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Coding through on the disconnected graph $nK_{1,m}$, $n>1$ a super mean labeling graph

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Abstract
In this paper we introduce the disconnected graph $nK_{1,m},\ n>1$ a super mean labeling graph and Sedge to connect to two, three, four up to n components. Now a technique of coding a message is presented using the super mean labeling graph on the disconnected graph $nK_{1,m}$, $n>1$ (two connected graph, three connected graph etc.).

Keywords
Connected graph,Disconnected graph, star graph, labeling graph, mean labeling graph, Sedge, Super mean labeling, GMJ coding, OFEB, PSNF, EBOF.

AMS Subject Classification
03E72.

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Contents

1. Introduction

A few techniques for coding a message secretly using super mean labeling on a the disconnected graph $nK_{1,m}$, $n>1$ presented through GMJ code. GMJ code stands for: (1) Graph Message Jumble code. A coding technique to communicate a message through graphs jumbling letters is named as GMJ code. (2) It also refers to the name of one of the researchers of this paper (Gabriel Margaret Joan) who has conceived this method of coding. Also, a rule for the super mean labeling on the disconnected graph $nK_{1,m}$, $n>1$ is provided which will facilitate the assignment of numbers. The communication becomes very much limited between the sender and the receiver and not to be understood by others. Cryptography is the science containing methods to transform an intelligible message into one it is unintelligible and transforming the message back to its form. For more details on this theory and its applications, we suggest the reader to refer [1–9].

Definition 1.1 (Graph)**.** *A graph $G=(V,E)$ consists of a finite set denoted by V, or by $V(G)$ if one wishes to make clear which graph is under consideration, and a collection E or $E(G)$, of unordered pairs $\{u,v\}$ of distinct elements from V. Each elements of V is called a vertex or a point or a node, and each element of E is called an edge or a line or a link.*

Definition 1.2 (Subgraph)**.** *Let G be a graph with vertex set $V(G)$ and edge-list $E(G)$ a subgraph of G is a graph all of whose vertices belong to $V(G)$ and all of whose edges belong to $E(G)$.*

Definition 1.3 (Connected and Disconnected graph)**.** *A graph G is connected if there is a path in G between any given pair of vertices, otherwise it is disconnected. Every disconnected graph can be split up into a number of connected subgraphs, called components.*

Definition 1.4 (Star graph)**.** *A star graph $K_{1,n}$ is a tree with n vertices of degree 1 and root vertex has degree n.*

Definition 1.5 (Sedge)**.** *A Sedge is defined as an edge connecting two components of a graph $K_{1,m}\cup K_{1,n}$ is a two star graph and is a two component or disconnected graph and is denoted by $\dot{\c{S}}$ which means adding a Sedge to a disconnected graph with components becomes a connected or a single component graph.*

$K_{1,l} \cup K_{1,m} \cup K_{1,n}$ is a three star graph and disconnected graph with three components and two Sedges becomes a connected or a single component graph.

Note: In this paper all Sedge is considered to connect the non-pendent vertices.

Definition 1.6 (Mean labeling graph)**.** *A graph p vertices and q edges is said to be a mean graph if there exists a function f from the vertex set of G to $\{0,1,2,\dots,q\}$ such that the induced map f^* from the edge set of G to $\{1,2,\dots,q\}$ defined by*

$$f^*(e{=}uv) = \begin{cases} \dfrac{f(u)+f(v)}{2}, & \text{if } f(u)+f(v) \text{ is even and} \\ \dfrac{f(u)+f(v)+1}{2}, & \text{if } f(u)+f(v) \text{ is odd.} \end{cases}$$

Then the resulting edges get distinct labels from the set $\{1,2,\dots q\}$.

Definition 1.7 (Super mean labeling)**.** *Let G be a (p,q) graph and $f: V(G) \to \{1,2,3\dots,p+q\}$ be an injection. For each edge $e = uv$,*

$$f^*(e) = \begin{cases} \dfrac{f(u)+f(v)}{2}, & \text{if } f(u)+f(v) \text{ is even and} \\ \dfrac{f(u)+f(v)+1}{2}, & \text{if } f(u)+f(v) \text{ is odd.} \end{cases}$$

Then f is called super mean labeling if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{1,2,3,\dots,p+q\}$. A graph that admits super mean labeling is called a super mean graph.

