



# On maximize fuzzy net present value in project scheduling in fuzzy environment

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## Abstract

In this paper, fuzzy cash flow weight and fuzzy discounted cash flow weight have been studied with the aid of interval valued fuzzy numbers in project scheduling. Relevant arithmetic operations of interval valued fuzzy numbers are defined to justify the notion. A numerical example is also included.

## Keywords

Project Scheduling, Cash Flow, Net Present Value, Interval Valued Fuzzy Numbers.

## AMS Subject Classification

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## 1. Introduction

Net Present Value (NPV) is used to evaluating the effec-

tiveness of investment projects. NPV is the sum of discounted net benefits over the investment project, cash flow analysis, investigating the outcome of an economical project. Project scheduling in the process where the various activities that need to be undertaken that can be manage the activities that need to be undertaken during the development of a project.

Christenson [4] introduced the construction of present value tables for use in evaluation of capital investment opportunities. Elmaghraby [6] the net present value of projects in the scheduling activities. Chiu [3], introduced fuzzy cash flow analysis using present worth criterion. Baroum [1], Kucha [9] and Neumann [11] develop the cash flow weight procedures for maximizing net present value of a project in fuzzy capital budgeting. Mika [10] simulated the multi-mode resource constrained project scheduling with positive discounted cash flows and different payment models. Bushan Rao [2] discussed the discrete-continuous project scheduling with discounted cash flows. Sung [12] and Creemers [5] used Markovian PERT networks with stochastic activity duration to maximize expected net present value. Hartmann [7] employed the maximizing the net present value of a project under uncertainty. Stephen Dinagar [13] discussed interval valued fuzzy numbers with new ranking.

In this work, Section 1 is introductory in nature. In Section 2, the basic definitions are discussed which used in this whole paper. Section 3 deals with the cash flow weight as well as the discounted cash flow weight. Moreover fuzzy cash flow weight and fuzzy discounted cash flow weight are introduced. The corresponding algorithm are studied with the

aid of Interval Valued Fuzzy Number. In Section 4, numerical illustrations are given. Conclusion given in Section 5.

## 2. Preliminaries

In this section, some basic definitions and arithmetic operations are reviewed.

**Definition 2.1.** A fuzzy set  $\tilde{A}$  in a universe of discourse  $X$  is defined as the following set of pairs  $\tilde{A} \equiv \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ . Here  $\mu_{\tilde{A}}: x \rightarrow [0, 1]$  is a mapping called the membership value of  $x \in X$  in a fuzzy set  $\tilde{A}$ .

**Definition 2.2.** The  $\alpha$ -level set (or interval of confidence at level  $\alpha$  (or  $\alpha$ -cut) of the fuzzy set  $\tilde{A}$  of  $X$  is a crisp set  $A_\alpha$  that contains all the elements of  $X$  that have membership values in  $A$  greater than or equal to  $\alpha$ . i.e.,  $A_\alpha = \{x : \mu_{\tilde{A}}(x) \geq \alpha, x \in X, \alpha \in [0, 1]\}$

**Definition 2.3.** A fuzzy set  $\tilde{A}$ , defined on the set of real numbers  $R$  is said to be a fuzzy number if its membership function has the following characteristics.

1.  $\tilde{A}$  is convex, i.e.,  $\tilde{A}(\lambda X_1 + (1 - \lambda)X_2) = \min\{\tilde{A}(X_1), \tilde{A}(X_2)\}$ , for all  $X_1, X_2 \in R$  and  $\lambda \in [0, 1]$
2.  $\tilde{A}$  is normal i.e., there exists an  $X_0 \in R$  such that  $\tilde{A}(X_0) = 1$
3.  $\tilde{A}$  is piecewise continuous.

**Definition 2.4.** An Interval valued fuzzy number  $\tilde{A}$  on  $R$  is given by  $\tilde{A} = \{x, (\mu_{\tilde{A}}^L(x), \mu_{\tilde{A}}^U(x)), x \in R\}$

and  $\mu_{\tilde{A}}^L(x) \leq \mu_{\tilde{A}}^U(x)$  for all  $x \in R$ .

And it is denoted by  $\tilde{A} = [\tilde{A}^L, \tilde{A}^U]$ , where  $\tilde{A}^L = (a_1^L, a_2^L, a_3^L, a_4^L)$  and  $\tilde{A}^U = (a_1^U, a_2^U, a_3^U, a_4^U)$  are the trapezoidal fuzzy numbers.

It is also noted that  $a_1^U \leq a_1^L, a_2^U \leq a_2^L, a_3^U \leq a_3^L, a_4^U \leq a_4^L$ .

