A new method for solving fully fuzzy linear programming problems

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Abstract
The point of this paper is to propose another technique for locate the fully fuzzy ideal answer for the completely fuzzy straight programming issues. It is helpful to apply the proposed new technique to analyze the current strategy for tackling fully fuzzy linear programming (FFLP) issues. This ground-breaking system is delineated by a model.

Keywords
Fuzzy Triangular number, Ranking capacity, completely fuzzy straight programming issues, ideal arrangement, customary simplex technique.

AMS Subject Classification
03E72.

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Article History: Received 01 February 2020; Accepted 29 March 2020

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1. Introduction

The issue of straight programming (LP) is one of the most punctual detailed issues in numerical programming where a direct capacity must be augmented (limited) over an arched imperative polyhedron $X$. In ordinary methodology parameters of direct programming models must be characterized and exact. Be that as it may, in actuality, circumstances, this is definitely not a reasonable supposition. Bellman and Zadeh \cite{1} proposed the idea of dynamic in fuzzy situations. From that point forward, a considerable lot of analysts have displayed their enthusiasm to tackle the fuzzy straight programming issues \cite{2-6}. Lotfiet al. \cite{11} called attention to that there is no technique in writing for finding the fuzzy ideal arrangement of completely fuzzy direct programming (FFLP) issues and proposed another strategy to locate the fuzzy ideal arrangement of FFLP issues with uniformity imperatives. Dehghal et al. \cite{5} proposed a fuzzy straight programming approach for finding the specific arrangement of Fully Fuzzy Linear System (FFLS) of conditions by utilizing the current technique, the got arrangements are surmised not definite and furthermore it is extremely hard to apply the current strategy to discover fuzzy ideal arrangement of FFLP issues.

Right now, deficiency of existing general type of completely fuzzy direct programming issues are survived and new broad type of completely fuzzy straight programming issues is proposed. The target work in fuzzy structure can be changed over into fresh structure by utilizing positioning capacities. An elective way to deal with the simplex technique for arrangements of straight writing computer programs is recommended. This methodology generally includes less emphasis than in the simplex technique or at the most equivalent number. To delineate the proposed strategy, numerical models and their outcomes are examined.
2. Preliminaries

Definition 2.1. The trademark capacity of a fresh set appoints a worth either 0 or 1 to each number in X. This capacity can be summed up to a capacity – to such an extent that the worth doled out to the component of the widespread set X fall inside a predetermined range.

i.e., the relegated esteem shows the participation evaluation of the component in the set A.

The capacity \( \mu_A \) is known as the participation work and the set \( \hat{A} = \{ (x, \mu_A(x)) : x \in X \} \) characterized by \( \mu_A(x) \) for each is known as a fuzzy set.

The function \( \mu_A \) is called the membership function and the set \( \hat{A} = \{ (x, \mu_A(x)) : x \in X \} \) defined by \( \mu_A(x) \) for each \( x \in X \) is called a fuzzy set.

Definition 2.2. A fuzzy set A, characterized on the general arrangement of genuine number R, is said to be a fuzzy number if and just if its enrolment work has the accompanying qualities.

A is convex, that is

1. A is convex, that is
   \[ \mu_A (\lambda x_1 + (1 - \lambda) x_2) \geq \min (\mu_A (x_1), \mu_A (x_2)) \]
   for all \( x_1, x_2 \in R, \lambda \in [0, 1] \).

2. \( \hat{A} \) is normal there exists \( x_0 \in R \) that \( \mu_{\hat{A}} (x_0) = 1 \).

3. \( \mu_A \) is piecewise continuous

Definition 2.3. A fuzzy number is said to be a triangular fuzzy number if its participation work is given by

\[ \mu_A(x) = \begin{cases} 
\frac{(x - x_1)}{(x_2 - x_1)}, & x_1 \leq x \leq x_2 \\
\frac{(x_2 - x)}{(x_2 - x_3)}, & x_2 \leq x \leq x_3 \\
0, & \text{otherwise}
\end{cases} \]

Definition 2.4. A triangular fuzzy number \( \hat{A} = (x_1, x_2, x_3) \) is said to be non-negative fuzzy number if and only if \( x_1 \geq 0 \).

Definition 2.5. Two triangular fuzzy numbers \( \hat{A} = (x_1, x_2, x_3) \) and \( \hat{B} = (y_1, y_2, y_3) \) are said to be equivalent if and just if \( x_1 = y_1, x_2 = y_2, x_3 = y_3 \).

