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New results on harmonic mean graphs

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Abstract

A graph G with p vertices and q edges is called a harmonic mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $\{1,2,...q+1\}$ in such a way that each edge $e = uv$ is labeled with

$$f(uv) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \quad \text{(or)} \quad \left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$$

then the edge labels are distinct. In this case f is called Harmonic mean labeling of G. In this paper we prove that some families of graphs such as zig-zag triangle $Z(T_n)$, alternate zig-zag triangle $AZ(T_n)$, spiked snake graph $SS(4,n)$ are harmonic mean graphs.

Keywords

Harmonic mean graph, zig-zag triangle $Z(T_n)$, alternate zig-zag triangle $AZ(T_n)$, spiked snake graph $SS(4,n)$.

AMS Subject Classification

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1. Introduction

Let $G = (V,E)$ be a (p,q) graph with $p = |V(G)|$ vertices and $q = |E(G)|$ edges, where $V(G)$ and $E(G)$ respectively denote the vertex set and edge set of the graph G. In this paper, we consider the graphs which are simple, finite and undirected. Harary's graph theory used for theoretic terminology and notations [3].

The concept of graph labeling was introduced by Rosa in 1967. A detailed survey of graph labeling is available in Gallian [2]. S.Somasundaram, R.Ponraj and S.S.Sandhya were introduced the concept of harmonic mean labeling of graphs. They investigated the existence of harmonic mean labeling of several family of graphs such as path, comb, cycle C_n, complete graph K_n complete bipartite graph $K_{2,2}$, triangular snake T_n, quadrilateral snake Q_n, alternate triangular snake $A(T_n)$, alternate quadrilateral snake $A(Q_n)$, crown $C_n \odot K_1, C_n \odot \bar{K}_2, C_n \odot \bar{K}_3$, dragon, wheel in [8–10]. The harmonic mean labeling of step ladder $S(T_n, P_n \odot K_2, C_n \odot K_2$, flower graph, $L_n \odot \bar{K}_2$, triangular ladder, double triangular snake $D(T_n)$, alternate double triangular snake $A(DT_n)$ double quadrilateral snake $D(Q_n)$, alternate double quadrilateral snake $A(DQ_n), Q_n \odot K_1, (C_m \odot K_1) \cup C_n, (C_m \odot K_1) \cup P_n$, are investigated by C.Jayasekaran, C.David raj and S.S.Sandhya [11, 12]. We have proved Harmonic mean labeling of subdivision graphs such as $P_n \odot K_1, P_n \odot \bar{K}_2$, H-graph, crown, $C_n \odot K_1, C_n \odot \bar{k}_2$, quadrilateral snake, Triangular snake and also proved Harmonic mean labeling of some graphs such as Triple triangular snake $T(T_n)$, Alternate Triple triangular snake $A[T(Q_n)]$, Triple quadrilateral snake $T(Q_n)$, Alternate Triple quadrilateral snake $A[T(Q_n)]$, Twig graph $T(n)$, balloon triangular snake $T_n(C_m)$, and key graph $Ky(m,n)$, [6, 7]. The following definitions are useful for the present investigation.

Notations:

$\lfloor x \rfloor$ - Largest integer less than or equal to x.

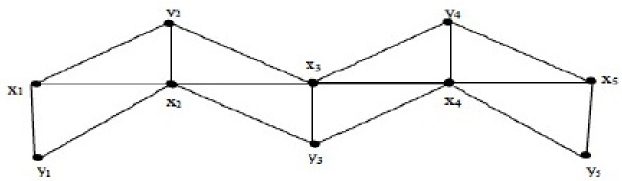
Definition 1.1. *[8] A Graph $G = (V,E)$ with p vertices and q edges is called a Harmonic mean graph if it is possible to label the vertices $v \in V$ with distinct labels $f(v)$ from $\{1,2,...,q+$*

1} *in such a way that when each edge $e = uv$ is labeled with*

$$f(uv) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \quad (or) \quad \left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$$

then the resulting edge labels are distinct. In this case f is called Harmonic mean labeling of G.

