Mathematical modeling in comparative relation of single queue multi server and multi queue multi server queuing models

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Abstract
Customers waiting to get service from server are portrayed by queue and also called waiting line. Unsatisfied customer due to lengthy queue can be potential loss to any service organization. In single queue single server, the above mentioned potential loss is higher. So, we deal with single queue multi server. This motivates us for the study among single queue multi server and multi queue multi server which one is best according to the expected total cost. In this paper, we revealed the total cost with assumption of service cost and waiting cost is better in single queue multiple server than multiple queue multiple server in supermarket checkouts counters.

Keywords
M/M/1 model, M/M/Y model, Single queue–multi server queuing model, service cost, waiting cost, expected total cost.

AMS Subject Classification
65C20.

1. Introduction
The concept of a self-serving bias grocery store was developed by speculator Clarence Saunders and his Piggly Wiggly stores. His first store opened in 1916. He was rewarded a number of patents for the ideas he implemented into his stores. The grocery stores were a financial success and Saunders began to offer franchises. A grocery business obtains and organizes a large assortment of individual suppliers’ products. It organizes and distributes to a chain of retail stores for sale to local customers.

A supermarket is a self service store that offers a wide range of food, drinks, and household goods, arranged into parts and shelves. It is bigger and has a wider selection than previous grocery stores, but is smaller in the product range and more confined than a hypermarket or big-box market.

Big Bazaar is the best supermarkets in India with 200 stores present across India. It has a strong appeal for the urban middle-class. In a supermarket, if we pay for the product in a checkout is known as self checkout. Self-checkout machines also known as self-service checkout and Semi Attended Customer Activated Terminal (SACAT) machines provide consumers with a system to process their own materiel transactions from a supplier [2].
The most intricate of the tasks accomplished by the self-inspect are running concurrently. When the common shoppers look into the initial item, he or she is incited to place the item in a bag. Each item in the ache is linked to this self-inspect system through information accessed by scanning the UPC on the product [3].

2. Literature Review

Waiting lines are a common phenomenon in our day to day life, especially in the province of organizations that are for profit making. This paper explains the analysis of queuing system for the supermarket checkout service unit using queuing models [4]. Recent days waiting time and service systems in queue are important parts of the business world. To get the best service, waiting is unavoidable one, for example in super market, banks, Ticket booking centres, Automated Teller Machine (ATM), etc., Customers are often to wait in queue whenever the service is busy. We have compute to minimize the waiting time of Farmers in Regulated Market Committee (FRMG) at Ulundurpet of Vilupuram district in Tamil Nadu using M/M/1 and M/D/1 models. [4]

Checkout counters are the service gates of supermarkets, which are not only reflecting supermarkets images but associate with supermarkets service quality and business efficiency. Thiyagarajan and Mohan Kumar [5] created approach of On Demand queuing system to detecting the need for patients to be sorted out in a queue and optimized by modern multi-speciality hospital. It involves increased waiting time in queues, patient time delay and individuals queue cutting due to unorganized queuing systems. This helps to right of entry error free data rapidly and also reduce the usage of fraud data given by the persons. This paper the queue sorting is performed by IoT devices through the optimized study of various types of queuing methods, such as single server queue, multi-server parallel queue, and On Demand queue [5]. The multi server multi queue (Isolated queue) initiated and derived by Vijay Prasad and Badshah [6, 7] and also studied M/M/s queuing model for decision making coauthored with Pradeep Porwal [8]. In supermarkets single queue is better than multiple queue is investigated by Xia and Zhang [9].

3. Procedure

In this paper, the queuing system is used in the supermarkets. The following assumptions were made for the queuing system at Thanjavur supermarkets, in accordance with the queuing theory.

Following symbols and terminology are used in the formulation and evolution of queuing models:

3.1 Different Types of Queuing Models

Various types of queuing models are in the queuing theory. They can be depicted as in Figures 1, 2 and 3.

3.2 M/M/1 MODEL

In this model arrivals are confirmed by Poisson probability distribution and come from an infinite population. Queue discipline is FCFS and service time constitutes an exponential distribution. is the probability distribution of the queue length and can be estimated the queuing system as a Poisson process.

\[
L_s = \sum_{n=0}^{\infty} nP_n = \sum_{n=0}^{\infty} n(1 - \rho)\rho^n = \frac{\rho}{(1 - \rho)}
\]

\[
\lambda = \frac{\rho}{\mu - \lambda}
\]

3.3 M/M/Y MODEL

In this section, we are going to discuss the mode as M/M/Y: (α/FCFS) multi server queuing model [2]. It is an extension form of single server model where regular shopper (customer) in a waiting line can be served by more than one server simultaneously.