Definition 2.6. A positioning capacity is a capacity, where is a lot of fuzzy numbers characterized on set of genuine numbers which maps each fuzzy number into the genuine line, where a characteristic request exists. Let \( \hat{A} = (x_1, x_2, x_3) \) be a triangular fuzzy number that \( \mathfrak{R} : F(R) \rightarrow R \).

\[ \mathfrak{R}(\hat{A}) = \frac{x_1 + x_2 + x_3}{3} \]

2.1 Arithmetic Operations

Let \( \hat{A} = (x_1, x_2, x_3) \) and \( \hat{B} = (y_1, y_2, y_3) \) be two triangular fuzzy numbers then

1. \( \hat{A} + \hat{B} = (x_1 + y_1, x_2 + y_2, x_3 + y_3) \)
2. \( -\hat{A} = (-x_1, -x_2, -x_3) \)
3. \( \hat{A} + \hat{B} = (x_1 + y_1, x_2 + y_2, x_3 + y_3) \)
4. Let \( \hat{A} = (e, f, g) \) be any triangular fuzzy number and \( \hat{B} = (l, m, n) \) be a non-negative fuzzy number then

\[ \hat{A} \cdot \hat{B} = \begin{cases} 
(\hat{e}l, \hat{f}m, \hat{g}n), & e \geq 0 \\
(\hat{e}m, \hat{f}g, \hat{g}n), & e < 0, g \geq 0 \\
(\hat{e}g, \hat{f}m, \hat{g}l), & g < 0
\end{cases} \]

3. Completely (Fully) Fuzzy Linear Programming Problem

Direct writing computer programmes is the customary methodology and the estimation of the parameters of straight programming model must be all around characterized and exact. Notwithstanding, in viable condition, this is definitely not a practical suspicion. In the genuine issues there may exists vulnerability about the parameter. Like that circumstance the parameters of direct programming issues might be spoken to as fuzzy numbers. Completely Fuzzy Linear Programming (FFLP) issues with \( m \) fuzzy imperatives and \( n \) fuzzy factors might be detailed. Like that situation the parameters of linear programming problems may be represented as fuzzy numbers. Fully Fuzzy Linear Programming (FFLP) problems with \( m \) fuzzy constraints and \( n \) fuzzy variables may be formulated as follows.

Maximize (or Minimize) \( z = \mathcal{C}^T \cdot \hat{X} \)

Subject to \( \hat{A} \cdot \hat{X} = \hat{b}, \hat{X} \) is a non-negative fuzzy number where

\[ \mathcal{C}^T = [C_j]_{1 \times n}, \quad \hat{X} = [\hat{x}_j]_{n \times 1} \]

\[ \hat{A} = [\hat{a}_{ij}]_{m \times n}, \quad \hat{b} = [\hat{b}_i]_{m \times 1} \]

and \( \hat{a}_{ij}, \hat{c}_j, \hat{x}_j, \hat{b}_i \in F(R) \)

The fuzzy ideal arrangement of FFLP issue will be a fuzzy number in the event that it fulfills the accompanying attributes.

1. \( \hat{X} \geq 0 \) and is a fuzzy number
2. \( \hat{A} \hat{X} = \hat{b} \)
3. In the event that there exists any non-negative fuzzy number with the end goal that, at that point \( \hat{X} \) such that \( \hat{A} \hat{X} = \hat{b} \), then

\( \mathfrak{R}(\mathcal{C}^T \hat{X}) > \mathfrak{R}(\mathcal{C}^T \hat{X}'), \) (Maximization case)

and \( \mathfrak{R}(\mathcal{C}^T \hat{X}) < \mathfrak{R}(\mathcal{C}^T \hat{X}'), \) (Minimization case).
Let $\hat{X}$ is a fuzzy optimal solution of fully fuzzy linear programming problem. If there exists a fuzzy number $\hat{y}$ such that

1. $\hat{y} \geq 0$ and is a fuzzy number
2. $\hat{A}\hat{y} = \hat{b}$
3. $\Re(\hat{C}^T \hat{X}) = \Re(\hat{C}^T \hat{Y})$

at that point is said to be an option fuzzy ideal arrangement of the above mentioned.

### 4. Proposed technique to locate the fuzzy ideal arrangement of FFLP issues

Another strategy is proposed to locate the fuzzy ideal arrangement of the accompanying kind of FFLP issues

$max \text{ (or min)} \quad z = \left( \hat{C}^T \hat{X} \right)$

Subject to $\hat{A}\hat{X} = \hat{b}$, $\hat{X}$ is a non-negative fuzzy number

**Step 1:** Substituting $\hat{C}^T = [c_{ij}]_{i \times m}, \hat{X} = [x_{ij}]_{n \times 1}, \hat{A} = [a_{ij}]_{m \times n},$ $\hat{b} = [b_i]_{m \times 1}$ the above FFLP issue might be composed as

$$\max \text{ (or min)} \quad \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j$$

$s.t. \sum_{j=1}^{n} \tilde{a}_{ij} \tilde{x}_j = \tilde{b}_i, i = 1, 2, \ldots, m, \tilde{x}_j \geq 0$ and is a fuzzy triangular number.