Definition 1.2. *[4] Let G be the graph obtained from the path $P_n = x_1, x_2, ..., x_n$ adding a new vertices $y_1, y_2, ..., y_n$ and new edges $y_1x_2, y_nx_{n-1}; x_iy_i$ for $1 \le i \le n, y_ix_{i-1}, y_ix_{i+1}$ for $2 \le i \le n-1$. The family of graph is called zig-zag triangle $Z(T_n)$.*



Definition 1.3. *Let G be the graph obtained from the path $P_n = x_1, x_2, ..., x_n$ and let $s_1, s_2, ..., s_k, r_1, r_2, ..., r_k, t_1, t_2, ..., t_k$ where $k = \lfloor \frac{n}{3} \rfloor$ and byu adding new edges $\{s_ix_{3i-1}, s_ix_{3i-2}, s_ix_{3i} / 1 \le i \le k\} \cup \{r_ix_{3i-1}, r_ix_{3i-2} / 1 \le i \le k\} \cup \{t_ix_{3i}, t_ix_{3i+1} / 1 \le i \le k\}$ if $n \equiv 1, 2 (mod\ 3)$. The family of graph is called alternate zig-zag triangle $AZ(T_n)$.*

Definition 1.4. *[1] A spiked snake graph $SS(4,n)$ is a graph obtained from a cyclic snake $S(4,n)$ with Additional edges. It is a graph with vertex set $V(SS(4,n)) = \{v_1, v_2, ..., v_{n+1}\} \cup \{x_iw_i, u_iy_i : 1 \le i \le n\}$ and the edge set*
$E(SS(4,n)) = \{v_iw_i, w_iv_{i+1}, v_{i+1}u_i, u_iv_i, w_ix_i, u_iy_i : 1 \le i \le n\}$.

In this paper we prove that zig-zag triangle $Z(T_n)$, alternate zig-zag triangle $A\ Z(T_n)$, spiked snake graph $SS(4,n)$ are harmonic mean graphs.

2. Harmonic mean labeling of graphs

Theorem 2.1. *The zig-zag triangle $Z(T_n)$ is a harmonic mean graph.*

Proof. Let $x_1, x_2, ..., x_n$ be the vertices of the path P_n and let $G = Z(T_n)$ be the zig-zag triangle graph.

Let $V(G) = \{x_iy_i / 1 \le i \le n\}$ and

$E(G) = \{y_ix_{i-1}, y_ix_{i+1} / 2 \le i \le n-1\} \cup \{y_ix_i / 1 \le i \le n\} \cup \{y_1, x_2, y_nx_{n-1}\}$.

Define a function $f : V \to \{1, 2, ..., q+1\}$ by

$f(x_1) = 3$
$f(x_i) = 4i - 3$ for $3 \le i \le n$ if i is odd.
$f(x_2) = 5$
$f(x_i) = 4i$ for $4 \le i \le n$ if i is even.
$f(y_1) = 1$
$f(y_2) = 7$
$f(y_3) = 8$
$f(y_i) = 4i - 5$ for $4 \le i \le n$

Then the resulting edge labels are distinct.

$f(x_1x_2) = 3$
$f(x_ix_{i+1}) = 4i$ for $3 \le i \le n-2$ if i is odd.
$f(x_2x_3) = 7$
$f(x_ix_{i+1}) = 4i + 1$ for $4 \le i \le n-1$ if i is even.
$f(x_1y_2) = 4$
$f(x_iy_{i+1}) = 4i - 2$ for $3 \le i \le n-2$ if i is odd.
$f(y_2x_3) = 8$
$f(y_ix_{i+1}) = 4i - 2$ for $4 \le i \le n-1$ if i is even.
$f(y_ix_i) = 4i - 3$ for $2 \le i \le n-1$ if i is even.
$f(x_1y_1) = 1$
$f(x_3y_3) = 9$
$f(x_iy_i) = 4i - 4$ for $5 \le i \le n$ if i is odd.
$f(y_1x_2) = 2$
$f(y_ix_{i+1}) = 4i - 1$ for $3 \le i \le n-2$ if i is odd.
$f(x_2y_3) = 6$
$f(x_iy_{i+1}) = 4i - 1$ for $4 \le i \le n-1$ if i is even.

Thus f provides a harmonic mean labeling of graph G. Hence G is a harmonic mean graph. □

Example 2.2. *A harmonic mean labeling of zig-zag triangle $Z(T_{11})$ is given in fig 2.1.1*

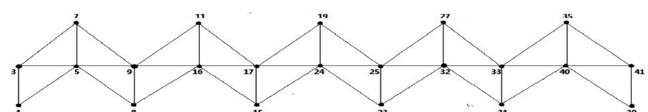


fig 2.1.1

Theorem 2.3. *The alternate zig-zag triangle $A\ Z(T_n)$ is a harmonic mean graph.*

Proof. Let $G = AZ(T_n)$ be an alternate zig-zag triangle snake.

