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The Co-bondage (Bondage) number of fuzzy graphs and its properties

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Abstract

In this paper, we define the Co-bondage number $b_c(G)$and new type of non-bondage$\{b_{en}$ and $b_{tn}\}$ for any fuzzy graph,and fuzzy strong line graph. A characterization is obtained for fuzzy strong line graphs $L_s(G)$ such that $L_s(G)$ is tree . A necessary condition for a fuzzy double strong line graph of cycle is a fuzzy trees and the exact value of $b_n(G)$ for any graph G is found and exact values of b_c, b_{en} and b_{tn} for some standard graphs are found and some bounds are obtained. Also,find the exact value of $b_{tn}(G)$ for any graph G is found. Moreover we define neighbourhood extension also analysis it properties by using bondage arcs and we also obtained relationships between b_c, $b_{tn}(G)$ and b_t.

Keywords: $\gamma(G)$- Minimum dominating set ,$b_c(G)$- maximum co-bondage number ,b(G)- minimum bondage number , b_{tn}-maximum total non-bondage number, b_{en}- maximum efficient-bondage number ,$L_s(G)$ - strong line graph,$L_s^*(G)$ - double strong line graph.

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1 Introduction

Fuzzy graph theory was introduced by A. Rosenfeld[8] in 1975. Fuzzy graph theory is now finding numerous applications in modern science and technology especially in the fields of neural networks, expert systems, information theory, cluster analysis, medical diagnosis, control theory, etc. Sunil Mathew, Sunitha M.S [10] has obtained the fuzzy graph-theoretic concepts like f- bonds, paths, cycles, trees and connectedness and established some of their properties. V.R. Kulli and B. Janakiram [7] have established the non-bondage number of a graph. First we give the definitions of basic concepts of fuzzy graphs and define the non-bondage and it is properties. All graphs consider here are finite, undirected, distinct labeling with no loop or multi arcs and p nodes and q (fuzzy) arcs. Any undefined term in this paper may be found in Harary[5]. Among the various applications of the theory of domination that have been considered, the one that is perhaps most often discussed concerns a communication network. Such a network consists of existing communication links between a fixed set of sites. The problem is to select a smallest set of sites at which to place transmitters so that every site in the network that does not have a transmitter is joined by a direct communication link to one that does have a transmitter. This problem reduces to that of finding a minimum dominating set in the graph corresponding to the network. This graph has a node representing each site and an arc between two nodes if the corresponding sites have a direct communications link joining them. To minimize the direct communication links in the network, it is non-bondage but in case we want minimum number site to control all other location sites that mean reduce the number of transmitting station. It is Co- bondage,and more

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general graph are connected but need not internal connected so determine say graph connected internal or not is called Neighbourhood Extendable we introduce the following section.

2 Preliminaries

Definition 2.1. *A fuzzy subset of a non-empty set V is a mapping* $\sigma : V \to [0,1]$. *A fuzzy relation on V is a fuzzy subset of* $E(V \times V)$. *A fuzzy graph* $G = (\sigma, \mu)$ *is a pair of function* $\sigma : V \to [0,1]$ *and* $\mu : V \times V \to [0,1]$, *where* $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$*for all* $u, v \epsilon V$.

Definition 2.2. *The underlying crisp graph of G=*(σ,μ) *is denoted by* $G^* = (V, E)$, *where* $V = \{u\epsilon V : \sigma(u) > 0\}$ *and* $E = \{(u,v)\epsilon V \times v : \mu(u,v) > 0\}$.

Definition 2.3. *The order* $P = \sum_{v\epsilon D} \sigma(v)$. *The graph* $G = (\sigma, \mu)$ *is denoted by G, if unless otherwise mentioned. Let be a fuzzy graph on. The degree of a vertex u is* $d_G(u) = \sum\limits_{(u \neq v)} \mu(uv)$*and The minimum degree of G is* $\delta(G) = \wedge\{d_G(u), u\epsilon V\}$*and the maximum degree of G is* $\delta(G) = \vee d_G(u), \forall \epsilon V$

Definition 2.4. *The strength of connectedness between two nodes u and v in a fuzzy graph G is define as the maximum of the strength of all paths between u and v and is denoted by* $CONN_G(u,v)$.

