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On solving fuzzy transportation problem based on distance based defuzzification method of various fuzzy quantities using centroid

A. Hari Ganesh,[1]* M. Suresh[2] and G. Sivakumar[3]

Abstract

In this paper, we introduce a method for solving simple and multi objective transportation problems with imprecise information based on various fuzzy quantities such as trapezoidal, pentagonal and hexagonal fuzzy numbers by the conversion into crisp information. First, we convert the imprecise information of the Transportation Problem in the form of trapezoidal, pentagonal and hexagonal fuzzy numbers based on proposed fuzzification techniques, and then the fuzzy transportation problems with various fuzzy quantities are converted into crisp transportation problems using appropriate proposed ranking functions introduced in this paper to find its basic and optimum solutions with the help of traditional method. Finally some numerical examples are given to make an effective analysis about the various fuzzy quantities through finding the solutions of transportation problem of imprecise information.

Keywords

Fuzzification, Defuzzification, Ranking function, Transportation Problem, Trapezoidal Fuzzy Number, Pentagonal Fuzzy Number, Hexagonal Fuzzy Number.

AMS Subject Classification

03E72.

[1] *Department of Mathematics, Poompuhar College, (Affiliated to Bharathidasan University), Melaiyur–609107, Tamil Nadu, India.*
[2] *Department of Mathematics, SRM Institute of Science and Technology, Ramapuram, Chennai–600089, Tamil Nadu, India.*
[3] *Department of Mathematics, A.V.V.M. Sri Pushpam College, (Affiliated to Bharathidasan University), Poondi–613503, Tamil Nadu, India.*
***Corresponding author**: [1]ahariganesh84@gmail.com; [2]sureshmaths.kumar@gmail.com

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1. Introduction

In real life situations most of the information is obtained in vague nature. In particular, mostly the data is observed in the form of intervals. The optimization problems involving such type of interval data have to be solved in an effective way to find best solution on today's competitive world. Recently many authors are being involved in solving such type of optimization problems through the concept of fuzzy set theory. The fuzzy set theory was introduced by Zadeh [12] in 1965 to deal this type of imprecise and vague information. In order to have an idea about solving optimization problems under uncertain environment, here we have made a recent survey about solving special type of fuzzy optimization problem such as fuzzy transportation problem based on various fuzzy numbers and its ranking.

There are several papers in the literature in which the fuzzy transportation problems have been solved using various ranking methods of fuzzy numbers. In 2011, Amarpreet Kaur, Amit Kumar [2] extend a classical method to propose a new method for solving fuzzy transportation problems with the help of ranking function. In 2014, Ali Ebrahimnejad [1] proposed a new ranking function for solving fuzzy transportation problem by converting into crisp one. The main contribution in his work is the reduction of the computational complexity of the existing ranking method proposed by Karu and Kumar [3] in the year 2012. In 2015, Iden Hasan Hussein and Anfal Hasan Dheyab [7] have introduced a new algorithm using ranking function for normal and abnormal triangular fuzzy numbers to find solution of fuzzy transportation problem using Vogel's modified distribution algorithm. In 2017, Darunee Hunwisai and Poom Kumam [5] introduced the method for solving fuzzy transportation problem using Robust's ranking technique for the representative value of the fuzzy number. In addition they used allocation table method to find an initial basic feasible solution for the Fuzzy Transportation Problem. In 2018, Mahananda Babasaheb Bhopale [8] proposed a new ranking method for converting fuzzy transportation problem into crisp valued transportation problem to find its optimum solution using MODI method. In the same year, Vidhya and Ganesan [11] presented a methodology for the solution of multi-objective fuzzy transportation problem using a new fuzzy arithmetic on parametric form of trapezoidal fuzzy numbers and a new ranking method. In 2019, Nagar et al. [9] proposed a new fuzzification and defuzzification method to convert Interval data based transportation problem into fuzzy transportation problem to find its optimum solution. Furthermore, they have made a comparative study with the available methods to show the effectiveness of the proposed algorithm in their work. In 2020, Ashok Sahebrao Mhaske and Kirankumar Laxmanrao Bondar [4] have introduced a method to convert the crisp transportation problem into the fuzzy transportation problem by using various types of fuzzy numbers such as triangular, pentagonal, and heptagonal fuzzy numbers. Moreover, they compare the minimum fuzzy transportation cost obtained from the different methods in their work. Dinesh C.S. Bisht and Pankaj Kumar Srivastava [6] proposed a method for solving Interval data based transportation problems through converting it into fuzzy transportation problem and then further conversion into crisp transportation problem using trisectional approach of fuzzification and newly proposed ranking technique based on in-centre concept based on traditional methods in the same year.

In this paper, we propose a methodology to solve the single and multi-objective transportation problem using proposed fuzzification and defuzzification methods of various fuzzy quantities such as trapezoidal, pentagonal and hexagonal fuzzy quantities. Moreover, we have planned to analysis the effectiveness of these fuzzy quantities based on the results obtained using the proposed methods for solving single and bi-objective transportation problems.

2. Preliminaries

Definition 2.1 (Fuzzy Set)**.** *A fuzzy set $\tilde{A}$ in the universal set X is defined as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$. Here $\mu_{\tilde{A}} : A \to [0,1]$ is the grade of the membership function and $\mu_{\tilde{A}}(x)$ is the grade value of $x \in X$ in the fuzzy set A.*

Definition 2.2 (Fuzzy Number)**.** *A fuzzy number $\tilde{A}$ is a subset of real line R, with the membership function satisfying the following properties:*

(i) $\mu_{\tilde{A}}(x)$ *is piecewise continuous in its domain.*

(ii) $\tilde{A}$ *is normal, i.e., there is a* $x_0 \in X$ *such that* $\mu_{\tilde{A}}(x_0) = 1$.

(iii) $\tilde{A}$ *is convex, i.e.,*
$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$,
$\forall x_1, x_2$ *in* X.

Definition 2.3 (Triangular Fuzzy Number)**.** *The fuzzy set $\tilde{A} = (a_1, a_2, a_3; w)$, where $a_1 < a_2 < a_3$ and defined on R, is called the generalized triangular fuzzy number, if the membership function of $\tilde{A}$ is given by*

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1 \\ w\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \leq x \leq a_2 \\ w\left(\frac{a_3-x}{a_3-a_2}\right), & a_2 \leq x \leq a_3 \\ 0, & x > a_3 \end{cases}$$

Definition 2.4 (Trapezoidal Fuzzy Number)**.** *The fuzzy set $\tilde{A} = (a_1, a_2, a_3, a_4; w)$, where $a_1 < a_2 < a_3 < a_4$ and defined on R, is called the generalized trapezoidal fuzzy number, if the membership function of $\tilde{A}$ is given by*

$$\mu_{\tilde{A}}(x) = \begin{cases} w\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \leq x \leq a_2 \\ w, & a_2 \leq x \leq a_3 \\ w\left(\frac{x-a_4}{a_3-a_4}\right), & a_3 \leq x \leq a_4 \\ 0, & otherwise \end{cases}$$

Definition 2.5 (Pentagonal Fuzzy Number)**.** *The fuzzy set $\tilde{A} = (a_1, a_2, a_3, a_4, a_5; w)$, where $a_1 < a_2 < a_3 < a_4 < a_5$ and defined on R, is called the generalized pentagonal fuzzy num-*

ber, if the membership function of $\tilde{A}$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1 \\ w\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ w\left(\frac{x-a_2}{a_3-a_1}\right), & a_2 \le x \le a_3 \\ w, & x = a_3 \\ w\left(\frac{a_4-x}{a_4-a_3}\right), & a_3 \le x \le a_4 \\ w\left(\frac{a_5-x}{a_5-a_4}\right), & a_4 \le x \le a_5 \\ 0, & x > a_5 \end{cases}$$

Definition 2.6 (Hexagonal Fuzzy Number)**.** *The fuzzy set $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6; w)$, where $a_1 < a_2 < a_3 < a_4 < a_5 < a_6$ and defined on R, is called the generalized hexagonal fuzzy number, if the membership function of $\tilde{A}$ is given by*

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), & a_2 \le x \le a_3 \\ 1, & a_3 \le x \le a_4 \\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), & a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right), & a_5 \le x \le a_6 \\ 0, & x > a_6 \end{cases}$$

Definition 2.7 (Fuzzification)**.** *Let the interval data be $[m, M]$. Then the tetra-section of this interval is considered as*

$$d = \frac{(M-m)}{4}$$

Thus the trapezoidal fuzzy number will be taken as

$$(m, m+d, m+3d, M) \tag{2.1}$$

where $M = m+4d$.