### 2.1 Pictorial Representation

Let  $\tilde{A} = [(2, 4, 5, 7), (1, 3, 6, 8)]$

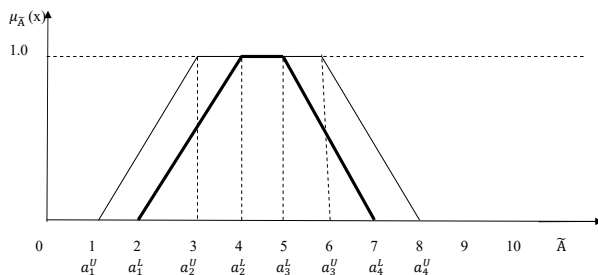


Figure 2.1. IVFN  $\tilde{A}$

**Definition 2.5.** An efficient for comparing the fuzzy number is by the use of ranking function defined  $\mathcal{R} : F(R) \rightarrow R$ , where  $F(R)$  is a set of fuzzy numbers defined on a set of real numbers, which maps each fuzzy number into a real number where a natural order exists.

For  $\tilde{A} = (a_1^L, a_1^U) \in F(R)$ , then the ranking function  $\mathcal{R} : F(R) \rightarrow R$  is defined as:

$$\mathcal{R}(\tilde{A}) = (a_1^L + a_2^L + a_3^L + a_4^L + a_1^U + a_2^U + a_3^U + a_4^U)/8$$

**Definition 2.6.** A fuzzy number  $\tilde{A}$  is said to be trapezoidal fuzzy number if its membership function  $\mu_{\tilde{A}} : x \rightarrow [0, 1]$  has the following characteristic function:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x - a_1)}{(a_2 - a_1)}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \frac{(x - a_4)}{(a_3 - a_4)}, & a_3 \leq x \leq a_4 \\ 0, & \text{otherwise} \end{cases}$$

**Definition 2.7.** A fuzzy number  $\tilde{A}$  is said to be an Interval valued fuzzy number if its membership function  $\mu_{\tilde{A}} : x \rightarrow [0, 1]$  has the following characteristic function:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1^L}{a_2^L - a_1^L}, & a_1^L \leq x \leq a_2^L \\ \frac{a_4^L - x}{a_4^L - a_3^L}, & a_3^L \leq x \leq a_4^L \\ 1, & a_2^U \leq x \leq a_3^U \text{ and } a_2^L \leq x \leq a_3^L \\ \frac{x - a_1^U}{a_2^U - a_1^U}, & a_1^U \leq x \leq a_2^U \\ \frac{a_4^U - x}{a_4^U - a_3^U}, & a_3^U \leq x \leq a_4^U \\ 0, & \text{otherwise} \end{cases}$$

### 2.2 Arithmetic Operations on IVFNS

In this section, arithmetic operations between two Interval valued fuzzy numbers, defined on universal set of real numbers  $R$ .

$$\text{Let } \tilde{A} = [(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)]$$

$$\text{and } \tilde{B} = [(b_1^L, b_2^L, b_3^L, b_4^L), (b_1^U, b_2^U, b_3^U, b_4^U)]$$

then we define

(i) **Addition:**

$$\tilde{A} \oplus \tilde{B} = \left[ (a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L), (a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U) \right]$$

(ii) **Subtraction:**

$$\tilde{A} \ominus \tilde{B} = \left[ (a_1^L - b_4^L, a_2^L - b_3^L, a_3^L - b_2^L, a_4^L - b_1^L), (a_1^U - b_4^U, a_2^U - b_3^U, a_3^U - b_2^U, a_4^U - b_1^U) \right]$$



**(iii) Multiplication:**

$$\tilde{A} \otimes \tilde{B} = \left[ \begin{array}{l} (a_1^L \cdot \mathcal{R}(\tilde{B}), a_2^L \cdot \mathcal{R}(\tilde{B}), a_3^L \cdot \mathcal{R}(\tilde{B}), a_4^L \cdot \mathcal{R}(\tilde{B})), \\ (a_1^U \cdot \mathcal{R}(\tilde{B}), a_2^U \cdot \mathcal{R}(\tilde{B}), a_3^U \cdot \mathcal{R}(\tilde{B}), a_4^U \cdot \mathcal{R}(\tilde{B})) \end{array} \right],$$

if  $\mathcal{R}(\tilde{B}) \geq 0$ .

$$\tilde{A} \otimes \tilde{B} = \left[ \begin{array}{l} (a_4^L \cdot \mathcal{R}(\tilde{B}), a_3^L \cdot \mathcal{R}(\tilde{B}), a_2^L \cdot \mathcal{R}(\tilde{B}), a_1^L \cdot \mathcal{R}(\tilde{B})), \\ (a_4^U \cdot \mathcal{R}(\tilde{B}), a_3^U \cdot \mathcal{R}(\tilde{B}), a_2^U \cdot \mathcal{R}(\tilde{B}), a_1^U \cdot \mathcal{R}(\tilde{B})) \end{array} \right],$$

if  $\mathcal{R}(\tilde{B}) < 0$ .

