tilde{B}}^{i^U}\right)\right)^q$$

$S_F\left(\tilde{\tilde{A}}^i,\tilde{\tilde{B}}^i\right) = 0.7941$, *Which is the similarity measure between* $\tilde{\tilde{A}}^i$ *and* $\tilde{\tilde{B}}^i$.

Example 3.2. *Let* $\tilde{\tilde{A}}^i, \tilde{\tilde{B}}^i \in GIVTIFN(R)$

$$\begin{aligned}\tilde{\tilde{A}}^i &= [\tilde{\tilde{A}}^{i^L}, \tilde{\tilde{A}}^{i^U}] \\ &= ([(-0.6,-0.42,0.8,0.86;0.3)(-0.53,-0.4,0.84,0.89;0.7)]; \\ &\quad [(-0.7,-0.5,0.85,0.88;0.1)(-0.64,-0.4,0.86,0.94;0.2)])\end{aligned}$$

$$\begin{aligned}\tilde{\tilde{B}}^i &= [\tilde{\tilde{B}}^{i^L}, \tilde{\tilde{B}}^{i^U}] \\ &= ([(-0.45,-0.37,0.58,0.7;0.2)(-0.25,-0.14,0.69,0.8;0.6)]; \\ &\quad [(-0.5,-0.4,0.63,0.8;0.3)(-0.4,-0.2,0.7,0.89;0.4)])\end{aligned}$$

Using step 1 and 2 of algorithm, we get

$$S_F\left(\tilde{\tilde{A}}^{i^L},\tilde{\tilde{B}}^{i^L}\right) = 0.5853$$

$S_F\left(\widetilde{\widetilde{A}}^{iU}, \widetilde{\widetilde{B}}^{iU}\right) = 0.5111$

taking $p = q = \frac{1}{2}$ *in step 3*

i.e. $S_F\left(\widetilde{\widetilde{A}}^{i}, \widetilde{\widetilde{B}}^{i}\right) = \left(S_F\left(\widetilde{\widetilde{A}}^{iL}, \widetilde{\widetilde{B}}^{iL}\right)\right)^p \cdot \left(S_F\left(\widetilde{\widetilde{A}}^{iU}, \widetilde{\widetilde{B}}^{iU}\right)\right)^q$

we obtain

$S_F\left(\widetilde{\widetilde{A}}^{i}, \widetilde{\widetilde{B}}^{i}\right) = 0.5469$, *Which is the similarity measure between* $\widetilde{\widetilde{A}}^{i}$ *and* $\widetilde{\widetilde{B}}^{i}$.

4. Similarity Measure between GIVTFN based on Chen [4]

A similarity measure between $\widetilde{A}, \widetilde{B} \in GIVTFN(R)$

$\widetilde{A} = \left[\widetilde{A}^L, \widetilde{A}^U\right] = [\left(a_1, a_2, a_3, a_4; w_{\widetilde{A}}\right)_L \left(a_1, a_2, a_3, a_4; w_{\widetilde{A}}\right)_U];$

$\widetilde{B} = \left[\widetilde{B}^L, \widetilde{B}^U\right] = [\left(b_1, b_2, b_3, b_4; w_{\widetilde{B}}\right)_L \left(b_1, b_2, b_3, b_4; w_{\widetilde{B}}\right)_U]$

is given as

$$S_{ch}\left(\widetilde{A}, \widetilde{B}\right) = \frac{1}{2} S\left(\widetilde{A}^U, \widetilde{B}^U\right) \times \left(1 + S\left(\widetilde{A}^\Delta, \widetilde{B}^\Delta\right)\right)$$

where

$$S(\tilde{A}^U, \tilde{B}^U) = \left(1 - \sqrt{\tfrac{1}{4}\Sigma_{i=1}^4 \left(a_i^U - b_i^U\right)^2}\right) \times \frac{\min\left(w_{\widetilde{A}}^U, w_{\widetilde{B}}^U\right)}{\max\left(w_{\widetilde{A}}^U, w_{\widetilde{B}}^U\right)}$$

$$S(\widetilde{A}^\Delta, \widetilde{B}^\Delta) = \left(1 - \sqrt{\tfrac{1}{4}\sum_{i=1}^4 \left(|a_i^U - a_i^L| - |b_i^U - b_i^L|\right)^2}\right) \times \left(1 - |w_{\widetilde{A}}^L - w_{\widetilde{B}}^L|\right)$$