Let the interval data be $[m, M]$. Then the hexa-section of this interval is considered as

$$d = \frac{(M-m)}{6}$$

Thus the pentagonal fuzzy number will be taken as

$$(m, m+d, m+3d, m+5d, M) \tag{2.2}$$

where $M = m+6d$.

Let the interval data be $[m, M]$. Then the octa-section of this interval is considered as

$$d = \frac{(M-m)}{8}$$

Thus the hexagonal fuzzy number will be taken as

$$(m, m+d, m+3d, m+5d, m+7d, M) \tag{2.3}$$

where $M = m+8d$.

3. Proposed Methods of Defuzzification for Various Fuzzy Quantities

3.1 Ranking Function of Generalized Trapezoidal Fuzzy Number for Defuzzification

This section proposes a new distance based ranking method for ordering generalized trapezoidal fuzzy number by converting fuzzy number into a crisp number. In order to introduce a new distance based ranking for ordering fuzzy numbers, the new balancing point of trapezoid is introduced using centroid of centroids. First, the trapezoid corresponding to the generalized trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4; w)$, is divided into three triangles ABC, AED and ECD. The reason for selecting this proposed centroid as a point of reference is that each centroid points ($G_1 = \left(\frac{a_1+a_2+a_3}{3}, \frac{2w}{3}\right)$ of ABC, $G_2 = \left(\frac{3a_1+a_3+2a_4}{6}, \frac{w}{6}\right)$ of AED and $G_3 = \left(\frac{a_1+3a_3+2a_4}{6}, \frac{w}{2}\right)$ of ECD) are balancing points of three triangles. Therefore, the centroid of these centroids would be a better balancing point of trapezoid.

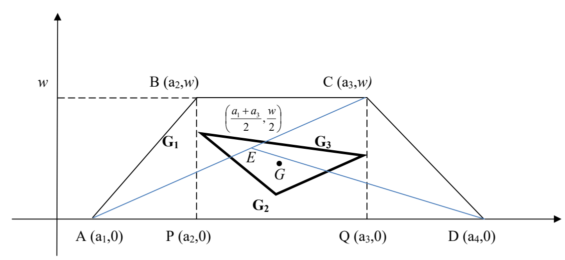

Figure 3.1. Centroid of Generalized Trapezoidal Fuzzy Number

Consider the generalized trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4; w)$. The centroid of centroids of the three triangles is

$$G = (x_0, y_0) = \left(\frac{3a_1+a_2+3a_3+2a_4}{9}, \frac{4w}{9}\right) \tag{3.1}$$

As a special case, for triangular fuzzy number $\tilde{A} = (a_1, a_2, a_4; w)$, i.e., $a_3 = a_2$ the centroid of centroids is given by

$$G = (x_0, y_0) = \left(\frac{3a_1+4a_2+2a_4}{9}, \frac{4w}{9}\right) \tag{3.2}$$

3.2 Ranking Function of Generalized Pentagonal Fuzzy Number for Defuzzification

This section proposes a new distance based ranking method for ordering generalized pentagonal fuzzy number by converting fuzzy number into a crisp number. In order to introduce a new distance based ranking for ordering pentagonal fuzzy numbers, the new balancing point of pentagonal is introduced using centroid of centroids. First, the pentagon corresponding to the generalized pentagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5; w)$, is divided into triangle BCD and trapezoid ABDE. Furthermore, the trapezoid is divided into three triangles ABD, AFE and DEF for finding the balancing point G' of the trapezoid ABDE as per the procedure introduced in Section 3.1. Finally, the average point of the balancing point G' of the trapezoid ABDE and the centroid G'' of the triangle is made as a reference point of pentagonal. The reason for selecting this proposed point as a point of reference is that each centroid points G' are balancing points of the trapezoid and triangle. Therefore, the average of centroids would be a better balancing point of pentagon.

Consider the generalized pentagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5; w)$. The average of centroids of trapezoid and triangle is

$$G = (x_0, y_0) = \left(\frac{3a_1 + 4a_2 + 3a_3 + 6a_4 + 2a_5}{18}, \frac{4w}{9} \right) \quad (3.3)$$

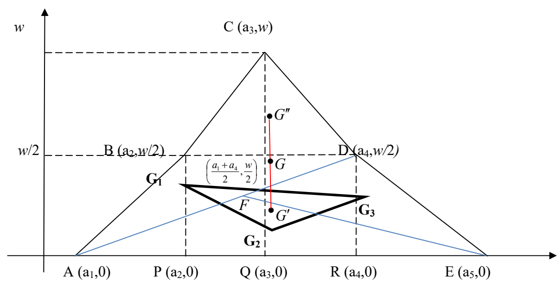

Figure 3.2. Centroid of Generalized Pentagonal Fuzzy Number

3.3 Ranking Function of Generalized Hexagonal Fuzzy Number for Defuzzification

This section proposes a new distance based ranking method for ordering generalized hexagonal fuzzy number by converting fuzzy number into a crisp number. In order to introduce a new distance based ranking for ordering hexagonal fuzzy numbers, the new balancing point of hexagon is introduced using centroid. First, the hexagon corresponding to the generalized hexagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6; w)$, is divided into two trapezoids ABEF and BCDE. Furthermore, the trapezoids are divided into three triangles (ABE, AHF, EFH for the first trapezoid ABEF and BCD, BIF, DEI for the second trapezoid BCDE) for finding the balancing point G' of the first trapezoid ABEF and G'' for the second trapezoid BCDE as per the procedure introduced in Section 3.1. Finally, the average point of the balancing point (centroid of centroids) G' and G'' of the trapezoids ABEF and BCDE is made as a reference point of hexagonal. The reason for selecting this proposed point as a point of reference is that each centroid points G' and G'' is balancing point of the trapezoids. Therefore, the average of these centroids would be a better balancing point of pentagon.

Consider the generalized hexagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6; w)$. The average of centroids of trapezoids is

$$G = (x_0, y_0) = \left(\frac{3a_1 + 4a_2 + a_3 + 3a_4 + 5a_5 + 2a_6}{18}, \frac{17w}{36} \right) \quad (3.4)$$

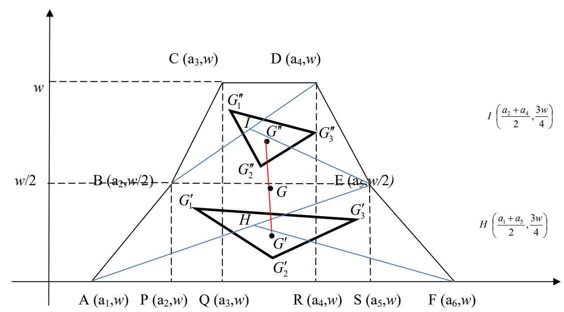

Figure 3.3. Centroid of Generalized Hexagonal Fuzzy Number

Definition 3.1 (Ranking Function)**.** *For mapping the set of all generalized triangular, trapezoidal, pentagonal and hexagonal fuzzy numbers to a set of all real number, the ranking function is defined based on the distance between the original point and the proposed centroids of these fuzzy numbers* (3.1)*,* (3.2)*,* (3.3) *and* (3.4) *as follows:*

$$R(\tilde{A}) = \sqrt{\left(x_0(\tilde{A})\right)^2 + \left(y_0(\tilde{A})\right)^2} \quad (3.5)$$

4. Formulation of Fuzzy Single & Multi Objective Transportation Model

In this section, we present single and multi-objective transportation model with fuzzy cost or/and fuzzy time.