**(iv) Division:**

$$\begin{aligned} \frac{\tilde{A}}{\tilde{B}} &= [1/\mathcal{R}(\tilde{B})(a_1^L, a_2^L, a_3^L, a_4^L), 1/\mathcal{R}(\tilde{B})(a_1^U, a_2^U, a_3^U, a_4^U)], \\ &\text{if } \mathcal{R}(\tilde{B}) > 0 \\ &= [1/\mathcal{R}(\tilde{B})(a_4^L, a_3^L, a_2^L, a_1^L), 1/\mathcal{R}(\tilde{B})(a_4^U, a_3^U, a_2^U, a_1^U)], \\ &\text{if } \mathcal{R}(\tilde{B}) < 0. \end{aligned}$$

**(v) Scalar Multiplication:**

If  $k \geq 0$  and  $k \in \mathcal{R}$ , then

$$k\tilde{A} = [(ka_1^L, ka_2^L, ka_3^L, ka_4^L), (ka_1^U, ka_2^U, ka_3^U, ka_4^U)].$$

If  $k < 0$  and  $k \in \mathcal{R}$  then

$$k\tilde{A} = [(ka_4^L, ka_3^L, ka_2^L, ka_1^L), (ka_4^U, ka_3^U, ka_2^U, ka_1^U)].$$

### 3. Fuzzy Net Present Value [8]

**3.1 Power Rule For IVFNs**

$\tilde{A} = [(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)]$  and  $\lambda$  is any scalar, then

$$\begin{aligned} &[(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)]^\lambda \\ &= \begin{cases} \left[ \begin{array}{l} ((a_1^L)^\lambda, (a_2^L)^\lambda, (a_3^L)^\lambda, (a_4^L)^\lambda), \\ ((a_1^U)^\lambda, (a_2^U)^\lambda, (a_3^U)^\lambda, (a_4^U)^\lambda) \end{array} \right], & \text{if } \lambda \geq 0 \\ \left[ \begin{array}{l} ((a_4^L)^\lambda, (a_3^L)^\lambda, (a_2^L)^\lambda, (a_1^L)^\lambda), \\ ((a_4^U)^\lambda, (a_3^U)^\lambda, (a_2^U)^\lambda, (a_1^U)^\lambda) \end{array} \right], & \text{if } \lambda < 0 \end{cases} \end{aligned}$$

$\forall \lambda \in \mathcal{R}$ .

**3.2 Cash Flow Weight**

A project is a network with activities ( $A, i = 1, 2, \dots, N$ ) represented as nodes, relations between activities represented as arcs, the resources required by activities denoted by  $r_{ik}$  ( $i = 1, 2, \dots, N$  and  $k = 1, 2, \dots, m$ ) the total resources available for the project denoted by  $r_{ik}$  ( $k = 1, 2, \dots, m$ ), and durations of the activities denoted by  $d_i$  ( $i = 1, 2, \dots, N$ ). Net cash flows of activities occur at the beginning or end of the related activity

and the value of it is independent of the starting or ending moment of the activity. The sum of all the cash flows from different activities starting or finishing in moment  $j$  will be denoted as  $CF_j$  ( $j = 1, 2, \dots, T^H$ ) where  $T^H$  denotes time horizon). Present value ( $PV$ ) of a single future payment occurred in the end of  $n^{th}$  year from now is given in (3.1) where  $F$  stands for amount of the payment and  $r$  denotes the interest rate (cost of capital).

$$PV = \frac{F}{(1+r)^n} \tag{3.1}$$

The goal is to find a schedule with a maximal  $NPV$  which is sum of all discounted cash flows formulated on (3.2):

$$NPV = \sum_{j=0}^n \frac{CF_j}{(1+r)^j} \tag{3.2}$$

Cash flow weight ( $CFW$ ) heuristic is a heuristic which dynamically selects a high priority activity from available activities for the assignment of resources. In the considered heuristic procedure, the priority of an activity is linked to the cash flows linked to the very activity and all the activities which follow it. The priority is measured by means of cash flow weighting.

**3.3 Cash Flow Weighting**

Cash flow weighting is an assignment of a weight to each activity with respect to the cash flow creating potential of the activity which means the sum of the cash flows occurred from the activity and its successor activities. The cash flow weight heuristic is a forward pass heuristic which selects the activity with the largest  $CFW$  from the list of available activities and attempts to assign it to the earliest possible period with considering precedence and resource constraints. After assignment of an activity, the resource constraints are updated. When the last activity is assigned, the procedure stops.