**Step 2:** If all the parameters $\tilde{c}_j, \tilde{x}_j, \tilde{a}_{ij}$ and $\tilde{b}_i$ are spoken to by triangular fuzzy numbers $(p_j, q_j, r_j), (x_j, y_j, z_j), (a_{ij}, b_{ij}, c_{ij})$ and $(b_i, g_i, h_i)$ separately then the FFLP issue may be composed as

$$\max \text{ (or min)} \quad \sum_{j=1}^{n} (p_j, q_j, r_j) \cdot (x_j, y_j, z_j)$$

$s.t. \sum (a_{ij}, b_{ij}, c_{ij}) \cdot (x_j, y_j, z_j) = (b_i, g_i, h_i),$

$$\forall i = 1, 2, \ldots, m (x_j, y_j, z_j) \geq 0$$

and is a fuzzy triangular number.

**Step 3:** Assuming $(a_{ij}, b_{ij}, c_{ij}) \cdot (x_j, y_j, z_j) = (m_{ij}, n_{ij}, o_{ij})$

The given issue takes the structure

$$\max \text{ (or min)} \quad \Re \left( \sum_{j=1}^{n} (p_j, q_j, r_j) \cdot (x_j, y_j, z_j) \right)$$

$s.t. \sum (m_{ij}, n_{ij}, o_{ij}) = (b_i, g_i, h_i), \forall i = 1, 2, \ldots, m,$

$(x_j, y_j, z_j) \geq 0$ and is a fuzzy triangular number.

**Step 4:** Equivalent of two fuzzy triangular numbers and their arithmetic operators, the FFLP is changed over the Crisp Linear programming (CLP) issue.

$$\max(\text{or min}) \quad \Re \left( \sum_{j=1}^{n} (p_j, q_j, r_j) (x_j, y_j, z_j) \right)$$

$s.t.$

$$\sum_{j=1}^{n} m_{ij} = b_i, \forall i = 1, 2, \ldots, m; \sum_{j=1}^{n} n_{ij} = g_i, \forall i = 1, 2, \ldots, m$$

$$\sum_{j=1}^{n} o_{ij} = h_i, \forall i = 1, 2, \ldots, m; y_j - x_j \geq 0, z_j - y_j \geq 0,$$

$$\forall j = 1, 2, \ldots, n$$

**Step 5:** Find the ideal arrangement by talking the CLP issue utilizing the accompanying calculation.

**Step 5a:** Obtain an underlying fundamental practical answer for the issue in the structure and put it in the principal segment of the simplex table. In elective technique for answer for CLP initial four stages are same.

**Step 5b:** Compute the net-developments by utilizing the connection where

1. If all then the underlying fundamental practical arrangements is an ideal essential plausible arrangement.

2. If at least one, continue on to the subsequent stage

**Step 5c:** If there are more than one negative at that point pick the entering vector comparing to which, is generally negative. Leave it alone for some $j = r$ and rest of the technique is same as that of simplex strategy.

It is demonstrated that on the off chance that we pick the entering vector with the end goal that is generally negative, at that point the co-operations required are less in certain issues.

This elective methodology indicated that either the emphases required are same or less yet cycles required are never more than those of the simplex technique.

In the event that there are more than one negative, at that point pick the most negative of them. Leave it alone for some $j = r$.

1. If all, then there is an unbounded answer for the given issue.

2. If at least one, then the relating vector enters the premise

**Step 5d:** Compute the proportions and pick the base of them. Leave the base of these proportions alone. At that point the co-operations required are less in certain issues.

**Step 5e:** Convert the main component to solidarity by separating it’s line by the main component itself and every other component in it’s segment to zeros by utilizing the relations

$$\hat{y}_{ij} = y_{ij} - \frac{y_{kj}}{y_{kr}} y_{kr}, \quad i = 1, 2, \ldots, m + 1, \quad i \neq k$$

$$\hat{y}_{kj} = \frac{y_{kj}}{y_{kr}}, \quad j = 1, 2, \ldots, n$$
**Step 5f:** Go to advance 5b and recurrent the computational technique until either an ideal arrangement is acquired or there is an unbounded arrangement.