Transportation problem is one of the subclass of Linear Programming Problem in which the objective is to transport various quantities of a single homogeneous commodity, that are initially stored at various origins, to different destinations in such a way that the total transportation cost or/and time is minimum. To achieve this objective we must know the amount and location of available supplies and the quantities demanded. We must know the cost or/and time that result from transporting one unit of commodity from various origins to various destinations.

Mathematically, the single and multi-objective transportation problems can be written as follows:

4.1 Mathematical Formulation of Fuzzy Single Objective Transportation Problem

Mathematically, the single objective transportation problem may be stated as a linear programming problem as follows:

$$\text{Minimize} \quad \tilde{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij} \quad (4.1)$$

$$\text{Subject to} \quad \sum_{j=1}^{n} x_{ij} \cong \tilde{a}_i, \quad i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} \cong \tilde{b}_j, \quad j = 1, 2, \ldots, n$$

Table 4.1. Fuzzy Single Objective Transportation Model with Fuzzy Cost or Fuzzy Time

Source/ Destination	1	2	…	j	…	n	Supply
1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	…	$\tilde{c}_{1j}$	…	$\tilde{c}_{1n}$	$\tilde{a}_1$
2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	…	$\tilde{c}_{2j}$	…	$\tilde{c}_{2n}$	$\tilde{a}_2$
⋮			⋮		⋮		⋮
i	$\tilde{c}_{i1}$	$\tilde{c}_{i2}$	…	$\tilde{c}_{ij}$	…	$\tilde{c}_{in}$	$\tilde{a}_i$
⋮			⋮		⋮		⋮
m	$\tilde{c}_{m1}$	$\tilde{c}_{m2}$	…	$\tilde{c}_{mj}$	…	$\tilde{c}_{mn}$	$\tilde{a}_m$
Demand	$\tilde{b}_1$	$\tilde{b}_2$		$\tilde{b}_j$		$\tilde{b}_n$	

$$\text{and} \quad x_{ij} \geq 0, \ \forall i \text{ and } j$$

where $\tilde{c}_{ij}, \tilde{t}_{ij}, \tilde{a}_i$ and $\tilde{b}_j$ are fuzzy cost, fuzzy time, fuzzy supply and fuzzy demand from ith source to jth designation respectively.

All denotes $\tilde{c}_{ij}, \tilde{t}_{ij}, \tilde{a}_i, \tilde{b}_j$ are non-negative fuzzy numbers.

The same problem may be represented in the form of $m \times n$ fuzzy matrix (Table 4.1) where each cell having a fuzzy cost or fuzzy time.

4.2 Mathematical Formulation of Multi-Objective Transportation Problem

Mathematically, the fuzzy multi-objective transportation problem can be stated as:

$$\text{Minimize} \quad \tilde{z}_k = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\tilde{p}_{ij}^{k}\right)x_{ij} \tag{4.2}$$

$$\text{Subject to} \quad \sum_{j=1}^{n} x_{ij} \cong \tilde{a}_i, \quad i = 1,2,\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} \cong \tilde{b}_j, \quad j = 1,2,\ldots,n$$

where $\tilde{z}_k = \{\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_k\}$.

If the objective function $\tilde{z}_1$ denotes the fuzzy cost function, then it can be started as

$$\tilde{z}_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}x_{ij}$$

If the objective function $\tilde{z}_2$ denotes the fuzzy time function, then it can be started as

$$\tilde{z}_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{t}_{ij}x_{ij}$$

Then it is a bi-objective fuzzy transportation problem, which is represented by using weights of objectives to consider the priorities of the objective as follows:

$$\tilde{z} = w_1\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij} + w_2\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{t}_{ij}x_{ij} \tag{4.3}$$

Table 4.2. Fuzzy Bi-Objective Transportation Model with Fuzzy Cost and Fuzzy Time

Source/ Destination	1	2	…	j	…	n	Supply
1	$\tilde{c}_{11};\tilde{t}_{11}$	$\tilde{c}_{12};\tilde{t}_{12}$	…	$\tilde{c}_{1j};\tilde{t}_{1j}$	…	$\tilde{c}_{1n};\tilde{t}_{1n}$	$\tilde{a}_1$
2	$\tilde{c}_{21};\tilde{t}_{21}$	$\tilde{c}_{22};\tilde{t}_{22}$	…	$\tilde{c}_{2j};\tilde{t}_{2j}$	…	$\tilde{c}_{2n};\tilde{t}_{2n}$	$\tilde{a}_2$
⋮			⋮		⋮		
i	$\tilde{c}_{i1};\tilde{t}_{i1}$	$\tilde{c}_{i2};\tilde{t}_{i2}$	…	$\tilde{c}_{ij};\tilde{t}_{ij}$	…	$\tilde{c}_{in};\tilde{t}_{in}$	$\tilde{a}_i$
⋮			⋮		⋮		⋮
m	$\tilde{c}_{m1};\tilde{t}_{m1}$	$\tilde{c}_{m2};\tilde{t}_{m2}$	…	$\tilde{c}_{mj};\tilde{t}_{mj}$	…	$\tilde{c}_{mn};\tilde{t}_{mn}$	$\tilde{a}_m$
Demand	$\tilde{b}_1$	$\tilde{b}_2$		$\tilde{b}_j$		$\tilde{b}_n$	

$$\text{Subject to} \quad \sum_{j=1}^{n} x_{ij} \cong \tilde{a}_i, \ i = 1,2,\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} \cong \tilde{b}_j, \ j = 1,2,\ldots,n$$

$$x_{ij} \geq 0, \ i = 1,2,\ldots,m; \ j = 1,2,\ldots,n$$

$$\text{and} \quad w_1 + w_2 = 1$$

where $\tilde{c}_{ij}, \tilde{t}_{ij}, \tilde{a}_i$ and $\tilde{b}_j$ are fuzzy cost, fuzzy time, fuzzy supply and fuzzy demand from ith source to jth designation respectively.

All denotes $\tilde{c}_{ij}, \tilde{t}_{ij}, \tilde{a}_i, \tilde{b}_j$ a non-negative fuzzy numbers.

$\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij}$: Total fuzzy cost for shipping from ith source to jth destination.

$\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{t}_{ij}x_{ij}$: Total fuzzy time for shipping from ith source to jth destination.

The same fuzzy transportation problem with two objectives may be represented in the form of $m \times n$ fuzzy matrix (Table 4.2) where each cell having a fuzzy cost, and fuzzy time.

5. Algorithm for solving Interval Valued Transportation Problem (IVTP)

Step 1: Convert the given IVTP into tabular form.
Step 2: Fuzzify the cost, supply and demand in the form of Interval as Trapezoidal, Pentagonal and Hexagonal Fuzzy Numbers using quadra, hexa and octa-sectional approach defined in Section 2 respectively.
Step 3: Apply ranking technique proposed in Section 3 to convert the Fuzzy Transportation Problem as a crisp one.
Step 4: If it is multi-objective Transportation Problem then convert it into single objective Transportation Problem using equation (4.3), then go to the next step. If it is a single objective Transportation Problem then go directly to the next step.
Step 5: Apply any of the classical method such as North West Corner Rule, Least Cost Method and Vogal's Approximation

Method to find initial basic feasible solution and then apply modified distribution (MODI) technique to get the optimal solution.

6. Numerical Illustrations

Illustration 1

A company has factories at F_1, F_2 and F_3 which supply warehouses at W_1, W_2 and W_3. Weekly factory capacities are 200, 160 and 90 units respectively. Weekly warehouses requirements are 180, 120 and 150 units respectively. Unit shipping costs in rupees are as follows:

Table 6.1

Factory	Warehouse			Supply
	W_1	W_2	W_3	
F_1	[11,21]	[5,40]	[6,18]	[100,300]
F_2	[7,23]	[2,14]	[9,28]	[80,280]
F_3	[21,32]	[12,36]	[8,32]	[45,135]
Demand*	[90,310]	[50,190]	[110,200]	

Determine the optimum distribution for this company to minimize shipping costs.