Theorem 1.8. *The disconnected graph $nK_{1,7}$, $n > 1$ is a super mean graph.*

Proof. The super mean labeling of $2K_{1,7}$ and $3K_{1,7}$ are given in Figure 1.1.

Let $G_1 = 2K_{1,7}$ and $G_2 = 3K_{1,7}$. Now $G_1 \cup G_1 = 4K_{1,7}$ and $G_1 \cup G_2 = 5K_{1,7}$. By Union of two super mean graphs is a super mean graph, $4K_{1,7}$ and $5K_{1,7}$ are super mean graphs. Again $G_1 \cup G_1 \cup G_1 = G_1 \cup 4K_{1,7} = 6K_{1,7}$ and $G_1 \cup G_1 \cup G_2 = G_1 \cup 5K_{1,7} = 7K_{1,7}$. By Union of two super mean graphs is a super mean graph, $6K_{1,7}$ and $7K_{1,7}$ are super mean graphs. Now for any integer $m \geqslant 2$, $mG_1 = 2mK_{1,7}$ and $(m-1)G_1 \cup G_2 = (2m+1)K_{1,7}$ are super mean graphs. Hence $nK_{1,7}$ is a super mean graph for all $n > 1$. □

Example 1.9.

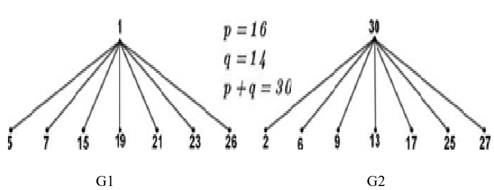

Figure 1.1. Disconnected graph with two components (Two-connected graph). Disconnected graph with two components G_1 and G_2.

Theorem 1.10. *The disconnected graph $nK_{1,m}$, $n > 1$ is a super mean graph.*

Proof. The proof follows by using Theorem 1.8. □

2. A Rule for Labeling

1. Some observations on super mean labeling of K_1, $m \cup K_1$, n, $m \leq n$ are listed. Here p and q represent the number of vertices and edges, $p = 2+m+n$, $q = m+n$, $p+q = 2+2m+2n$. The numbers from 1 to $2+2m+2n$ must be assigned to the top vertices and the pendant vertices, and in the process, the edge values get allotted. Repetition is not permitted. Here $f(u)$, $f(v)$, $f(u_i)$, and $f(v_j)$ are the numbers assigned to the top vertices and the pendant vertices, and $f^*(uu_i)$ and $f^*(vv_j)$ are the numbers assigned to the edges of the first and the second star, respectively. The rule for getting the edge values $f^*(uu_i) = (f(u)+f(v))/2$ or $f^*(uu_i) = (f(u)+f(v)+1)/2$ if $f(u)+f(v)$ where the edge connects u and u_i. Note that the edge value can be the actual or adjusted mean. The average of the largest and the previous number is $\dfrac{(2+2m+2n)+(2+2m+2n-1)}{2} = 2+2m+2n$. As repetition is not allowed, this combination is not considered. Hence neither the edge value nor the pendant vertices can exceed $2+2m+2n$. So it becomes possible to label a $K_{1,m} \cup K_{1,n}$ through super mean labeling for all values of m and n without omitting any number between 1 and $(p+q)$ with $m \leq n$.
2. If $\big(f(u)$ and $f(u_i)\big)$ or $\big(f(v)$ and $f(v_i)\big)$ are both odd or both even, then the edge value is the actual mean. If they are not alike, then the edge value assumes the adjusted mean.
3. When $f(u)$ is odd and if $f(u_i) = 2s$, $f(u_i+1) = 2s+1$ they lead to same edge value and hence to be avoided, that is, $\dfrac{1+6}{2}$ and $\dfrac{1+7}{2}$ have the same edge value.
4. When $f(u) = 1$. When $f(u_i) = 2s+1$ and $f(u_i+1)$ different edge values and hence can be assigned. That is, give different edge values.