**3.4 Cash Flow Weight Algorithm**

There are three steps on cash flow weight procedure. In the first step, the cash flow weights of each activity are determined and all activities are included to the list of available activities in an order of  $i$  ( $i = 1, 2, \dots, N$ ) without taking into account the predecessors. In the second step, the activity with the highest  $CFW$  is selected from the top of the list of available activities. In case of a tie, the lowest numbered task is assigned first. If the selected task has predecessors, in order to assign the selected activity as soon as possible, the predecessors of the selected activity are assigned respectively in the increasing order of their indices  $i$  ( $i = 1, 2, \dots, N$ ) and as soon as possible with respect to the resources available. After assignment of the selected activity the available resources are updated. In the third step if there is any unassigned activity second step is repeated, otherwise the project schedule is completed.

**3.5 Discounted Cash Flow Weight**

Discounted cash flow algorithm has the same procedure with cash flow weight algorithm while it deals with discounted



cash flow weights (*DCFWS*) instead of *CFW*s. *DCFWS* for an activity is determined by the summation of cash flow of the activity and the discounted value of all future cash flows of successor activities.

### 3.6 Fuzzy Cash Flow

We consider a project with fuzzy cash flows, linked to the beginning or ending of activities independent of their time setting, fuzzy interest rate. The goal is to find a schedule with a maximal fuzzy NPV, where in comparing the fuzzy NPV we choose one of the relations defined in Section 3.2. Fuzzy present value (*PV*) of a single future payment occurred in the end of *n*th year from now is given in (3.3) where  $\tilde{F}$  stands for fuzzy amount of the payment and  $\tilde{i}$  denotes the fuzzy interest rate.

$$\tilde{P}V = \frac{\tilde{F}}{(1 + \tilde{i})^n} \tag{3.3}$$

The general formula of fuzzy net present value  $N\tilde{P}V$  is given in (3.4), where  $C\tilde{F}_j$  denotes net fuzzy cash flows occurred at time *j*, *n* denotes the useful life of the project and  $\tilde{i}$  denotes the fuzzy interest rate.

$$N\tilde{P}V = \sum_{j=0}^n \frac{C\tilde{F}_j}{(1 + \tilde{i})^j} \tag{3.4}$$

Fuzzy net present value formula for *IVFNS*s is generated on (3.3):

$$N\tilde{P}V = \sum_{\substack{j=0 \\ C\tilde{F}_j < 0}}^n \left\{ \left( \frac{C\tilde{F}_{ja_1^L}}{(1 + \tilde{i}_{a_1^L})^j}, \frac{C\tilde{F}_{ja_2^L}}{(1 + \tilde{i}_{a_2^L})^j}, \frac{C\tilde{F}_{ja_3^L}}{(1 + \tilde{i}_{a_3^L})^j}, \frac{C\tilde{F}_{ja_4^L}}{(1 + \tilde{i}_{a_4^L})^j} \right), \right. \\ \left. \left( \frac{C\tilde{F}_{ja_1^U}}{(1 + \tilde{i}_{a_1^U})^j}, \frac{C\tilde{F}_{ja_2^U}}{(1 + \tilde{i}_{a_2^U})^j}, \frac{C\tilde{F}_{ja_3^U}}{(1 + \tilde{i}_{a_3^U})^j}, \frac{C\tilde{F}_{ja_4^U}}{(1 + \tilde{i}_{a_4^U})^j} \right) \right\} \\ + \sum_{\substack{j=0 \\ C\tilde{F}_j < 0}}^n \left\{ \left( \frac{C\tilde{F}_{ja_1^L}}{(1 + \tilde{i}_{a_1^L})^j}, \frac{C\tilde{F}_{ja_2^L}}{(1 + \tilde{i}_{a_2^L})^j}, \frac{C\tilde{F}_{ja_3^L}}{(1 + \tilde{i}_{a_2^L})^j}, \frac{C\tilde{F}_{ja_4^L}}{(1 + \tilde{i}_{a_2^L})^j} \right), \right. \\ \left. \left( \frac{C\tilde{F}_{ja_1^U}}{(1 + \tilde{i}_{a_1^U})^j}, \frac{C\tilde{F}_{ja_2^U}}{(1 + \tilde{i}_{a_2^U})^j}, \frac{C\tilde{F}_{ja_3^U}}{(1 + \tilde{i}_{a_2^U})^j}, \frac{C\tilde{F}_{ja_4^U}}{(1 + \tilde{i}_{a_1^U})^j} \right) \right\} \tag{3.5}$$

where  $C\tilde{F}_j = [(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)]$  represents cash flow.

$\tilde{i} = [(\tilde{i}_{a_1^L}, \tilde{i}_{a_2^L}, \tilde{i}_{a_3^L}, \tilde{i}_{a_4^L}), (\tilde{i}_{a_1^U}, \tilde{i}_{a_2^U}, \tilde{i}_{a_3^U}, \tilde{i}_{a_4^U})]$  represent Interest rate.

### 3.7 Fuzzy Cash Flow weighting

Fuzzy cash flow weighting is an assignment of a fuzzy weight to each activity with respect to the fuzzy cash flow creating potential of the activity which means the sum of the cash flows occurred from the activity and its successor activities. In this procedure, the cash flows of the activities are assumed as either negative or positive fuzzy numbers.