Using this method we proposed a similarity measure for GIVTIFNs

4.1 Proposed Algorithm and Numerical Examples for GIVTIFNs

Step:1

$$S\left(\widetilde{\widetilde{A}}^{iU}, \widetilde{\widetilde{B}}^{iU}\right) = \left(1 - \sqrt{\frac{1}{4}\sum_{i=1}^4 \left(\frac{a_i^U + b_i^U}{2} - \frac{c_i^U + d_i^U}{2}\right)^2}\right) \times \frac{\min\left(\frac{w_{\widetilde{\widetilde{A}}^i}^U + u_{\widetilde{\widetilde{A}}^i}^U}{2}, \frac{w_{\widetilde{\widetilde{B}}^i}^U + u_{\widetilde{\widetilde{B}}^i}^U}{2}\right)}{\max\left(\frac{w_{\widetilde{\widetilde{A}}^i}^U + u_{\widetilde{\widetilde{A}}^i}^U}{2}, \frac{w_{\widetilde{\widetilde{B}}^i}^U + u_{\widetilde{\widetilde{B}}^i}^U}{2}\right)}$$

$$S\left(\widetilde{\widetilde{A}}^{i\Delta}, \widetilde{\widetilde{B}}^{i\Delta}\right) = \left(1 - \sqrt{\frac{1}{4}\Sigma_{i=1}^4 \left(|\left(\frac{a_i^U + b_i^U}{2} - \frac{a_i^L + b_i^L}{2}\right)| - |\frac{c_i^U + d_i^U}{2} - \frac{c_i^L + d_i^L}{2}|\right)^2}\right) \times \left(1 - |\frac{w_{\widetilde{\widetilde{A}}^i}^L + u_{\widetilde{\widetilde{A}}^i}^L}{2} - \frac{w_{\widetilde{\widetilde{B}}^i}^L + u_{\widetilde{\widetilde{B}}^i}^L}{2}|\right)$$

Step:2

Calculate the similarity measure between $\widetilde{\widetilde{A}}^{i}, \widetilde{\widetilde{B}}^{i} \in GIVTIFNs$

$$S_{ch}\left(\widetilde{\widetilde{A}}^{i}, \widetilde{\widetilde{B}}^{i}\right) = \frac{1}{2} S\left(\widetilde{\widetilde{A}}^{iU}, \widetilde{\widetilde{B}}^{iU}\right) \times \left(1 + S\left(\widetilde{\widetilde{A}}^{i\Delta}, \widetilde{\widetilde{B}}^{i\Delta}\right)\right)$$

Example 4.1. *Let* $\widetilde{\widetilde{A}}^{i}, \widetilde{\widetilde{B}}^{i} \in GIVTIFN(R)$

$\widetilde{\widetilde{A}}^{i} = \left[\widetilde{\widetilde{A}}^{iL}, \widetilde{\widetilde{A}}^{iU}\right]$

$= ([(0.3, 0.46, 0.5, 0.7; 0.4)(0.34, 0.48, 0.56, 0.74; 0.5)];$
$[(0.2, 0.34, 0.6, 0.8; 0.1)(0.28, 0.39, 0.62, 0.9; 0.3)])$

$\widetilde{\widetilde{B}}^{i} = \left[\widetilde{\widetilde{B}}^{iL}, \widetilde{\widetilde{B}}^{iU}\right]$

$= ([(0.2, 0.4, 0.49, 0.52; 0.3)(0.3, 0.52, 0.68, 0.74; 0.6)];$
$[(0.1, 0.3, 0.5, 0.6; 0.2)(0.23, 0.41, 0.7, 0.8; 0.4)])$

Using step 1 of algorithm, we find the values of

$S\left(\widetilde{\widetilde{A}}^{iU}, \widetilde{\widetilde{B}}^{iU}\right) = 0.7860$

$S\left(\widetilde{\widetilde{A}}^{i\Delta}, \widetilde{\widetilde{B}}^{i\Delta}\right) = 0.7851$

Using step 2 i.e.