Also when $f(u)$ is even, the situation is reversed. These are to be noted while labeling the numbers to the pendant vertices.
Step 1: Take 1 and $p+q$ as $f(u)$ and $f(v)$, respectively. $f(u_i) = 2$, for the edge value becomes 2, but $f(v_i) = 2$ is permitted.
Step 2: If $f(u_i) = 3$, then $f(v_i) = 4$ and if $f(u_i) = 5$, then $f(v_i) = 2$. That is, assign a value to u_1 and assign the next possible least integer to v_1 of the second star, the u_2 and v_2 are labeled proceeding in the same manner. Once or twice we may have to continue with assigning to u_is successively in order to avoid any repetition. This procedure makes labeling disconnected graph $nK_{1,m}$, $n > 1$ easy using super mean labeling.

3. GMJ Coding Method

By assigning numbers to the 26 alphabets of English in a different manner, choosing a suitable labeled graph with a given clue mathematical or non-mathematical, finding the number in the graph for each letter of each word of the given message, and presenting the letter codes in a unique way in some form such as a horizontal string and the codes by a picture shuffling the order of the letters in order to increase the secrecy of the coded message is named as GMJ coding method.

3.1 Procedure for Encoding

Step 1: A suitable disconnected graph has to be taken. A clue, mathematical or non-mathematical is stated to find the disconnected graph which is to be used.
Step 2: By using the super mean labeling on this disconnected graph, numbers from 1 to $(p+q)$ are assigned to the top vertices and the pendant vertices and there by the edge values get fixed. The rule given in Section 2 (Step 2) goes a long way in fixing the super mean labeling on the disconnected graph.
Step 3: The 26 alphabets of English are divided in some way (using GMJ coding), and the numbers attached to the alphabets are noted down.
Step 4: The Greek letters α and β are used to refer to the first and second components, respectively. T, E_i and P_i denote the top vertex, the ith pendant vertex, and the ith edge value in order. For example, $\alpha(P_i)$ denotes the number assigned to the ith pendant vertex of the first star.
Step 5: The message to be coded is written (wordwise).
Step 6: By using the notations stated in Step 4, the coding is written along a horizontal string with (1,1) denoting the space between the words.
Step 7: Present the coding in a shape desired, shuffling the order of the letters.
Step 8: For decoding the message, the instructions given for coding and a knowledge of super mean labeling on disconnected graph $nK_{1,m}$, $n > 1$ is a super mean graph

Example 3.1. *Consider Disconnected graph G_1 and G_2.*

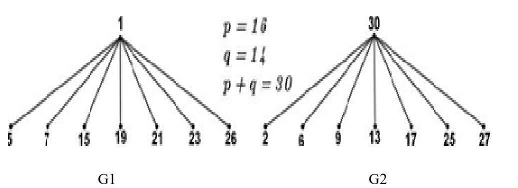

Figure 3.1

Example 3.2. ***Message***: *Valleyside Bamboo Bridge.*
Clue: *A special prime twinkling perfect one. (Special prime-2, it is the only even number which is prime, twinkling-referring to a Star. So, a two star is understood. First perfect number is 6. Therefore the required graph is $(K_{1,6} \cup K_{1,6}.)$*

Labeling: *The super mean labeling done for $K_{1,6} \cup K_{1,6}$ is shown below.*

$$K_{1,6} \cup K_{1,6},\ p = 14, q = 12, p+q = 26$$
$$f(u) = 1, f(u_1) = 5, f(u_2) = 7, f(u_3) = 15, f(u_4) = 19,$$
$$f(u_5) = 21, f(u_6) = 23, f(v) = 26, f(v_1) = 2,$$
$$f(v_2) = 6, f(v_3) = 9, f(v_4) = 13, f(v_5) = 17, f(v_6) = 24.$$