## 4. Fuzzy Cash Flow Weight Algorithm

There are the following four steps involved on fuzzy cash flow weight algorithm:

**Step 1:** The fuzzy cash flow weights of each activity which are denoted by  $C\tilde{F}W_i$ , are determined and all activities are added without predecessors to the available list.

**Step 2:**  $C\tilde{F}W$  values are ordered with a method from (3.1)

**Step 3:** The activity with the highest  $C\tilde{F}W$  is selected from the list of precedence available. In case of a tie, the lowest numbered task is assigned first. If the selected task has predecessors, in order to assign the selected activity as soon as possible, the predecessors of the selected activity are assigned respectively. After assignment of the selected activity the resource available list is updated.

**Step 4:** If there is any unassigned activity the third step is repeated, otherwise the project schedule is completed.

## 5. Fuzzy Discounted Cash Flow Weight Algorithm

Fuzzy discounted cash flow algorithm has the same procedure with fuzzy cash flow algorithm while it deals with fuzzy discounted cash flow weights  $DC\tilde{F}W$  instead of  $C\tilde{F}W$ .  $DC\tilde{F}W_i$  for an activity is determined by the summation of cash flow of the activity and the discounted value of all future cash flows of successor activities

## 6. Numerical Illustrations

The fuzzy cash flows occurred at the beginning of the activity, immediate predecessors, durations, and resource requirements for each task are given in Table 6.1. The number of available resources for this project is determined as 5. A network diagram of a project is given in Fig. 6.1 with the cash flows, resource requirements, and durations of the tasks. The project has just one type of resource which is limited to 5 over the project realization time.

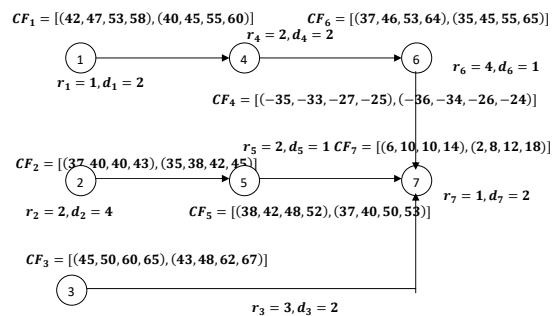


Figure 6.1. Network Diagram of the Project.

### 6.1 Fuzzy Cash Flow Weighting

The calculation of  $DC\tilde{F}W$  for the tasks 1 to 7 is given below. The results, their preference values calculated using the defuzzification value, their pessimistic and optimistic values are



**Table 6.1.** Project Data.

Task Number	Fuzzy Cash Flow	Immediate Predecessors	Duration	Resource Requirement
1	[(42,47,53,58), (40,45,55,60)]	–	2	1
2	[(37,40,40,43), (35,38,42,45)]	–	4	2
3	[(45,50,60,65), (43,48,62,67)]	–	2	3
4	[(-35,-33,-27,-25), (-36,-34,-26,-24)]	1	2	2
5	[(38,42,48,52), (37,40,50,53)]	2,4	1	2
6	[(36,47,54,63), (35,45,55,65)]	4	1	4
7	[(6,10,10,14), (2,8,12,18)]	3,5,6	2	1

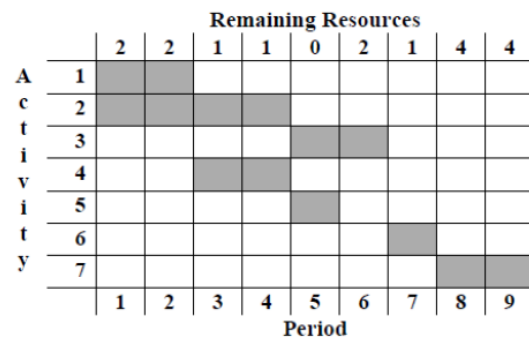
**Table 6.2.** Fuzzy Cash Flow Weights

Task No	Fuzzy Cash Flow	$C\tilde{F}W$	Preference Value	Pessimist Value	Optimist Value
1	[(42,47,53,58), (40,45,55,60)]	[(88,112,137,163), (78,104,146,172)]	125	83	168.5
2	[(37,40,40,43), (35,38,42,45)]	[(81,92,98,109), (74,86,104,116)]	5	77.5	113.5
3	[(45,50,60,65), (43,48,62,67)]	[(51,60,70,79), (45,56,74,85)]	75	48	83
4	[(-35,-33,-27,-25), (-36,-34,-26,-24)]	[(45,66,85,104), (38,59,91,112)]	65	42	109.5
5	[(38,42,48,52), (37,40,50,53)]	[(44,52,58,66), (39,48,62,71)]	60	41.5	69.5
6	[(36,47,54,63), (35,45,55,65)]	[(42,57,64,77), (37,53,67,83)]	55	40	81.5
7	[(6,10,10,14), (2,8,12,18)]	[(6,10,10,14), (2,8,12,18)]	10	8	17

given on Table 6.2.