$S_{ch}\left(\widetilde{\widetilde{A}}^{i}, \widetilde{\widetilde{B}}^{i}\right) = \frac{1}{2} S\left(\widetilde{\widetilde{A}}^{iU}, \widetilde{\widetilde{B}}^{iU}\right) \times \left(1 + S\left(\widetilde{\widetilde{A}}^{i\Delta}, \widetilde{\widetilde{B}}^{i\Delta}\right)\right)$

we get

$S_{ch}\left(\widetilde{\widetilde{A}}^{i}, \widetilde{\widetilde{B}}^{i}\right) = 0.7015,$

Which is the similarity measure between $\widetilde{\widetilde{A}}^{i}$ *and* $\widetilde{\widetilde{B}}^{i}$.

Example 4.2. *Let* $\widetilde{\widetilde{A}}^{i}, \widetilde{\widetilde{B}}^{i} \in GIVTIFN(R)$

$\widetilde{\widetilde{A}}^{i} = \left[\widetilde{\widetilde{A}}^{iL}, \widetilde{\widetilde{A}}^{iU}\right]$

$= ([(-0.6, -0.42, 0.8, 0.86; 0.3)(-0.53, -0.4, 0.84, 0.89; 0.7)]$
$[(-0.7, -0.5, 0.85, 0.88; 0.1)(-0.64, -0.4, 0.86, 0.94; 0.2)])$

$\widetilde{\widetilde{B}}^{i} = \left[\widetilde{\widetilde{B}}^{iL}, \widetilde{\widetilde{B}}^{iU}\right]$

$= ([(-0.45, -0.37, 0.58, 0.7; 0.2)(-0.25, -0.14, 0.69, 0.8; 0.6)];$
$[(-0.5, -0.4, 0.63, 0.8; 0.3)(-0.4, -0.2, 0.7, 0.89; 0.4)])$

Using step 1 of algorithm, we find the values of

$S\left(\widetilde{\widetilde{A}}^{iU}, \widetilde{\widetilde{B}}^{iU}\right) = 0.7806$

$S\left(\widetilde{\widetilde{A}}^{i\Delta}, \widetilde{\widetilde{B}}^{i\Delta}\right) = 0.7814$

Using step 2 i.e.

$S_{ch}\left(\widetilde{\widetilde{A}}^{i}, \widetilde{\widetilde{B}}^{i}\right) = \frac{1}{2} S\left(\widetilde{\widetilde{A}}^{iU}, \widetilde{\widetilde{B}}^{iU}\right) \times \left(1 + S\left(\widetilde{\widetilde{A}}^{i\Delta}, \widetilde{\widetilde{B}}^{i\Delta}\right)\right)$

We get

$S_{ch}\left(\widetilde{\widetilde{A}}^{i}, \widetilde{\widetilde{B}}^{i}\right) = 0.6953$

Which is the similarity measure between $\widetilde{\widetilde{A}}^{i}$ *and* $\widetilde{\widetilde{B}}^{i}$.

5. Comparison between two Proposed Similarities

Comparing these two similarity measures the degree of similarity is smaller in S_{ch} than S_F in one set of Examples i.e., [eg.3.1;4.1] and S_{ch} has the high degree of Similarity and in

Illustration	Proposed Algorithm Based on [5] S_F	Illustration	Proposed Algorithm Based on [4] S_{ch}
3.1	0.7941	4.2	0.7015
3.2	0.5469	4.3	0.6953

another set of examples i.e., [eg.3.2;4.2] degree of similarity is smaller in S_F than S_{ch} and S_F has the high degree of Similarity.

6. Conclusion

We have provided the algorithms for similarity measures between two generalized interval-valued trapezoidal intuitionistic fuzzy numbers and two similarity measures for the above said number are proposed. Also to explain the notion some numerical examples for the proposed method have been illustrated also it is important to note that similarity measures distance is smaller, it will be the higher degree of similarity and the larger distance will be the lower degree of similarity.

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