Solution

The basic and optimum solution of the given Interval based transportation problem is obtained using Trapezoidal, Pentagonal and Hexagonal Fuzzy Numbers based on fuzzification and defuzzification techniques as follows:

Case I: In this case, first the interval numbers in the given transportation problem (Table 6.1) are converted in to trapezoidal fuzzy numbers using proposed fuzzification formula (2.1) as shown in Table 6.7.

The fuzzy transportation problem (Table 6.7) with trapezoidal fuzzy numbers is converted into crisp transportation problem using the ranking function (3.5) of proposed centroid (3.1) as follows:

Table 6.2

Factory	Warehouse			Supply
	W_1	W_2	W_3	
F_1	16.01	22.50	12.01	200
F_2	15.01	8.01	18.51	180
F_3	26.50	24.00	20.00	90
Demand*	200	120	155	

Finally the IBFS and optimum solution are obtained using Step 5 as follows.

$$\begin{aligned}\text{Initial Basic Feasible Solution} &= (16.01\times 45)+(12.01\times 155)\\ &\quad +(15.01\times 60)+(8.01\times 120)\\ &\quad +(26.5\times 90)+(0\times 5)\\ &= 6828.8\end{aligned}$$

$$\begin{aligned}\text{Optimum Solution} &= (16.01\times 135)+(12.01\times 65)\\ &\quad +(15.01\times 60)+(8.01\times 120)\\ &\quad +(20\times 90)+(0\times 5)\\ &= 6603.8\end{aligned}$$

Case II:

In this case, first the interval numbers in the given transportation problem (Table 6.1) are converted in to pentagonal fuzzy numbers using proposed fuzzification formula (2.2) as shown in Table 6.8.

Then the fuzzy transportation problem (Table 6.8) with pentagonal fuzzy numbers is converted into crisp transportation problem using the ranking function (3.5) of proposed centroid (3.2) as follows:

Table 6.3

Factory	Warehouse			Supply
	W_1	W_2	W_3	
F_1	16.10	22.83	12.12	201.85
F_2	15.15	8.12	18.68	181.85
F_3	26.61	24.23	20.23	90.83
Demand*	202.04	121.30	155.83	

Finally the IBFS and optimum solution are obtained using Step 5 as follows.

$$\begin{aligned}\text{Initial Basic Feasible Solution} &= (16.1\times 46.02)+(12.12\times 155.83)\\ &\quad +(15.15\times 60.55)+(8.12\times 121.3)\\ &\quad +(26.61\times 90.83)+(0\times 4.64)\\ &= 6948.86\end{aligned}$$

$$\begin{aligned}\text{Optimum Solution} &= (16.1\times 136.85)+(12.12\times 65)\\ &\quad +(15.15\times 60.55)+(8.12\times 121.3)\\ &\quad +(20.23\times 90.83)+(0\times 4.64)\\ &= 6730.86\end{aligned}$$

Case III:

In this case, first the interval numbers in the given transportation problem (Table 6.1) are converted in to hexagonal fuzzy numbers using proposed fuzzification formula (2.3) as shown in Table 6.9.

The fuzzy transportation problem (Table 6.9) with hexagonal fuzzy numbers is converted into crisp transportation problem using the ranking function (3.5) of proposed centroid (3.4) as follows:

Finally the basic and optimum solutions are obtained using Step 5 as follows:

$$\begin{aligned}\text{Initial Basic Feasible Solution} &= (16.08\times 45.76)+(12.09\times 155.63)\\ &\quad +(15.12\times 60.42)+(8.1\times 120.97)\end{aligned}$$

Table 6.4

Factory	Warehouse			Supply
	W_1	W_2	W_3	
F_1	16.08	22.75	12.09	201.39
F_2	15.12	8.10	18.64	181.39
F_3	26.58	24.17	20.17	90.63
Demand	201.53	120.97	155.63	

$$
\begin{aligned}
&+(26.58\times 90.63)+(0\times 4.72)\\
&=6919.74\\
\text{Optimum Solution} &= (16.08\times 136.39)+(12.09\times 65)\\
&+(15.12\times 60.42)+(8.1\times 120.97)\\
&+(20.17\times 90.63)+(0\times 4.72)\\
&=6700.42
\end{aligned}
$$

The basic and optimum solutions obtained from the above three cases are summarized in Table 6.5.

In order to analyse the effectiveness of various fuzzy numbers through the obtained basic and optimization solutions, the solutions are plotted in the XY plane as shown in Figure 6.1.

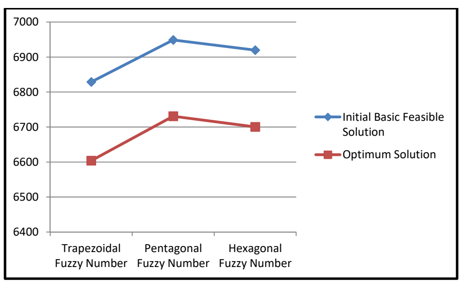

Figure 6.1. Comparative Analysis between Various Fuzzy Quantities

Figure 6.1 shows that the trapezoidal fuzzy number giving the best basic and optimum solution than pentagonal and hexagonal fuzzy numbers as per the proposed fuzzification and defuzzification techniques.

Illustration 2

A district has medicine warehouses with medicines of various treatments like Allopathy, Ayruvedic, Homeopathy, Unani, Yoga and Naturopathy. The medicines are to be distributed to the disease population who has been affected by the diseases like Swine Flu, Ebola, Dengue, Malaria, and Tuberculosis in winter season. The availability of these medicines are (52000, 54000, 57000), (33000, 35000, 37000), (11200, 13700, 15300), (6400, 8700, 10800), (2500, 3200, 4100) and (1540, 1630, 1750) units respectively. The size of the patients affected by the communicable diseases Swine Flu, Ebola, Dengue, Malaria, and Tuberculosis are (22700, 23400, 26500), (15250, 17500, 19250). (11350, 13550, 16550), (6340, 7520, 8450) and (4230, 5420, 6320) respectively. Unit treatment cost in rupees and dosage in gram are as given in Table 6.10.

Determine the optimum distribution of medicines for the district to minimize treatment costs and dosage.

Solution:

The basic and optimum solutions of the given Interval based optimization problem is obtained using Trapezoidal, Pentagonal and Hexagonal Fuzzy Numbers based on fuzzification and defuzzification techniques as follows:

Case I:

In this case, first the interval numbers in the given optimization problem (Table 6.10) are converted into trapezoidal fuzzy numbers using proposed fuzzification formula (2.1) as shown in Table 6.11.

The fuzzy transportation problem (Table 6.11) with trapezoidal fuzzy numbers is converted into crisp transportation problem using the ranking function (3.5) of the proposed centroid (3.1) as shown in Table 6.2.

In order to find the basic and optimum solutions, the bi-objective optimization problem (Table 6.12) is converted into single objective optimization problem using the equation (4.3) with $w_1 = 0.5$ and $w_2 = 0.5$ as shown in Table 6.13.