$$\begin{aligned}
 C\tilde{F}W_1 &= C\tilde{F}_1 + C\tilde{F}_4 + C\tilde{F}_5 + C\tilde{F}_6 + C\tilde{F}_7 \\
 &= [(42, 47, 53, 58), (40, 45, 55, 60)] \\
 &\quad + [(-35, -33, -27, -25), \\
 &\quad \quad (-36, -34, -26, -24)] \\
 &\quad + [(38, 42, 48, 52), (37, 40, 50, 53)] \\
 &\quad + [(36, 47, 54, 63), (35, 45, 55, 65)] \\
 &\quad + [(6, 10, 10, 14), (2, 8, 12, 18)]
 \end{aligned}$$

Ranking of  $C\tilde{F}W$  values of activities are found as:  $C\tilde{F}W_1 > C\tilde{F}W_2 > C\tilde{F}W_3 > C\tilde{F}W_4 > C\tilde{F}W_5 > C\tilde{F}W_6 > C\tilde{F}W_7$  for the optimistic and neutral ranking methods. Activity 1 which has the highest value is scheduled first and the available resources updated as 4 for periods 1–2. Activity 2 which has the next highest value is scheduled in periods 1–4 and available resources are updated as 2 for the periods 1–2, and as 3 for



**Figure 6.2.** Project Schedule Resulting From  $C\tilde{F}W$  Heuristic by Neutral and Optimistic Ranking Methods

the periods 3–4. Activity 4 which has the third highest  $C\tilde{F}W$  value is scheduled in periods 3–4 and available resources are updated as 1 for the 3–4. Activity 3 which has the next highest value is scheduled in periods 5–6 and available resources are



updated as 2 for periods 5–6. Activity 6 which has the next highest value is scheduled in period 7 and available resources for period 7 are updated as 1. Activity 5 which has the next highest value is scheduled in period 5 and available resources for period 5 are updated as 0 and the last activity, Activity 7 is scheduled in periods 8–9 and available resources are updated for periods 8–9 as 4. After scheduling the last activity the algorithm is stopped.

		Remaining Resources									
		2	2	0	0	3	3	3	1	4	4
Activity	1	■	■								
	2	■	■	■	■						
	3			■	■						
	4					■	■				
	5							■			
	6								■	■	
	7									■	■
		1	2	3	4	5	6	7	8	9	10
		Period									

Figure 6.3. Project Schedule Resulting From  $C\tilde{F}W$  Heuristic by Pessimistic Ranking Method

The project schedules resulting from the neutral and optimistic ranking methods for  $C\tilde{F}W$  heuristic is given in Fig. 6.2 The net present value based on  $C\tilde{F}W$  Heuristic by Neutral and Optimistic Ranking Methods is calculated for the fuzzy interest rate  $\tilde{i} = [(0.07, 0.09, 0.09, 0.11) (0.06, 0.8, 0.10, 0.12)]$  as follows:

$$\begin{aligned}
 NPV = & \frac{[(42, 47, 53, 58), (40, 45, 55, 60)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^0} \\
 & + \frac{[(37, 40, 40, 43), (35, 38, 42, 45)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^0} \\
 & + \frac{[(45, 50, 60, 65), (43, 48, 62, 67)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^4} \\
 & + \frac{[(-35, -33, -27, -25), (-36, -34, -26, -24)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^2} \\
 & + \frac{[(38, 42, 48, 52), (37, 40, 50, 53)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^4} \\
 & + \frac{[(36, 47, 54, 63), (35, 45, 55, 65)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^6} \\
 & + \frac{[(6, 10, 10, 14), (2, 8, 12, 18)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^7} \\
 = & [(132.95, 157.72, 183.70, 210.64), \\
 & (123.218, 147.77, 193.64, 218.214)]
 \end{aligned}$$

Ranking of  $C\tilde{F}W$  values of activities are found as:  $C\tilde{F}W_1 > C\tilde{F}W_2 > C\tilde{F}W_3 > C\tilde{F}W_5 > C\tilde{F}W_6 > C\tilde{F}W_4 > C\tilde{F}W_7$  for the pessimistic ranking method. Activity 1 which has the highest value is scheduled first and the available resources updated as 4 for periods 1–2. Activity 2 which has the next highest value is scheduled in periods 1–4 and available resources are updated as 2 for the periods 1–2, and as 3 for the periods 3–4. Activity 3 which has the third highest  $C\tilde{F}W$  value is scheduled in periods 3–4 and available resources are

updated as 0 for the 3–4. Activity 5 which has the next highest value but because of the predecessors, Activity 4 is scheduled in periods 5–6 and available resources are updated as 3 for periods 5–6. Activity 5 which has no predecessor constraint any more is scheduled in period 7 and available resources are updated as 3 for period 7. Activity 6 which has the next highest value is scheduled in period 8 and available resources for period 8 are updated as 1, and the last activity, Activity 7 is scheduled in periods 9–10 and available resources are updated for periods 9–10 as 4. After scheduling the last activity the algorithm is stopped. The project schedules resulting from the pessimistic ranking method for  $C\tilde{F}W$  heuristic is given in Fig. 6.3.