1. If \(n < Y\) (number of regular shoppers in the system is
The service level in queuing / waiting facility is a function of obtained from giving that service. The combined service rate will be \( \mu_n = n \mu; n < Y \).

2. If \( n \geq Y \) (number of regular shoppers in the system is greater than or equal to the number of servers) then all servers will be busy and the most number of consumers in the queue will be \((n - Y)\). The collective service rate will be \( \mu_n = Y \mu; n \geq Y \).

3. The performance measures for M/M/Y model are given below.

\[
\lambda_n = \lambda \text{ for all } n \geq 0 \\
\mu_n = \begin{cases} 
    n \mu; & n < Y \\
    Y \mu; & n \geq Y 
\end{cases} \\
P_n = \begin{cases} 
    \left( \frac{\mu}{\mu - \lambda} \right)^n P_0; & n \leq Y \\
    \frac{\mu}{\mu - \lambda} P_0; & n > Y 
\end{cases} \\
P_0 = \sum_{n=0}^{Y} \left( \frac{\lambda}{\mu} \right)^n + \sum_{n=Y+1}^{\infty} \lambda \mu \left( \frac{\lambda}{\mu - \lambda} \right)^n P_0 \\
L_q = \frac{1}{(Y - 1)!} \left( \frac{\lambda}{\mu} \right)^Y \frac{\lambda N}{(Y - \lambda)^2} P_0 \\
L_s = L_q + \frac{\lambda}{\mu}.
\]

### 3.4 Formula for Expected Total Cost in M/M/1 and M/M/Y Models

The service level in queuing / waiting facility is a function of the service rate \( \mu \) that balances the following two conflicting cost [1].

- Cost of offering the service
- Cost incurred due to delay in offering the service.

Economic examine of these costs helps the management to make a trade-off between the increased costs of providing best service and reducing waiting time cost \( s \) of customers obtained from giving that service.

Expected total cost

- in M/M/1 and M/M/Y model

\[
E(TC) = E(SC) + E(WC) = C_s + L_s C_w
\]

- in Single–queue–Single server model

\[
E(TC) = E(SC) + E(WC) = SC_s + L_s C_w
\]

- in Single–queue–Multi server model

\[
E(TC) = S[E(SC) + E(WC)] = S[C_s + L_s C_w]
\]

### 3.5 Recitation of the Problem

If we examine \( Y \) number of checkout counters in the supermarket, and the regular shoppers arrival rate is \( \lambda \) and the service rate of all checkout counters in \( \mu \). We have to calculate and compare all the values in both models.

- If there is only one queue, then the system taken as M/M/Y queuing system and using multi server queuing model to find all the performance measures.
- If there are \( Y \) queues in the system taken as \( Y \) isolated M/M/1 queuing systems, then we will apply single server queuing model to calculate (or) regular shoppers arrival rate becomes \( \lambda / S \).

### 4. Comparative Study

Suppose we have \( Y = 2 \) and \( 3 \) checkout counters in the supermarket, the regular shopper’s arrival rate is \( \lambda = 90 \) and the service rate is \( \mu = 70 \) to evaluate the expected total cost assumed that service cost is Rs. 250 and waiting cost is Rs. 300 in both models. Measured performance outcome is shown in Table 1.

<table>
<thead>
<tr>
<th>No</th>
<th>make tracks counters</th>
<th>( \lambda )</th>
<th>( \mu )</th>
<th>Single Queue</th>
<th>Isolated Queue</th>
<th>Total cost for M/M/1</th>
<th>Total cost for M/M/Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>2</td>
<td>90</td>
<td>70</td>
<td>0.2173</td>
<td>0.9051</td>
<td>2.1908</td>
<td>45</td>
</tr>
<tr>
<td>2.</td>
<td>3</td>
<td>90</td>
<td>70</td>
<td>0.2679</td>
<td>0.1245</td>
<td>1.4102</td>
<td>30</td>
</tr>
</tbody>
</table>

### 5. Conclusion

We evaluated and compared the expected total cost for single queue multi server model and multi queue multi server model in supermarket checkout counters. Hence, we have concluded that the expected total cost is lesser in M/M/1 model while than M/M/Y model.

In supermarkets, during festival seasons single queue multiple server is preferable. For multiple queue more spaces needed, to overcome such a problem the management can avail single queue multiple server model instead of multiple queue multiple server model. Then the management can utilize that space to display for their products which may profit to the management.
References