Finally the basic and optimum solutions are obtained using the proposed algorithm as follows:

Initial Basic Feasible Solution

$$
\begin{aligned}
\text{Total treatment cost} &= (3100\times 12920)+(2600\times 17250)\\
&+(0\times 24330)+(2000\times 11680)\\
&+(4600\times 13950)+(6300\times 7395)\\
&+(24250\times 1975)+(0\times 13250)\\
&+(0\times 8600)+(19200\times 3300)\\
&+(0\times 1645)\\
&=330274250\\
\text{Total dosage} &= (5.52\times 12920)+(10.01\times 17250)\\
&+(0\times 24330)+(257.50\times 11680)\\
&+(1525\times 13950)+(1610\times 7395)\\
&+(950\times 1975)+(0\times 13250)\\
&+(0\times 8600)+(61\times 3300)\\
&+(0\times 1645)\\
&=38508840.9
\end{aligned}
$$

Optimum Solution

$$
\begin{aligned}
\text{Total treatment cost} &= (2600\times 17250)+(0\times 37250)\\
&+(2000\times 24600)+(4600\times 700)\\
&+(6300\times 7395)+(24250\times 1975)\\
&+(0\times 330)+(3900\times 13250)\\
&+(0\times 8600)+(19200\times 3300)\\
&+(0\times 1645)\\
&=258919725\\
\text{Total dosage} &= (10.01\times 17250)+(0\times 37250)
\end{aligned}
$$

Table 6.5. Comparative Analysis between various fuzzy quantities

	Initial basic feasible solution (VAM)	Optimum solution	Difference between IBFS & optimum solution	Effectiveness
Trapezoidal fuzzy number	6828.80	6603.80	225	I
Pentagonal fuzzy number	6948.86	6730.86	218	III
Hexagonal fuzzy number	6919.74	6700.42	219.32	II

$$+(257.5\times 24600)+(1525\times 700)+(1610\times 7395)+(950\times 1975)+(0\times 330)+(45\times 13250)+(0\times 8600)+(61\times 3300)+(0\times 1645) = 22154422.5$$

Case II:

In this case, first the interval numbers in the given optimization problem (Table 6.10) are converted into pentagonal fuzzy numbers using proposed fuzzification formula (2.2) as shown in Table 6.14.

The fuzzy optimization problem (Table 6.14) with pentagonal fuzzy numbers is converted into crisp optimization problem using the ranking function (3.5) of the proposed centroid (3.3) as shown in Table 6.15.

In order to find the basic and optimum solutions, the bi-objective optimization problem (Table 6.15) is converted into single objective optimization problem using the equation (4.3) with $w_1 = 0.5$ and $w_2 = 0.5$ as shown in Table 6.16.

Finally the basic and optimum solutions are obtained using the proposed algorithm as follows:

Initial Basic Feasible Solution

$$\text{Total treatment cost} = 3111.11\times 12990.38+2611.11\times 17287.04+0\times 24268.88+2009.26\times 11644.81+4607.41\times 13998.15+6316.67\times 7414.54+24291.67\times 1979.54+0\times 13287.96+0\times 8640.74+19240.74\times 3314.81+0\times 1646.94 = 332142828$$

$$\text{Total dosage} = 5.55\times 12990.38+10.07\times 17287.04+0\times 24268.88+260.60\times 11644.81+1531.02\times 13998.15+1615.74\times 7414.54+951.85\times 1979.54+0\times 13287.96+0\times 8640.74+61.24\times 3314.81+0\times 1646.94 = 38779455.2$$

Optimum Solution

$$\text{Total treatment cost} = (2611.11\times 17287.04)+(0\times 37259.26)+(2009.26\times 24635.19)+(4607.41\times 710.19)+(6316.67\times 7414.54)+(24291.67\times 1979.54)+(0\times 297.58)+(3912.96\times 13287.96)+(0\times 8640.74)+(19240.74\times 3314.81)+(0\times 1646.94) = 300912457$$

$$\text{Total dosage} = (10.07\times 17287.04)+(0\times 37259.26)+(260.60\times 24635.19)+(1531.02\times 710.19)+(1615.74\times 7414.54)+(951.85\times 1979.54)+(0\times 297.58)+(45.17\times 13287.96)+(0\times 8640.74)+(61.24\times 3314.81)+(0\times 1646.94) = 22348736.2$$

Case III:

In this case, first the interval numbers in the given optimization problem are converted into hexagonal fuzzy numbers using proposed fuzzification (2.3) as shown in Table 6.17.

The above fuzzy optimization problem with hexagonal fuzzy numbers is converted into crisp optimization problem using the ranking function (3.5) of proposed centroid (3.4) as shown in Table 6.18.

In order to find the basic and optimum solutions, the bi-objective optimization (Table 6.18) is converted into single objective optimization problem using the equation (4.3) with $w_1 = 0.5$ and $w_2 = 0.5$ as follows.

Finally the basic and optimum solutions are obtained using the proposed algorithm as follows:

Initial Basic Feasible Solution

$$\text{Total treatment cost} = (3108.33\times 12963.51)+(2608.33\times 17277.78)+(0\times 24293.43)+(2006.94\times 11662.88)+(4605.56\times 13986.11)+(6312.50\times 7409.65)+(24291.67\times 1978.4)+(0\times 13278.47)+(0\times 8630.56)+(19230.56\times 3311.11)+(0\times 1646.46) = 331688143.2$$

$$\text{Total dosage} = (5.54\times 12963.51)+(10.05\times 17277.78)+(0\times 24293.43)+(259.83\times 11662.88)+(1529.51\times 13986.11)+(1614.31\times 7409.65)$$

Table 6.6. Comparative Analysis between various fuzzy quantities

Fuzzy Number	Initial Basic Feasible Solution (VAM)		Optimum Solution		Difference Between IBFS & Optimum Solution		Effectiveness
	Total Treatment Cost	Total Dosage	Total Treatment Cost	Total Dosage	Total Treatment Cost	Total Dosage	
Trapezoidal	330,274,250	38,508,840.9	258,919,725	22,154,422.5	71,354,525	16,354,418.40	I
Pentagonal	332,142,828	38,779,455.2	300,912,457	22,348,736.2	31,230,371	16,430,719.00	III
Hexagonal	331,688,143.2	38,713,996.53	347,232,465.4	26,679,466.27	15,544,322	12,034,530.26	II

$$+(951.39\times 1978.4)+(0\times 13278.47) +(0\times 8630.56)+(61.18\times 3311.11) +(0\times 1646.46) = 38713996.53$$

Optimum Solution

$$\text{Total treatment cost} = (2608.33\times 17277.78)+(0\times 37256.94) +(2006.94\times 24626.39)+(4605.56\times 707.64) +(6312.50\times 7409.65)+(24291.67\times 1978.4) +(0\times 314.96)+(3909.72\times 13278.47) +(0\times 8630.56)+(31027.78\times 3311.11) +(0\times 1646.46) = 347232465.4$$

$$\text{Total dosage} = (10.05\times 17277.78)+(0\times 37256.94) +(259.83\times 24626.39)+(1529.51\times 707.64) +(1614.31\times 7409.65)+(951.39\times 1978.4) +(0\times 314.96)+(45.13\times 13278.47) +(0\times 8630.56)+(1383.78\times 3311.11) +(0\times 1646.46) = 26679466.27$$

The basic and optimum solutions obtained from the above three cases are summarized in Table 6.6.

In order to analyse the effectiveness of various fuzzy numbers through the obtained basic and optimization solutions, the solutions are plotted in the XY plane as shown in Figures 6.2 and 6.3.

The basic and optimum solutions obtained by converting the interval based optimization problem as a fuzzy optimization problem with trapezoidal fuzzy number is minimum. So the analysis suggests that the trapezoidal fuzzy numbers might give the effective solution than the other fuzzy numbers such as pentagonal and hexagonal fuzzy numbers as per the proposed fuzzication and defuzzication techniques.

