Analytic even mean labeling of splitting graphs

T. Sajitha kumari, 1* M. Regees 2 and S. Chandra kumar 3

Abstract
Let $G(V,E)$ be a graph with $p$ vertices and $q$ edges. $A(p,q)$-graph $G$ is called an analytic even mean graph if there exist an injective function $f: V \rightarrow \{0, 2, 4, ..., 2q\}$ with an induced edge labeling $f^*: E \rightarrow \mathbb{Z}$ such that when each edge $e = uv$ with $f(u) < f(v)$ is labeled with

$$f^*(uv) = \begin{cases} \left\lfloor \frac{f(v)^2 - (f(u)+1)^2}{2} \right\rfloor & \text{if } f(u) \neq 0 \\ \left\lfloor \frac{f(v)^2}{2} \right\rfloor & \text{if } f(u) = 0 \end{cases}$$

if $f(u) \neq 0$ and

$$f^*(uv) = \left\lfloor \frac{f(v)^2}{2} \right\rfloor$$

if $f(u) = 0$, all the edge labels are even and distinct. In this paper we prove that the splitting graphs $spl(P_n)$ and $spl(C_n)$ admits analytic even mean labeling.

Keywords
Mean labeling, Analytic mean labeling, Analytic even mean labeling, Splitting graph.

AMS Subject Classification
05C78.

1. Introduction

By a graph $G = (V,E)$ with $p$ vertices and $q$ edges we mean a simple and undirected graph. The idea of graph labeling was bring in by Rosa in 1967 [1]. Somasundaram and Ponraj [2] have set up the conception of mean labeling of graphs. A detailed survey of graph labeling can be found in [3]. P.Jeyanthi R.Gomathy and Gee-Choon Lau [4] called a graph $G$ is analytic odd mean if there exist an injective function $f: V \rightarrow \{0, 2, 4, ..., 2q-1\}$ with an induced edge labeling $f^*: E \rightarrow \mathbb{Z}$ such that for every edge $uv$ with $f(u) < f(v)$,

$$f^*(uv) = \begin{cases} \left\lfloor \frac{f(v)^2 - (f(u)+1)^2}{2} \right\rfloor & \text{if } f(u) \neq 0 \\ \left\lfloor \frac{f(v)^2}{2} \right\rfloor & \text{if } f(u) = 0 \end{cases}$$

is injective.

$A(p,q)$-graph $G$ is called an analytic even mean graph if there exist an injective function $f: V \rightarrow \{0, 2, 4, ..., 2q\}$ with an induced edge labeling $f^*: E \rightarrow \mathbb{Z}$ such that when each edge $e = uv$ with $f(u) < f(v)$ is labeled with

$$f^*(uv) = \left\lfloor \frac{f(v)^2 - (f(u)+1)^2}{2} \right\rfloor$$

if $f(u) \neq 0$ and

$$f^*(uv) = \left\lfloor \frac{f(v)^2}{2} \right\rfloor$$

References

1. Scott Christian College(Autonomous) Nagercoil-629001, Kanyakumari, Tamil Nadu, India.
2. Department of Mathematics, Malankara Catholic College, Maranagiri, Kaliakavilai-629153, Tamil Nadu, India.
3. Department of Mathematics, Scott Christian College(Autonomous), Nagercoil-629001, Kanyakumari, Tamil Nadu, India.
*Corresponding author: 1 sajithasabu09@gmail.com

©2020 MJM.
if \( f(u) = 0 \), all the edge labels are even and distinct. This labeling \( f \) is called an analytic even mean labeling [5]. The splitting graph \( spl(G) \) is obtained by adding a new vertex \( v' \) corresponding to each vertex \( v \) of \( G \) such that \( N(v) = N(v') \) [6].

### 2. Main Results

Here we show that the splitting graphs \( spl(P_n) \) and \( spl(C_n) \) admit analytic even mean labeling.

