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A study on rw-closed sets in topological spaces

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Abstract

The aim of this is to introduce a new class of sets namely rw-closed sets in topological spaces and their properties. Furthermore we discuss $rw\mu$-closed sets and generalized regular sets in topological spaces.

Keywords

Topology, spaces.

AMS Subject Classification

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Contents

1. Introduction

The branch of mathematics that studies topological spaces in their won right is called Topology. Topology is the Modern version of geometry the study of all different sorts of spaces. The word topology is used both for the mathematical discipline and for a family of sets with certain properties that are used to define a topological spaces a basic object of topology.

Topology is a major area of mathematics concerned with properties that are preserved under continuous deformation of objects such as *rw*-closed sets, *rw*-locally closed sets and $rw\mu$-closed sets.

2. Peliminaries

Definition 2.1. *A Topology on a set X is a collection τ of subsets of X having the following properties*

1. *$\phi$and X are in τ.*
2. *The union of the elements of any sub collection of τ is in τ.*
3. *The intersection of the elements of any finite sub collection of τ is in τ.*

A set X for which a topology τ has been specified is called a Topological space.

Example 2.2. *Let $X=\{a,\ b,\ c\}$ then this set has 2^3 elements then*

$$t=\{\{\},\ \{a\},\ \{b\},\ \{c\},\ \{a,\ b\},\ \{b,\ c\},\ \{c,\ a\},\ \{a,\ b,\ c\}\}$$

Definition 2.3. *If X is a topological space with topology t, say that a subset $\cup$of X is an open set of X if $\cup \in \tau$.*

Definition 2.4. *A subset A of a topological space X is said to be closed if the set $X-A$ is open.*

Example 2.5. *The subset $[a,\ b]$ of $\mathbb{R}$ is closed because it is complement $\mathbb{R}-[a,\ b]=(-\infty,\ a)\cup(b,\ +\infty)$. Is an open set. (The union of open sets is open).*

Definition 2.6. *A subset A of a topological space (X,τ) is called Regular open, if $A=int\ [cl\,(A)]$.*

Definition 2.7. *A subset A of a topological space (X,τ) is called Regular closed, if $A=cl\ [int\ \ (A)]$.*

Definition 2.8. *A subset A of a space (X,τ) is called Regular Semi-open if there is a regular open set U such that $U\subset A\subset cl\ \ (U)$. The family of all regular semi-open sets of X is denoted by $RSO\,(x)$.*

Definition 2.9. *A subset A of a topological space (X,τ) is called is pre-open if $A\subseteq int\ [cl\ \ (A)]$.*

Definition 2.10. *A subset A of a topological space (X,τ) is called Semi-openif $A\subseteq int\ [cl\ \ (A)]$.*

Definition 2.11. *A subset A of a space (X,τ) is said to be Semi-regular open if it is both semi-open and semi closed.*

Definition 2.12. *A clopen set in a topological space is a set which is both open and closed.*

A set is defines to be closed if its complement is open, which leaves the probability an open set whose complement is itself also open, making the first set both open and closed and therefore clopen.

Example 2.13. *In any topological space X, the empty set and the whole space X are both clopen.*

Consider the space X which consists of the union of the two intervals $[0,\ 1]$ *and* $[2,\ 3]$ *of R.*

The topology on X is inherited as the subspaces topology from the ordinary topology on the real in R. In X, the set $[0,\ 1]$ *is clopen, as is the set* $[2,\ 3]$.

Definition 2.14. *A subset of A of a space* (X,τ) *is called Generalized closed, if* $cl\ (A) \subseteq \cup$ *whenever* $A \subseteq U$ *and* $U \in \tau$.

Definition 2.15. *A subset A of a space* (X,τ) *is called Regular Generalized Closed, if* $cl\ (A) \subseteq \cup$ *whenever* $A \subseteq U \in RO(x)$.

Definition 2.16. *A subset A of a space* (X,τ) *is called Generalized pre-regular closed (or) pre-regular generalized closed if* $cl\ (A) \subseteq \cup$ *whenever* $A \subseteq U \in RO(x)$.

Definition 2.17. *A subset A of a space* (X,τ) *is called Regular Weekly Generalized Closed Set if* $cl\ [Int\ (A)]$ *whenever* $A \subseteq U$ *and U is regular open in X.*

Definition 2.18. *A Bi-topological is a set endowed with two topological typically, if the set is X and the topologies are* σ *and* τ *the refer to the bi-topological spaces as* (σ,τ).