**Step 6:** Find the fuzzy ideal arrangements by putting the estimations of in \( \tilde{x}_j = (x_j, y_j, z_j) \).

**Step 7:** Find the fuzzy ideal incentive by putting \( \tilde{x}_j \) in
\[
\sum_{j=1}^{n} \tilde{c}_j \cdot \tilde{x}_j
\]

### 5. Numerical Example

Consider the accompanying completely fuzzy straight programming issue and understand it by the proposed strategy.

Maximize \( (1, 2, 3) \cdot \tilde{x}_1 + (2, 3, 4) \cdot \tilde{x}_2 \).

Subject to \((0, 1, 2) \cdot \tilde{x}_1 + (1, 2, 3) \cdot \tilde{x}_2 \leq (2, 10, 24) \)
\((1, 2, 3) \cdot \tilde{x}_1 + (0, 1, 2) \cdot \tilde{x}_2 \leq (1, 8, 21) \)

\( \tilde{x}_1 \) and \( \tilde{x}_2 \) are non-negative triangular fuzzy numbers.

**Solution**

Let \( \tilde{x}_1 = (x_1, y_1, z_1) \) and \( \tilde{x}_2 = (x_2, y_2, z_2) \) then the given FFLP problem can be written as

Maximize \( Z = (1, 2, 3) \cdot (x_1, y_1, z_1) + (2, 3, 4) \cdot (x_2, y_2, z_2) \)

Subject to \((0, 1, 2) \cdot (x_1, y_1, z_1) + (1, 2, 3) \cdot (x_2, y_2, z_2) = (2, 10, 24) \)
\((1, 2, 3) \cdot (x_1, y_1, z_1) + (0, 1, 2) \cdot (x_2, y_2, z_2) = (1, 8, 21) \)

\( (x_1, y_1, z_1) \) and \( (x_2, y_2, z_2) \) are non-negative triangular fuzzy numbers.

Using step (3), given FFLP problem may be written as

Maximize \( 9(1x_1 + 2x_2, 2y_1 + 3y_2, 3z_1 + 4z_2) \)

Subject to \((0x_1 + x_2, y_1 + y_2, 2z_1 + 3z_2) = (2, 10, 24) \)
\((x_1 + 0x_2, 2y_1 + y_2, 3z_1 + 2z_2) = (1, 8, 21) \)

\( (x_1, y_1, z_1) \) and \( (x_2, y_2, z_2) \) are non-negative triangular fuzzy numbers.

Using step (4) and ranking, the above FFLP problem is converted into the following CLP problem.

Maximize \( \frac{1}{3} (x_1 + 2x_2, 2y_1 + 3y_2 + 3z_1 + 4z_2) \)

Subject to \(0x_1 + x_2 = 2 \)
\(x_1 + 0x_2 = 1 \)
\(y_1 + 2y_2 = 10 \)
\(2y_1 + y_2 = 8 \)
\(2z_1 + 3z_2 = 24 \)
\(3z_1 + 2z_2 = 21 \)
\(y_1 - x_1 \geq 0, z_1 - y_1 \geq 0, y_2 - x_2 \geq 0, z_2 - y_2 \geq 0 \)

By applying steps subsequently, we get iteration tables one by one. Now all the net-assessments \((z_j - c_j) \geq 0 \).

Consequently the ideal solution is reached
\[\text{max esteem} = 18, \text{when } x_1 = 1, x_2 = 2, y_1 = 2, y_2 = 4, z_1 = 3, z_2 = 6.\]

Utilizing stage (6) the fuzzy ideal arrangement is given by
\( x_1 = (1, 2, 3), x_2 = (2, 4, 6). \)

Utilizing stage (7) the fuzzy ideal estimation of the given FFLP issue is \((5, 16, 33)\).

### 6. Conclusion

In view of the current examination it tends to be inferred that another strategy is proposed to locate the fuzzy ideal arrangement of FFLP issues by changing over into CLP. Likewise, utilizing elective way to deal with comprehend the CLP rather than conventional strategy, in order to show up the ideal arrangement in least or equivalent number of emphases. To show the proposed technique by a numerical model and are settled.

### References


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Table 1

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$z_j - c_j$ | $\sum y_j$ | $\sum z_j - c_j$ |
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| $5 \rightarrow$ | $1/1$ | $3/1$ | $3/1$ | $5$ | $5$ | $1$ | $1$ | $1$ | $1$ | $1$ |

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$\sum y_j$ | $\sum z_j - c_j$ |
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Proceed like for 6 iterations, the final iterations table by

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$z_j - c_j$ | $\sum y_j$ | $\sum z_j - c_j$ |
| $54/3$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $2/3$ | $1/3$ | $4/9$ | $1/9$ | $2/5$ | $2/15$ |

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ISSN(P):2319 – 3786
Malaya Journal of Matematik
ISSN(O):2321 – 5666
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