The net present value based on  $C\tilde{F}W$  Heuristic by Neutral and Optimistic Ranking Methods is calculated for the fuzzy interest rate  $\tilde{i} = [(0.07, 0.09, 0.09, 0.11) (0.06, 0.8, 0.10, 0.12)]$  as follows:

$$\begin{aligned}
 NPV = & \frac{[(42, 47, 53, 58), (40, 45, 55, 60)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^0} \\
 & + \frac{[(37, 40, 40, 43), (35, 38, 42, 45)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^0} \\
 & + \frac{[(-35, -33, -27, -25), (-36, -34, -26, -24)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^4} \\
 & + \frac{[(38, 42, 48, 52), (37, 40, 50, 53)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^6} \\
 & + \frac{[(45, 50, 60, 65), (43, 48, 62, 67)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^2} \\
 & + \frac{[(36, 47, 54, 63), (35, 45, 55, 65)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^7} \\
 & + \frac{[(6, 10, 10, 14), (2, 8, 12, 18)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^8} \\
 = & [(137.55, 165.25, 187.78, 210.74), \\
 & (128.1, 151.99, 196.93, 220.81)]
 \end{aligned}$$

### 6.2 Fuzzy Discounted Cash Flow Weighting

The calculation of  $DC\tilde{F}W$  for the tasks 1 to 7 is given below. The results, their preference values calculated using the ranking function, their pessimistic and optimistic values are given on Table 6.3.

$$\begin{aligned}
 DC\tilde{F}W_1 = & DC\tilde{F}_1 + DC\tilde{F}_4 + DC\tilde{F}_5 + DC\tilde{F}_6 + DC\tilde{F}_7 \\
 = & [(63.29, 80.63, 98.38, 114.55), \\
 & (57.19, 74.38, 103.88, 121.17)]
 \end{aligned}$$

Rankings of  $DC\tilde{F}W$  values of activities are found as:

$$\begin{aligned}
 DC\tilde{F}W_1 > DC\tilde{F}W_2 > DC\tilde{F}W_5 > DC\tilde{F}W_3 \\
 > DC\tilde{F}W_4 > DC\tilde{F}W_6 > DC\tilde{F}W_7
 \end{aligned}$$

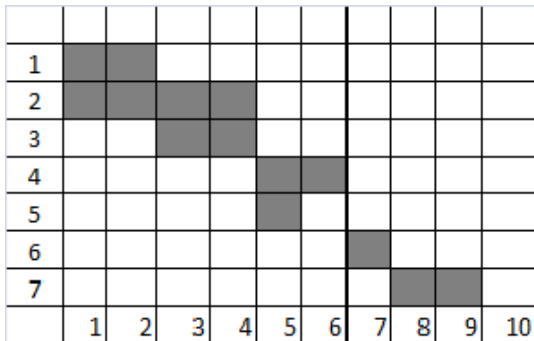
for the neutral ranking method, and  $DC\tilde{F}W_1 > DC\tilde{F}W_5 > DC\tilde{F}W_2 > DC\tilde{F}W_4 > DC\tilde{F}W_3 > DC\tilde{F}W_6 > DC\tilde{F}W_7$  for the



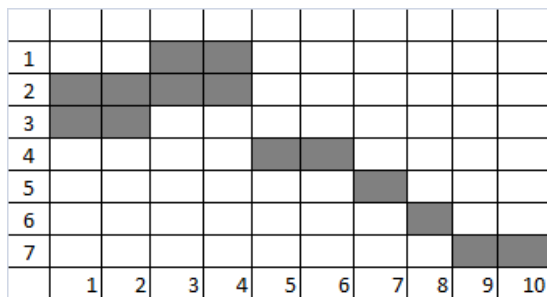
**Table 6.3.** Fuzzy Discounted Cash Flow Weight