Definition 2.19. *The intersection of all regular semi-open subsets of* (X,τ) *containing A is called the regular semi-Kernel of and is denoted by* $rsker\ (A)$.

Result 2.20. *For any subset A of* (X,τ), $A\ \subset rsker\ (A)$.

Definition 2.21. *A subset A in* (X,τ) *is called regular W-open in X if* A^c *is rw− closed in* (X,τ).

Definition 2.22. *Let* μ *be a general topology on a topological space* (X,τ) *then* $A \subseteq X$ *is called a Regular weakly* $\mu-$ *closed set (or) simply a* **r***w.*$\mu-$ *closed set, if* $C_\mu(A) \subseteq U$ *whenever* $A \subseteq U\ \in RSO(X)$.

The complement of anrw $\mu-$ *closed set is called an* **r***w* $\mu-$ *open set.*

Remark 2.23. *Let* μ *be a general topological space* (X,τ).

Then μ*-closed set* $\rightarrow W\mu-$ *closed set* $\rightarrow$ *rw.*μ*-closed set.*

Example 2.24. *Consider the topological space* (X,τ)*, Where* $X = \{a,\ b,\ c\}$ *and let* $\tau = \{X, \emptyset,\ \{b\}, \{c\},\ \{b,\ c\}\}$, $\mu = \{X, \emptyset,\ \{c\},\ \{a,\ b\},\ \{b,\ c\},\ \{a,\ c\}\}$ *be a general topology on the space X then* $\{b,c\}$ *is rw* $\mu-$ *closed in* (X,τ) *but neither a rw* $\mu-$ *closed set not a* $\mu-$ *closed set .*

Result 2.25. *The space* (X,τ) *in general topology* $\mu = \tau$ *then rw*$\mu-$ *closed sets become equivalent to rw− closed sets.*

Theorem 2.26. *The union of two rw− closed subsets of X is also rw− closed subsets of X.*

Proof. Assume that, A and B are $rw-$ closed sets in X.

To prove: $A \cup B$ is an $rw-$ closed subset of X.

Let U be regular semi-open in X. Such that $A \cup B \subset U$. Then $A \subseteq U$ *and* $B \subseteq U$. Since A and B are $rw-$ closed, $cl\ (A) \subset U$ and $cl\ (B) \subset U$. Hence $cl\ (A \cup B) = cl\ (A) \cup cl\ (B) \subset U$.

$$ie)\ cl\ \ (A \cup B) \subset U.$$

Therefore $A \cup B$ is a $rw-$ closed set in X. Hence the union of two $rw-$ closed subset of X is also a $rw-$ closed subset of X. □

Result 2.27. *The intersection of two rw− closed sets in X is generally not an rw− closed set in X.*

Example 2.28. *Let* $X = \{a,\ b,\ c,\ d\}$ *be with* $\tau = \{X, \emptyset,\ \{a\},\ \{b\},\ \{a,\ b\},\ \{a,\ b,\ c\}\}$.

If $A = \{a,\ \ b\}$ *and* $B = \{a,\ \ c,\ \ d\}$*. Then A and B are rw− closed sets in X, but* $A \cap B = \{a\}$ *is not anrw− closed set in X.*

Theorem 2.29. *For an element* $x \in X$ *the set* $X/\{x\}$ *is rw− closed or regular semi open.*

Theorem 2.30. *If A is regular open and rw− closed then A is regular closed and hence clopen.*

Theorem 2.31. *If A is a rw− closed subset of X such that* $A \subset B \subset cl\ (A)$*. Then A is a rw− closed set in X.*

Result 2.32. *Consider the topological space* (X,τ)*. Where* $X = \{a,\ \ b,\ \ c,\ \ d\}$ *be with the topology.*

$$t = \{X, \emptyset,\ \ \{a\},\ \{b\},\ \{a,\ \ b\},\ \{a,\ b,\ c\}\}.$$

Let $A = \{d\}$ *and* $B = \{c,\ \ d\}$*. Then A and B are rw− closed sets in* (X,τ) *but* $A \subset B$ *is not subset in* $cl\ (A)$.

Theorem 2.33. *Let A be rw− closed in* (X,τ)*. Then A is closed if and only if* $cl\,(A)/A$ *is regular semi – open.*

3. Conclusion

The mathematical technique resented in this project provides both on open sets and closed sets with which to understand the property of clopen set to discuss the theorems in different areas that are connected by the ir common assumptions of clopen set.

This project discussed the concepts of rw− closed sets and rw$\mu-$ closed sets some definitions withexample if $rw-$ closed sets extended togrw− closed sets, rw$g-$ closed sets in Topological space.

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