Consider a path $x_1, x_2, ..., x_n$ to construct G, join x_i, x_{i+1} (alternatively) with new vertices s_i, t_i, r_i, for $1 \le i \le n-1$. Then the resultant graph is alternate zig-zag triangle. Here we consider two different cases.

Case(i)

If the degree of x_1 is 3

Define a function $f : V(G) \to \{1, 2, ..., q+1\}$ by,

$f(x_1) = 2$
$f(x_i) = 10k$ if $i = 3k+1$ for $1 \le k \le n$
$f(x_2) = 8$
$f(x_i) = 10k + 6$ if $i = 3k+2$ for $1 \le k \le n$
$f(x_3) = 9$

$f(x_i) = 10k+9$ if $i = 3k+3$ for $1 \le k \le n$
$f(s_i) = 10i-6$ for $1 \le i \le \frac{n}{3}$
$f(r_1) = 1$
$f(r_i) = 10i-8$ for $2 \le i \le \frac{n}{3}$
$f(t_1) = 6$
$f(t_i) = 10i-2$ for $2 \le i \le \frac{n}{3}$

Then the resulting edge labels are distinct.

$f(x_1x_2) = 4$
$f(x_ix_{i+1}) = 10k+3$ if $i = 3k+1$ for $1 \le k \le n$
$f(x_2x_3) = 9$
$f(x_ix_{i+1}) = 10k+8$ if $i = 3k+2$ for $1 \le k \le n$
$f(x_ix_{i+1}) = 10k$ if $i = 3k$ for $1 \le k \le n$
$f(x_1s_1) = 3$
$f(x_is_{i-2k}) = 10k+2$ if $i = 3k+1$ for $1 \le k \le n-2$
$f(x_3s_1) = 5$
$f(x_is_{i-(2k+2)}) = 10k+6$ if $i = 3k+3$ for $1 \le k \le n$
$f(x_2s_1) = 6$
$f(x_is_{i-(2k+1)}) = 10k+5$ if $i = 3k+2$ for $1 \le k \le n-1$
$f(x_1r_1) = 2$
$f(x_ir_{i-2k}) = 10k+1$ if $i = 3k+1$ for $1 \le k \le n-2$
$f(x_2r_1) = 1$
$f(x_ir_{i-(2k+1)}) = 10k+4$ if $i = 3k+2$ for $1 \le k \le n-2$
$f(x_2t_1) = 7$
$f(x_it_{i-(2k+1)}) = 10k+7$ if $i = 3k+2$ for $1 \le k \le n-1$
$f(x_3t_1) = 8$
$f(x_it_{i-(2k+2)}) = 10k+9$ if $i = 3k+3$ for $1 \le k \le n$

Case(ii)

Define a function $f : V(G) \to \{1,2,...,q+1\}$ by

$f(x_1) = 1$
$f(x_i) = 10k$ if $i = 3k+1$ for $1 \le k \le n$
$f(x_2) = 3$
$f(x_i) = 10k+3$ if $i = 3k+2$ for $1 \le k \le n$
$f(x_3) = 7$
$f(x_i) = 10k+7$ if $i = 3k+3$ for $1 \le k \le n$
$f(s_i) = 10i-1$ for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$
$f(r_1) = 2$
$f(r_i) = 10i-9$ for $2 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$
$f(t_i) = 10i-5$ for $2 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$

Then the resulting edge labels are distinct.

$f(x_1x_2) = 1$
$f(x_ix_{i+1}) = 10k+1$ if $i = 3k+1$ for $1 \le k \le n$
$f(x_2x_3) = 4$
$f(x_ix_{i+1}) = 10k+4$ if $i = 3k+2$ for $1 \le k \le n$
$f(x_ix_{i+1}) = 10k-1$ if $i = 3k$ for $1 \le k \le n$
$f(x_2s_1) = 5$
$f(x_is_{i-(2k+1)}) = 10k+5$ if $i = 3k+2$ for $1 \le k \le n-2$
$f(x_3s_1) = 8$
$f(x_is_{i-(2k+2)}) = 10k+8$ if $i = 3k+3$ for $1 \le k \le n-1$
$f(x_is_{i-(2k+1)}) = 10k$ if $i = 3k+1$ for $1 \le k \le n$
$f(x_2r_1) = 2$
$f(x_ir_{i-(2k+1)}) = 10k+2$ if $i = 3k+2$ for $1 \le k \le n-2$
$f(x_3r_1) = 3$
$f(x_ir_{i-(2k+2)}) = 10k+3$ if $i = 3k+3$ for $1 \le k \le n-1$
$f(x_3t_1) = 6$
$f(x_it_{i-(2k+2)}) = 10k+6$ if $i = 3k+3$ for $1 \le k \le n-1$
$f(x_it_{i-(2k+1)}) = 10k-3$ if $i = 3k+1$ for $1 \le k \le n$