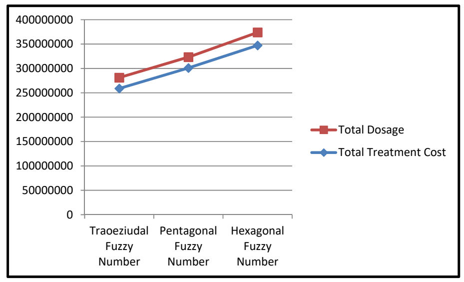

Figure 6.2. Optimum Solution

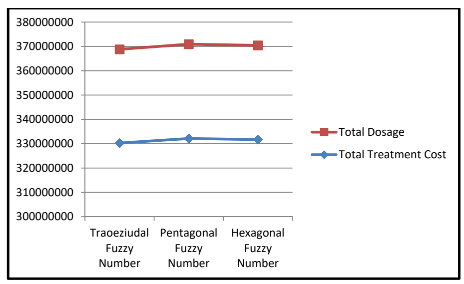

Figure 6.3. Initial Basic Feasible Solution (IBFS)

7. Conclusion

This paper made an attempt to solve the interval based transportation problem through the concept of fuzzy set theory using fuzzification and defuzzification of various fuzzy numbers. In this work, initially interval based transportation problem was converted into three different fuzzy transportation problems with various fuzzy numbers such as trapezoidal, pentagonal and hexagonal fuzzy numbers using proposed fuzzification techniques. Then the fuzzy transportation problems were converted to crisp transportation problems with the help of proposed defuzzification techniques of these fuzzy numbers using new distance between original point and centroid of centroids. Finally the three different basic and optimum solutions of interval based transportation problem are observed to analyse the effectiveness of various fuzzy numbers to obtain the minimum basic and optimum solution. The same analysis may be done for finding the effectiveness of various generalized fuzzy numbers through the same transportation problem. Moreover, this analysis may be extended to other fuzzy numbers through some other optimizations problems like assignment problem, game theoretical problem etc.

Table 6.7

Factory	Warehouse			Supply
	W_1	W_2	W_3	
F_1	(11,13.5, 18.5,21)	(5,13.75, 31.25,40)	(6,9, 15,18)	(100,150, 250,300)
F_2	(7,11, 19,23)	(2,5, 11,14)	(9,13.75, 23.25,28)	(80,130, 230,280)
F_3	(21,23.75, 29.25,32)	(12,18, 30,36)	(8,14, 26,32)	(45,67.5, 112.5,135)
Demand	(90,145, 255,310)	(50,85, 155,190)	(110,132.5, 177.5,200)	

Table 6.8

Factory	Warehouse			Supply
	W_1	W_2	W_3	
F_1	(11,12.67,16, 19.33,21)	(5,10.83,22.5, 34.17,40)	(6,8,12,16,18)	(100,133.33,200, 266.67,300)
F_2	(7,9.67,15, 20.33,23)	(2,4,8,12,14)	(9,12.17,18.5, 24.83,28)	(80,113.33,180, 246.67,280)
F_3	(21,22.83,26.5, 30.17,32)	(12,16,24,32,36)	(8,12,20,28,32)	(45,60,90,120,135)
D	(90,126.67,200,277.33,310)	(50,73.33,120, 166.67,190)	(110,125,155, 185,200)	

Table 6.9

Factory	Warehouse			Supply
	W_1	W_2	W_3	
F_1	(11,12.25,14.75, 17.25,19.75,21)	(5,9.38,18.13, 26.88,35.63,40)	(6,7.5,10.5,13.5, 16.5,18)	(100,125,175, 225,275,300)
F_2	(7,9,13, 17,21,23)	(2,3.5,6.5, 9.5,12.5,14)	(9,11.38,16.13, 20.88,25.63,28)	(80,105,155, 205,255,280)
F_3	(21,22.38,25.13, 27.88,30.63,32)	(12,15,21, 27,33,36)	(8,11,17, 23,29,32)	(45,56.25, 78.75,101.25,123.75,135)
Demand	(90,117.5,172.5, 227.5,282.5,310)	(50,67.5,102.5, 137.5,172.5,190)	(110,121.25,143.75, 166.25,188.75,200)	

Table 6.10

Treatment/ Diseases	Swine Flu (D_1)	Ebola (D_2)	Dengue (D_3)	Malaria (D_4)	Tuberculosis (D_5)	Supply (T_j)
Allopathy (T_1)	[2500,3700] [4,7]	[2000,3200] [7,13]	[8000,9000] [425,560]	[10000,4000] [250,300]	[28000,32000] [550,750]	[52000, 57000]
Ayurvedic (T_2)	[1500,2500] [90,425]	[2400,3500] [450,580]	[4200,5000] [1200,1850]	[5400,7200] [1300,1920]	[22000,26500] [850,1050]	[33000, 37000]
Homeopathy (T_3)	[3500,4600] [0.6, 1.2]	[4000,5000] [1.2, 2.3]	[3200,4600] [36,54]	[18000,24000] [55,80]	[32000,45000] [90,175]	[11200, 15300]
Unani (T_4)	[3200,4300] [190,390]	[3500,4500] [420,520]	[5000,5800] [1200,1520]	[12000,21000] [725,950]	[29000,33000] [1290,1475]	[6400, 10800]
Yoga (T_5)	[950,1500] [10,42]	[1000,1400] [14,24]	[2100,2700] [1300,2050]	[3200,5200] [545,850]	[17000,21400] [48,74]	[2500, 4100]
Naturopathy (T_6)	[4200,5200] [8,14]	[4000,6500] [19,23]	[6200,8200] [950,1450]	[12000,21000] [325,540]	[29000,33000] [37,53]	[1540, 1750]
Demand*	[22700,26500]	[15250,19250]	[11350,16550]	[6340,8450]	[4230,6320]	

*(no. of patients affected by the disease D_i to be taken the treatment)

Table 6.11

Treatment/ Diseases	Swine Flu (D_1)	Ebola (D_2)	Dengue (D_3)	Malaria (D_4)	Tuberculosis (D_5)	Supply (T_j)
Allopathy (T_1)	(2500, 2800, 3400, 3700) (4,4.75, 6.25,7)	(2000,2300, 2900,3200) (7,8.5, 11.5,13)	(8000,8250, 8750,9000) (425,458.75, 526.25,560)	(10000,11000, 13000,14000) (250,262.5, 287.5,300)	(28000,29000, 31000,32000) (550,600, 700,750)	(52000,53250, 55750,57000)
Ayurvedic (T_2)	(1500,1750, 2250,2500) (90,173.75, 341.25,425)	(2400,2675, 3225,3500) (450,482.5, 547.5,580)	(4200,4400, 4800,5000) (1200,1362.5, 1687.5,1850)	(5400,5850, 6750,7200) (1300,1455, 1765,1920)	(22000,23125, 25375,26500) (850,900, 1000,1050)	(33000,34000, 36000,37000)
Homeopathy (T_3)	(3500,3775, 4325,4600) (0.6,0.75, 1.05,1.2)	(4000,4250, 4750,5000) (1.2,1.475, 2.025,2.3)	(3200,3550, 4250,4600) (36,40.5, 49.5,54)	(18000,19500, 22500,24000) (55,61.25, 73.75,80)	(32000,35250, 41750,45000) (90,111.25, 153.75,175)	(11200,12225, 14275,15300)
Unani (T_4)	(3200,3475, 4025,4300) (190,240, 340,390)	(3500,3750, 4250,4500) (420,445, 495,520)	(5000,5200, 5600,5800) (1200,1280, 1440,1520)	(12000,14250, 18750,21000) (725,781.25, 893.75,950)	(29000,30000, 32000,33000) (1290,1336.25, 1428.75,1475)	(6400,7500, 9700,10800)
Yoga (T_5)	(950,1087.5, 1362.5,1500) (10,18,34,42)	(1000,1100, 1300,1400) (14,16.5, 21.5,24)	(2100,2250, 2550,2700) (1300,1487.5, 1862.5,2050)	(3200,3700, 4700,5200) (545,621.25, 773.75,850)	(17000,18100, 20300,21400) (48,54.5, 67.5,74)	(2500,2900, 3700,4100)
Naturopathy (T_6)	(4200,4450, 4950,5200) (8,9.5,12.5,14)	(4000,4625, 5875,6500) (19,20, 22,23)	(6200,6700, 7700,8200) (950,1075, 1325,1450)	(12000,14250, 18750,21000) (325,378.75,486.25,540)	(29000,30000, 32000,33000) (37,41,49,53)	(1540,1592.5, 1697.5,1750)
Demand*	(22700,23650, 25550,26500)	(15250,16250, 18250,19250)	(11350,12650, 15250,16550)	(6340,6867.5, 7922.5,8450)	(4230,4752.5, 5797.5,6320)	