**Theorem 2.1.** The splitting graph \( spl(P_n) \) is an analytic even mean graph.

**Proof.** Let \( G \) be the splitting graph \( spl(P_n) \).

Let
\[
V(G) = \{v_i, v'_i; \ 0 \leq i \leq n - 1\}
\]
and
\[
E(G) = \{v_iv_{i+1}, v_i'v'_i, v'_iv'_{i+1}; 0 \leq i \leq n - 2\}.
\]
Here
\[
|V(G)| = 2n
\]
and
\[
|E(G)| = 3(n - 1).
\]

We define an injective map
\[
f : V(G) \to \{0, 2, 4, ..., 6(-1)\}
\]
by
\[
f(v_0) = 0
\]
and
\[
f(v_i) = 2i; \ 1 \leq i \leq n - 1
\]
and
\[
f(v'_i) = 2n + 2i; \ 0 \leq i \leq n - 1
\]

Let \( f^* \) be the generated edge labeling of \( f \).

\[
f^*(v_iv_{i+1}) = \frac{4i + 3}{2}; \ 1 \leq i \leq n - 1,
\]
\[
f^*(v_{i+1}v'_i) = \frac{4n^2 + 8in - 12i - 9}{2}; \ 0 \leq i \leq n - 2,
\]
\[
f^*(v'_iv'_{i+1}) = \frac{4n^2 + 8in + 4i + 8n + 3}{2}; \ 0 \leq i \leq n - 2,
\]

The edges \( v_{i+1}v'_i \) are increased by 14 as \( i \) increases, the edges \( v'_iv'_{i+1} \) are increased by 22 as \( i \) increases and the edges \( v_iv_{i+1} \) are increased by 2 as \( i \) increases.

According to this, all the edge labels are even and distinct. Hence the splitting graph \( spl(P_n) \) is an analytic even mean graph.

---

**Example 2.2.** The analytic even mean labeling of the splitting graph \( spl(P_5) \) is shown in the following figure.

![Figure 2.1](image)

**Theorem 2.3.** The splitting graph \( spl(C_n) \) is an analytic even mean graph.

**Proof.** Let \( G \) be the splitting graph \( spl(C_n) \).

Let
\[
V(G) = \{v_iv'_i; 0 \leq i \leq n - 1\}
\]
and
\[
E(G) = \{v_iv_{i+1}, v'_iv'_{i+1}; 0 \leq i \leq n - 2\} \cup \{v_{n}v'_{n-1}, v'_nv'_{n-1}, v_{n-1}v'_n, v'_{n-1}v'_{n-1}\}
\]
Here
\[
|V(G)| = 2n
\]
and
\[
|E(G)| = 3n
\]

We define an injective map \( f : V(G) \to \{0, 2, 4, ..., 6n\} \) by
\[
f(v_0) = 0
\]
and
\[
f(v_i) = 2i; \ 1 \leq i \leq n - 1
\]
and
\[
f(v'_i) = 2n + 2i; \ 0 \leq i \leq n - 1
\]

Let \( f^* \) be the generated edge labeling of \( f \).

\[
f^*(v_0v_{n-1}) = 2(n - 1)^2
\]
\[
f^*(v_iv_{i+1}) = \frac{4i + 3}{2}; \ 0 \leq i \leq n - 2
\]
\[
f^*(v'_iv'_{i+1}) = \frac{4n^2 + 8in + 4i + 8n + 3}{2}; \ 0 \leq i \leq n - 2,
\]
\[
f^*(v_{n}v'_{n-1}) = \frac{16n^2 - 16n + 3}{2}
\]
\[
f^*(v'_{n-1}v'_{n}) = \frac{4n - 1}{2}
\]

According to this, all the edge labels are even and distinct. Hence the splitting graph \( spl(C_n) \) is an analytic even mean graph.
Example 2.4. The analytic even mean labeling of the splitting graph $spl(C_9)$ is shown in the following figure.

![Figure 2.2](image)

3. Conclusion

Here we proved the analytic even mean labeling of the splitting graphs $spl(P_n), spl(C_n)$. In future, we can construct many analytic even mean graphs using these ideas.

References