Task No	Fuzzy Cash Flow	$DC\tilde{F}W$	Preference Value	Pessimist Value	Optimist Value
1	[(42,47,53,58), (40,45,55,60)]	[(63.29,80.63,98.38,114.58), (57.19,74.38,103.88,121.17)]	89.18	57.24	117.86
2	[(37,40,40,43), (35,38,42,45)]	[(59.5,36.86,70.78,78.13), (54.98,62.76,74.87,82.67)]	65.06	60.24	63.74
3	[(45,50,60,65), (43,48,62,67)]	[(38.15,44.21,51.97,58.03), (34.42,41.57,54.61,61.76)]	43.35	26.68	46.83
4	[(-35,-33,-27,-25), (-36,-34,-26,-24)]	[(24.79,37.53,49.78,61.35), (20.59,33.18,53.48,66.13)]	35.21	22.69	59.89
5	[(38,42,48,52), (37,40,50,53)]	[(28.1,32.91,36.83,41.63), (25.27,30.51,39.22,44.47)]	48.09	36.28	80.4
6	[(36,47,54,63), (35,45,55,65)]	[(24.69,33.43,37.6,45.15), (21.89,31.15,39.28,48.52)]	34.86	23.29	43.05
7	[(6,10,10,14), (2,8,12,18)]	[(3.27,5.46,5.46,7.65), (1.09,4.37,6.55,9.83)]	5.46	2.18	8.74

optimistic ranking. The difference between neutral ranking method and optimistic ranking method is on Activity 2 and Activity 5. The Activity 2 should be scheduled first due to it is predecessor of Activity 5. So these two rankings result on the same schedule which is shown in Fig. 6.4. The project schedules resulting from the neutral and optimistic ranking methods for  $DC\tilde{F}W$  heuristic is given in Fig. 6.4.



**Figure 6.4.** Project Schedule Resulting From  $CFW$  Heuristic by Neutral and Optimistic Ranking Methods



**Figure 6.5.** Project Schedule Resulting From  $CFW$  Heuristic by Pessimistic Ranking Method

The net present value based on  $DC\tilde{F}W$  Heuristic by Neutral and Optimistic Ranking Methods is calculated for the fuzzy interest rate

$\tilde{i} = [(0.07, 0.09, 0.09, 0.11) (0.06, 0.8, 0.10, 0.12)]$  as follows:

$$\begin{aligned}
 NPV = & \frac{[(42, 47, 53, 58), (40, 45, 55, 60)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^0} \\
 & + \frac{[(37, 40, 40, 43), (35, 38, 42, 45)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^0} \\
 & + \frac{[(45, 50, 60, 65), (43, 48, 62, 67)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^2} \\
 & + \frac{[(-35, -33, -27, -25), (-36, -34, -26, -24)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^4} \\
 & + \frac{[(38, 42, 48, 52), (37, 40, 50, 53)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^4} \\
 & + \frac{[(36, 47, 54, 63), (35, 45, 55, 65)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^6} \\
 & + \frac{[(6, 10, 10, 14), (2, 8, 12, 18)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^7} \\
 = & [(144.02, 169.24, 196.4, 220.44), \\
 & (134.13, 159.04, 205.91, 230.93)]
 \end{aligned}$$

Ranking of  $DC\tilde{F}W$  values of activities are found as:

$$\begin{aligned}
 DC\tilde{F}W_2 > DC\tilde{F}W_1 > DC\tilde{F}W_5 > DC\tilde{F}W_3 \\
 > DC\tilde{F}W_6 > DC\tilde{F}W_4 > DC\tilde{F}W_7
 \end{aligned}$$

for the pessimistic ranking method. The project schedules resulting from the pessimistic ranking methods for  $DC\tilde{F}W$  heuristic is given in Fig. 6.5.

The net present value based on  $DC\tilde{F}W$  Heuristic by Optimistic Method is calculated for the fuzzy interest rate  $\tilde{i} = [(0.07, 0.09, 0.09, 0.11) (0.06, 0.8, 0.10, 0.12)]$  as follows:



$$\begin{aligned}
 N\tilde{P}V &= \frac{[(42, 47, 53, 58), (40, 45, 55, 60)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^2} \\
 &+ \frac{[(37, 40, 40, 43), (35, 38, 42, 45)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^0} \\
 &+ \frac{[(45, 50, 60, 65), (43, 48, 62, 67)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^2} \\
 &+ \frac{[(-35, -33, -27, -25), (-36, -34, -26, -24)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^4} \\
 &+ \frac{[(38, 42, 48, 52), (37, 40, 50, 53)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^6} \\
 &+ \frac{[(36, 47, 54, 63), (35, 45, 55, 65)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^7} \\
 &+ \frac{[(6, 10, 10, 14), (2, 8, 12, 18)]}{[(1.07, 1.09, 1.09, 1.11), (1.06, 1.08, 1.10, 1.12)]^8} \\
 &= [(138, 162, 188.75, 211.83), \\
 &\quad (128.5, 152.51, 197.9, 221.86)]
 \end{aligned}$$

## 7. Conclusion

In this paper, two distinct heuristic methods are proposed for project scheduling to maximize fuzzy net present value of a project. The scheduling resulting from  $C\tilde{F}W$  and  $DC\tilde{F}W$  heuristics are different which make differences on project's fuzzy net present value with the aid of interval valued fuzzy numbers (IVFNs).

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