*(no. of patients affected by the disease D_i to be taken the treatment)

Table 6.12

Treatment/ Diseases	Swine Flu (D_1)	Ebola (D_2)	Dengue (D_3)	Malaria (D_4)	Tuberculosis (D_5)	Supply (T_j)
Allopathy (T_1)	3100, 5.52	2600, 10.01	8500, 492.50	12000, 275	30000, 650	54500
Ayurvedic (T_2)	2000, 257.50	2950, 465	4600, 1525	6300, 1610	24250, 950	35000
Homeopathy (T_3)	4050, 1	4500, 1.81	3900, 45	21000, 67.50	38500, 132.50	13250
Unani (T_4)	3750, 290	4000, 470	5400, 1360	16500, 837.50	31000, 1382.50	8600
Yoga (T_5)	1225, 26	1200, 19.01	2400, 1675	4200, 697.50	19200, 61	3300
Naturopathy (T_6)	4700, 11.01	5250, 21	7200, 1200	16500, 432.50	31000, 45	1645
Demand*	24600	17250	13950	7395	5275	

*(no. of patients affected by the disease D_i to be taken the treatment)

Table 6.13

Treatment/ Diseases	Swine Flu (D_1)	Ebola (D_2)	Dengue (D_3)	Malaria (D_4)	Tuberculosis (D_5)	Supply (T_j)
Allopathy (T_1)	1552.76	1305	4496.25	6137.50	15325	54500
Ayurvedic (T_2)	1128.75	1707.50	3062.50	3955	12600	35000
Homeopathy (T_3)	2025.50	2250.90	1972.50	10533.75	19316.25	13250
Unani (T_4)	2020	2235	3380	8668.75	16191.25	8600
Yoga (T_5)	625.50	609.50	2037.50	2448.75	9630.50	3300
Naturopathy (T_6)	2355.50	2635.50	4200	8466.25	15522.50	1645
Demand*	24600	17250	13950	7395	5275	

*(no. of patients affected by the disease D_i to be taken the treatment)

Table 6.14

Treatment/ Diseases	Swine Flu (D_1)	Ebola (D_2)	Dengue (D_3)	Malaria (D_4)	Tuberculosis (D_5)	Supply (T_j)
Allopathy (T_1)	(2500,2700, 3100,3500,3700) (4,4.50, 5.5,6.50,7)	(2000,2200, 2600,3000,3200) (7,8.00,10, 12.00,13)	(8000,8166.67, 8500,8833.33,9000) (425,447.50, 492.5,537.50,560)	(10000,10666.67, 12000,13333.33,14000) (250,258.33, 275,291.67,300)	(28000,28666.67, 30000,31333.33,32000) (550,583.33, 650,716.67,750)	(52000, 52833.33, 54500,561666.67 57000)
Ayurvedic (T_2)	(1500,1666.67, 2000,2333.33, 2500) (90,145.83, 257.5,369.17,425)	(2400,2583.33, 2950,3316.67, 3500) (450,471.67, 515,558.33,580)	(4200,4333.33, 4600,4866.67, 5000) (1200,1308.33, 1525,1741.67,1850)	(5400,5700, 6300,6900, 7200) (1300,1403.33, 1610,1816.67,1920)	(22000,22750, 24250,25750, 26500) (850,883.33, 950,1016.67,1050)	(33000,33666.67, 35000,36333.33, 37000)
Homeopathy (T_3)	(3500,3683.33, 4050,4416.67, 4600) (0.6,0.70,0.9, 1.10,1.2)	(4000,4166.67, 4500,4833.33, 5000) (1.2,1.38,1.75, 2.12,2.3)	(3200,3433.33, 3900,4366.67, 4600) (36,39.00,45, 51.00,54)	(18000,19000, 21000,23000, 24000) (55,59.17,67.5, 75.83,80)	(32000,34166.67, 38500,42833.33, 45000) (90,104.17, 132.5,160.83,175)	(11200,11883.33, 13250,14616.67, 15300)
Unani (T_4)	(3200,3383.33, 3750,4116.67, 4300) (190,223.33, 290,356.67,390)	(3500,3666.67, 4000,4333.33, 4500) (420,436.67, 470,503.33,520)	(5000,5133.33, 5400,5666.67, 5800) (1200,1253.33, 1360,1466.67,1520)	(12000,13500, 16500,19500, 21000) (725,762.50, 837.5,912.50,950)	(29000,29666.67, 31000,32333.33, 33000) (1290,1320.83, 1382.5,1444.17,1475)	(6400, 7133.33, 8600, 10066.67, 10800)
Yoga (T_5)	(950,1041.67, 1225,1408.33, 1500) (10,15.33, 26,36.67,42)	(1000,1066.67, 1200,333.33, 1400) (14,15.67, 19,22.33,24)	(2100,2200, 2400,2600, 2700) (1300,1425.00, 1675,1925.00,2050)	(3200,3533.33, 4200,4866.67, 5200) (545,595.83, 697.5,799.17,850)	(17000,17733.33, 19200,20666.67, 21400) (48,52.33, 61,69.67,74)	(2500, 2766.67, 3300, 3833.33, 4100)
Naturopathy (T_6)	(4200,4366.67, 4700,5033.33, 5200) (8,9.00,11, 13.00,14)	(4000,4416.67, 5250,6083.33, 6500) (19,19.67,21, 22.33,23)	(6200,6533.33, 7200,7866.67, 8200) (950,1033.33,1200, 1366.67,1450)	(12000,13500, 16500,19500, 21000) (325,360.83, 432.5,504.17,540)	(29000,29666.67, 31000,32333.33, 33000) (37,39.67,45, 50.33,53)	(1540, 1575, 1645, 1715, 1750)
Demand*	(22700, 23333.33, 24600, 25866.67, 26500)	(15250, 15916.67, 17250, 18583.33, 19250)	(11350, 12216.67, 13950, 15683.33, 16550)	(6340, 6691.67, 7395, 8098.33, 8450)	(4230, 4578.33, 5275, 5971.67, 6320)	

*(no. of patients affected by the disease D_i to be taken the treatment)

Table 6.15

Treatments/Diseases	Swine Flu (D_1)	Ebola (D_2)	Dengue (D_3)	Malaria (D_4)	Tuberculosis (D_5)	Supply (T_j)
Allopathy (T_1)	3111.11, 5.55	2611.11, 10.07	8509.26, 493.75	12037.04, 275.46	30037.04, 651.85	54546.30
Ayurvedic (T_2)	2009.26, 260.60	2960.19, 465.28	4607.41, 1531.02	6316.67, 1615.74	24291.67, 951.85	35037.04
Homeopathy (T_3)	4060.19, 1.01	4509.26, 1.82	3912.96, 45.17	21055.56, 67.73	38620.37, 133.29	13287.96
Unani (T_4)	3760.19, 291.85	4009.26, 470.93	5407.41, 1362.96	16583.33, 839.58	31037.04, 1384.21	8640.74
Yoga (T_5)	1230.09, 26.30	1203.70, 19.10	2405.56, 1681.94	4218.52, 700.32	19240.74, 61.24	3314.81
Naturopathy (T_6)	4709.26, 11.06	5273.15, 21.04	7218.52, 1204.63	16583.33, 434.49	31037.04, 45.15	1646.94
Demand*	24635.19	17287.04	13998.15	7414.54	5294.35	

*(no. of patients affected by the disease D_i to be taken the treatment)