Thus f provides a harmonic mean labeling of graph G. Hence G is a harmonic mean graph. □

Example 2.4. *A harmonic mean labeling of zig-zag triangle $Z(T_{12})$ is given in fig 2.2.1*

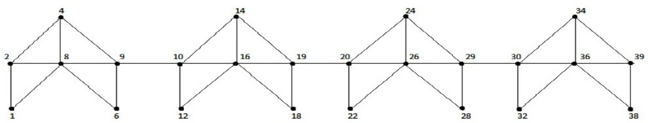


fig 2.2.1

Example 2.5. *A harmonic mean labeling of alternate zig-zag triangle A $Z(T_{14})$ is given in fig 2.2.2*

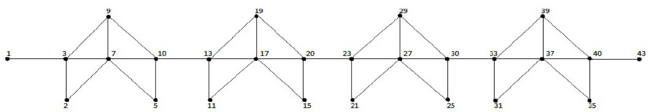


fig 2.2.2

Theorem 2.6. *The spiked snake graph $SS(4,n)$ is a harmonic mean labeling of graphs.*

Proof. Denote the vertices of a spiked snake graph $SS(4,n)$ as follows.

$V(SS(4,n)) = v_1 \cup v_2 \cup v_3 \cup v_5$.

$v_1 = \{v_i/1 \leq i \leq n+1\}$

$v_2 = \{u_i/1 \leq i \leq n\}$

$v_3 = \{w_i/1 \leq i \leq n\}$

$v_4 = \{x_i/1 \leq i \leq n\}$

$v_5 = \{y_i/1 \leq i \leq n\}$

Define a function $f : V(G) \to \{1,2,...,q+1\}$ by

$f(v_1) = 3$

$f(v_i) = 6i-5$ for $2 \leq i \leq n$.

$f(w_i) = 6i-1$ for $1 \leq i \leq n$.

$f(u_1) = 2$

$f(u_i) = 6i-3$ for $2 \leq i \leq n$.

$f(x_i) = 6i$ for $1 \leq i \leq n-1$.

$f(y_1) = 1$

$f(y_i) = 6i-4$ for $2 \leq i \leq n-1$.

Then the resulting edge labels are distinct.

$f(v_1w_1) = 4$

$f(v_{i+1}w_{i+1}) = 6i+3$ for $1 \leq i \leq n-1$.

$f(w_iv_{i+1}) = 6i$ for $1 \leq i \leq n-1$.

$f(w_ix_i) = 6i-1$ for $1 \leq i \leq n$.

$f(u_1y_1) = 1$

$f(u_{i+1}y_{i+1}) = 6i+2$ for $1 \leq i \leq n$.

$f(v_1u_1) = 2$

$f(v_{i+1}u_{i+1}) = 6i+1$ for $1 \leq i \leq n-1$.

$f(u_1v_2) = 3$

$f(u_{i+1}v_{i+2}) = 6i+4$ for $1 \leq i \leq n-1$.

Thus f provides a harmonic mean labeling of graph G. Hence G is a mean graph. □

Example 2.7. *A harmonic mean labeling of spiked snake graph $SS(4,n)$ is given in fig 2.6.1*

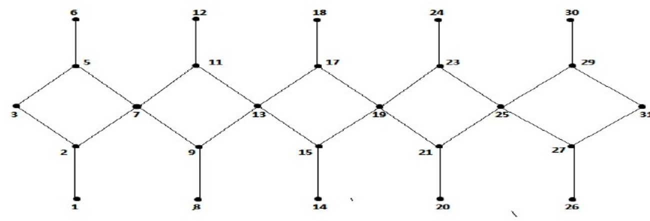


fig 2.3.1

3. Conclusion

We have presented some new results on Harmonic mean labeling of certain classes of graphs like the zig-zag triangle $Z(T_n)$, alternate zig-zag triangle A $Z(T_n)$, spiked snake graph $SS(4,n)$. Analogous work can be carried out for other families and in the context of different types of graph labeling techniques

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