1	26	2	25	3	24	4	23	5	22	6	21	7
A	B	C	D	E	F	G	H	I	J	K	L	M
20	8	19	9	18	10	17	11	16	12	15	13	14
N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Numbering of alphabets: *(OFEB) The numbers 1–13 and 14–26 are allotted to the odd-and even-positioned alphabets moving forward and backward from A to C to E and Z to X to V and so on; this method is named as OFEB (odds forward, evens backward).*

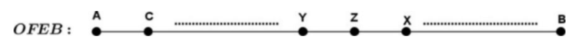

Figure 3.2

We express the numbering of alphabets in terms of a function for encoding. For decoding we reverse the process. Coding is done word by word using OFEB to get the number for any letter and search for it in the two star graph labeled.

$$g(2k+1) = k+1 \text{ for } k = 0, 1, \ldots, 12$$
$$g(2k) = (27-k) \text{ for } k = 1, 2 \ldots, 13$$

Coding: *(wordwise)*
Valleyside–$\beta(E_2)\alpha(T)\alpha(P_5)\alpha(P_5)\alpha(E_1)$
$\beta(P_4)\alpha(E_4)\alpha(P_1)\beta(E_6)\alpha(E_1)$
bamboo–$\beta(T)\alpha(T)\alpha(P_2)\beta(T)\alpha(E_3)\alpha(E_3)$
bridge–$\beta(T)\beta(E_3)\alpha(P_1)\beta(E_6)\alpha(E_2)\alpha(E_1)$
Horizontal string: $\beta(E_2)\alpha(T)\alpha(P_5)\alpha(P_5)\alpha(E_1)\beta(P_4)\alpha(E_4)$ $\alpha(P_1)\beta(E_6)\alpha(E_1)(1,1)\beta(T)\alpha(T)\alpha(P_2)\beta(T)\alpha(E_3)$ $\alpha(E_3)(1,1)\beta(T)\beta(E_3)\alpha(P_1)\beta(E_6)\alpha(E_2)\alpha(E_1)$

Example 3.3. *For the same shape, graph, and sentence, a different coding pattern is used just for comparison.*

1	6	7	2	8	9	10	11	3	12	13	14	15
A	B	C	D	E	F	G	H	I	J	K	L	M
16	17	4	18	19	20	21	22	23	24	25	5	26
N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Numbering of alphabets: PSNF The alphabets in the position 12, 22, 32, 42 and 52 are given the numbers 1, 2, 3, 4, 5.

Then the letter B gets the number 6, C gets the number 7, and E gets the number 8 and so on. This method is named as PSNf (perfect square numbered first).

The function for encoding is given below
$g(n^2) = n$, for $n = 1, 2, 3, 4, 5$. $g(n^2 + k) = 5 + k$, for $n = 1$, $k = 1, 2$
$g(n^2 + k) = 7 + k$, for $n = 2$, $k = 1, 2, \ldots, 4$

Figure 3.3

$g(n^2+k) = 11+k$, for $n=3$, $k=1,2,\ldots,6$
$g(n^2+k) = 17+k$, for $n=4$, $k=1,2,\ldots,8$
$g(n^2+k) = 25+k$, for $n=5$, $k=1$.

Coding is done word by word using PSNF to get the number for any letter and search for it in the two star graph labeled.

Horizontal string: $\alpha(P_6)\alpha(T)\beta(E_1)\beta(E_1)\alpha(E_3)\alpha(P_1)$ $\beta(E_4)\alpha(E_1)\beta(P_1)\alpha(E_3)(1,1)\beta(P_2)\alpha(T)\alpha(P_3)\beta(P_2)$ $\beta(P_5)\beta(P_5)(1,1)\beta(P_2)\alpha(P_4)\alpha(E_1)\beta(P_1)\alpha(E_4)\alpha(E_3)$

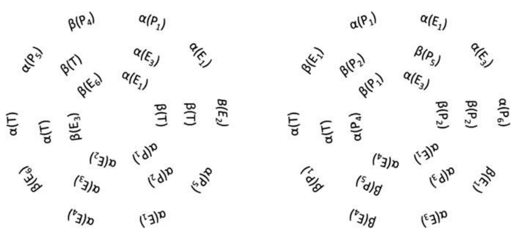

Figure 3.4. Coded message (OFEB) Coded message (PSNF)

For the same sentence the codings are different with respect to the two methods OFEB and PSNF are seen from Examples 3.2 and 3.3 represented by Fig. 3.4.