Table 6.16

Treatments/Diseases	Swine Flu (D_1)	Ebola (D_2)	Dengue (D_3)	Malaria (D_4)	Tuberculosis (D_5)	Supply (T_j)
Allopathy (T_1)	1558.33	1310.59	4501.51	6156.25	15344.45	54546.30
Ayurvedic (T_2)	1134.93	1712.73	3069.21	3966.21	12621.76	35037.04
Homeopathy (T_3)	2030.60	2255.54	1979.06	10561.65	19376.83	13287.96
Unani (T_4)	2026.02	2240.09	3385.19	8711.46	16210.63	8640.74
Yoga (T_5)	628.20	611.40	2043.75	2459.42	9650.99	3314.81
Naturopathy (T_6)	2360.16	2647.09	4211.58	8508.91	15541.10	1646.94
Demand*	24635.19	17287.04	13998.15	7414.54	5294.35	

*(no. of patients affected by the disease D_i to be taken the treatment)

Table 6.17

Treatments/Diseases	Swine Flu (D_1)	Ebola (D_2)	Dengue (D_3)	Malaria (D_4)	Tuberculosis (D_5)	Supply (T_j)
Allopathy (T_1)	(2500, 2650, 2950, 3250, 3550, 3700) (4, 4.375, 5.125, 5.875, 6.625, 7)	(2000, 2150, 2450, 2750, 3050, 3200) (7, 7.75, 9.25, 10.75, 12.25, 13)	(8000, 8125, 8375, 8625, 8875, 9000) (425, 441.88, 475.63, 509.38, 543.13, 560)	(10000, 10500, 11500, 12500, 13500, 14000) (250, 256.25, 268.75, 281.25, 293.75, 300)	(28000, 28500, 29500, 30500, 31500, 32000) (550, 575, 625, 675, 725, 750)	(5200, 52625, 53875, 55125, 56375, 57000)
Ayurvedic (T_2)	(1500, 1625, 1875, 2125, 2375, 2500) (90, 131.88, 215.63, 299.38, 383.13, 425)	(2400, 2537.50, 2812.50, 3087.50, 3362.50, 3500) (450, 466.25, 498.75, 531.25, 563.75, 580)	(4200, 4300, 4500, 4700, 4900, 5000) (1200, 1281.25, 1443.75, 1606.25, 1768.75, 1850)	(5400, 5625, 6075, 6525, 6975, 7200) (1300, 1377.50, 1532.50, 1687.50, 1842.50, 1920)	(22000, 22562.50, 23687.50, 24812.50, 25937.50, 26500) (850, 875, 925, 975, 1025, 1050)	(33000, 33500, 34500, 35500, 36500 37000)
Homeopathy (T_3)	(3500, 3637.50, 3912.50, 4187.50, 4462.50, 4600) (0.6, 0.68, 0.83, 0.98, 1.13, 1.2)	(4000, 4125, 4375, 4625, 4875, 5000) (1.2, 1.34, 1.61, 1.89, 2.16, 2.3)	(3200, 3375, 3725, 4075, 4425, 4600) (36, 38.25, 42.75, 47.25, 51.75, 54)	(18000, 18750, 20250, 21750, 23250, 24000) (55, 58.13, 64.38, 70.63, 76.88, 80)	(32000, 33625, 36875, 40125, 43375, 45000) (90, 100.63, 121.88, 143.13, 164.38, 175)	(11200, 11712.5, 123737.5, 13762.5, 14787.5, 15300)
Unani (T_4)	(3200, 3337.50, 3612.50, 3887.50, 4162.50, 4300) (190, 215, 265, 315, 365, 390)	(3500, 3625, 3875, 4125, 4375, 4500) (420, 432.50, 457.50, 482.50, 507.50, 520)	(5000, 5100, 5300, 5500, 5700, 5800) (1200, 1240, 1320, 1400, 1480, 1520)	(12000, 13125, 15375, 17625, 19875, 21000) (725, 753.13, 809.38, 865.63, 921.88, 950)	(29000, 29500, 30500, 31500, 32500, 33000) (1290, 1313.13, 1359.38, 1405.63, 1451.88, 1475)	(6400, 6950, 8050, 9150, 10250, 10800)
Yoga (T_5)	(950, 1018.75, 1156.25, 1293.75, 1431.25, 1500) (10, 14, 22, 30, 38, 42)	(1000, 1050, 1150, 1250, 1350, 1400) (14, 15.25, 17.75, 20.25, 22.75, 24)	(2100, 2175, 2325, 2475, 2625, 2700) (1300, 1393.75, 1581.25, 1768.75, 1956.25, 2050)	(3200, 3450, 3950, 4450, 4950, 5200) (545, 583.13, 659.38, 735.63, 811.88, 850)	(17000, 17550, 18650, 19750, 20850, 21400) (48, 51.25, 57.75, 64.25, 70.75, 74)	(2500, 2700, 3100, 3500, 3900, 4100)
Naturopathy (T_6)	(4200, 4325, 4575, 4825, 5075, 5200) (8, 8.75, 10.25, 11.75, 13.25, 14)	(4000, 4312.50, 4937.50, 5562.50, 6187.50, 6500) (19, 19.50, 20.50, 21.50, 22.50, 23)	(6200, 6450, 6950, 7450, 7950, 8200) (950, 1012.50, 1137.50, 1262.50, 1387.50, 1450)	(12000, 13125, 15375, 17625, 19875, 21000) (325, 351.88, 405.63, 459.38, 513.13, 540)	(29000, 29500, 30500, 31500, 32500, 33000) (37, 39, 43, 47, 51, 53)	(1540, 1566.25, 1618.75, 1671.25, 1723.75, 1750)
Demand*	(22700, 23175, 24125, 25075, 26025, 26500)	(15250, 15750, 16750, 17750, 18750, 19250)	(11350, 12000, 13300, 14600, 15900, 16550)	(6340, 6603.75, 7031.25, 7658.75, 8186.25, 8450)	(4230, 4491.25, 5013.75, 5536.25, 6058.75, 6320)	

*(no. of patients affected by the disease D_i to be taken the treatment)

Table 6.18

Treatments/ Diseases	Swine Flu (D_1)	Ebola (D_2)	Dengue (D_3)	Malaria (D_4)	Tuberculosis (D_5)	Supply (T_j)
Allopathy (T_1)	3108.33, 5.54	2608.33, 10.05	8506.94, 493.44	12027.78, 275.35	30027.78, 651.39	54534.72
Ayurvedic (T_2)	2006.94, 259.83	2957.64, 465.21	4605.56, 1529.51	6312.50, 1614.31	24291.67, 951.39	35037.04
Homeopathy (T_3)	4057.64, 1.02	4506.94, 1.82	3909.72, 45.13	21041.67, 67.68	38590.28, 133.09	13278.47
Unani (T_4)	3757.64, 291.39	4006.94, 470.69	5405.56, 1362.22	16562.50, 839.06	31027.78, 1383.78	8630.56
Yoga (T_5)	1228.82, 26.23	1202.78, 19.08	2404.17, 1680.21	4213.89, 699.62	19230.56, 61.18	3311.11
Naturopathy (T_6)	4706.94, 11.05	5267.36, 21.03	7213.89, 1203.47	16562.50, 433.99	31027.78, 45.11	1646.46
Demand*	24626.39	17277.78	13986.11	7409.65	5289.51	

*(no. of patients affected by the disease D_i to be taken the treatment)

Table 6.19

Treatments/ Diseases	Swine Flu (D_1)	Ebola (D_2)	Dengue (D_3)	Malaria (D_4)	Tuberculosis (D_5)	Supply (T_j)
Allopathy (T_1)	1556.94	1309.19	4500.19	6151.56	15339.58	54534.72
Ayurvedic (T_2)	1133.38	1711.42	3067.54	3963.40	12621.53	35037.04
Homeopathy (T_3)	2029.33	2254.38	1977.42	10554.67	19361.69	13278.47
Unani (T_4)	2024.51	2238.82	3383.89	8700.78	16205.78	8630.56
Yoga (T_5)	627.52	610.93	2042.19	2456.75	9645.87	3311.11
Naturopathy (T_6)	2359	2644.20	4208.68	8498.25	15536.45	1646.46
Demand*	24626.39	17277.78	13986.11	7409.65	5289.51	

*(no. of patients affected by the disease D_i to be taken the treatment)

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