Example 3.4. *Message Stop be silent for zero six days.*
Clue One more than perfect single two less than perfect double. (Here perfect single refers to the first perfect number 6, one more than 6 is 7, perfect double is $6\times 2 = 12$, 2 less than this is 10, so the graph is $K_{1,7}\cup K_{1,10}$.
Labeling The super mean labeling done for $K_{1,7}\cup K_{1,10}$ shown below.
$p=19$, $q=17$, $p+q=36$
$f(u)=1, f(u_i)=3, f(u_2)=9, f(u_3)=12, f(u_4)=14,$
$f(u_5)=30, f(u_6)=33, f(u_7)=35, f(v)=3, f(v_1)=4,$
$f(v_2)=6, f(v_3)=10, f(v_4)=11, f(v_5)=13, f(v_6)=15,$
$f(v_7)=19, f(v_8)=22, f(v_9)=27, f(v_{10})=31.$

Numbering of alphabets and writing the coding are done as in Example 3.3 using EBOF.
Coding(word wise) After coding Horizontal string is written.
Horizontal string $\beta(E_3)\beta(P_1)\beta(E_2)\beta(P_2)(1,1)\beta(P_5)\alpha(E_5)$ $(1,1)\beta(E_3)\alpha(E_7)\alpha(E_4)\alpha(E_5)\alpha(E_3)\beta(P_1)(1,1)\beta(P_4)$ $\beta(E_2)\alpha(E_2)(1,1)\alpha(T)\alpha(E_5)\alpha(E_2)\beta(E_2)(1,1)\beta(E_3)$ $\alpha(E_7)\alpha(E_1)(1,1)\alpha(P_3)\alpha(P_4)\beta(E_6)\beta(E_3)$.

Example 3.5. *For the same shape, graph, and sentence, different coding pattern is used just for comparison. By using PSNF and the super mean labeling on $K_{1,7}\cup K_{1,10}$ the message is encoded.*
Horizontal string $\beta(E_1)\beta(E_2)\alpha(E_6)\beta(P_1)(1,1)\beta(P_2)$ $\alpha(E_4)(1,1)\beta(E_1)\alpha(P_1)\alpha(P_4)\alpha(E_4)\alpha(E_5)\beta(E_2)(1,1)$ $\alpha(P_2)\alpha(E_6)\beta(P_7)(1,1)\beta(E_6)\alpha(E_4)\beta(P_7)\alpha(E_6)(1,1)$ $\beta(E_1)\alpha(P_1)\beta(E_5)(1,1)\alpha(E_1)\alpha(T)\alpha(E_2)\beta(E_1)$

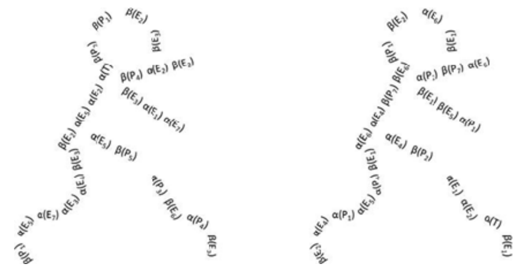

Figure 3.5. Coded message (EBOF) Coded message (PSNF)

For the same sentence, the codings are different with respect to the two methods EBOF and PSNF are seen from Examples 3.4 and 3.5 and represented by Fig. 3.5.

4. Conclusion

In this paper, we have used the super mean labeling on any suitable for the disconnected graph $nK_1, m, n > 1$ communicating some messages through different types of numbering of alphabets OFEB,PSNF, EBOF (and also used in three connected graph, that means three components RNRO,IPFO, and VSCC for coding). The coded messages are given in pictures in order to make the understanding of the messages a bit difficult.

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