Definition 2.5. *A u-v path P is called a strongest path if its strength equals* $CONN_G(u,v)$.

Definition 2.6. *A fuzzy graph* $H = (\tau, \rho)$ *is called a fuzzy sub graph of G if* $\tau(x) \leq \sigma(x)$*for all* $x\epsilon V$*and* $\rho(x,y) \leq \mu(x,y)$ *for all*$(x,y)\epsilon V$.

Definition 2.7. *A fuzzy sub graph H =*(τ, ρ) *is said to be a spanning fuzzy sub graph of G, if* $\tau(x) = \sigma(x)$*for all x.*

Definition 2.8. *A fuzzy G is said to be connected if there exists a strongest path A path P of length n is a sequence of distinct nodes* $u_0 u_1, u_2, u_n$ *such that* $(u_{i-1}, u_i) > 0$ *and degree of membership of a weakest arc is defined as its strength.*

Definition 2.9. *If* $u_0 = u_n$ *and* $n \geq 3$, *then P is called a cycle and it is a fuzzy cycle if there is more than one weak arc. Let u be a node in fuzzy graphs G then* $N(u) = \{v : (u,v)\}$ *is strong arc*$\}$*is called neighbourhood of u and N[u] = N(u)*$\bigcup$ *u is called closed neighbourhood of u. Neighbourhood degree of the node is defined by the sum of the weights of the strong neighbour node of u is denoted by* $d_s(u) = \sum\limits_{v\epsilon N(u)} \sigma(v)$

3 Fuzzy dominating set

Definition 3.10. *Let G be a fuzzy graph and u be a node in G then there exist a node v such that (u, v) is a strong arc then u dominates v .*

Definition 3.11. *Let G be a fuzzy graph. A subset D of V is said to be a fuzzy dominating set if for every node* $v\epsilon V \setminus D$, *there exists* $u\epsilon D$ *such that u dominates v.*

Definition 3.12. *The domination number of G is the minimum cardinality taken over all dominating sets in G and is denoted by* $\gamma(G)$,*where* $\gamma(G) = \sum\limits_{v\epsilon D} \sigma(v)$. *A dominating set with cardinality* $\gamma(G)$ *is called* γ *- set of G.But here consider cardinality of* γ *is total number of elements in set*

Definition 3.13. *Let G be a fuzzy graph without isolated node. A subset D of V is said to be fuzzy total dominating set if for every node* $v\epsilon V$, *there exists at least one u in D such that u dominates v.*

Definition 3.14. *The domination number of G is the minimum cardinality taken over all dominating sets in G and is denoted by* $\gamma_t(G)$,*where* $\gamma_t(G) = \sum\limits_{v\epsilon v} \sigma(v)$. *A dominating set with cardinality* $\gamma_t(G)$ *is called* $\gamma_t(G)$*- set of G.But here consider cardinality of* γ_t *is total number of elements in set*

Definition 3.15. *Let G be a fuzzy graph without isolated node. A subset D of V is said to be fuzzy efficient dominating set if for every node* $v\epsilon V - D$, *there exists exact one u in D such that u dominates v.*

Definition 3.16. *The domination number of G is the minimum cardinality taken over all dominating sets in G and is denoted by* $\gamma_e(G)$,*where* $\gamma_e(G) = \sum\limits_{v\epsilon v} \sigma(v)$. *A dominating set with cardinality* $\gamma_e(G)$ *is called* $\gamma_e(G)$*- set of G.But here consider cardinality of* γ_e *is total number of elements in set*

4 Fuzzy non bondage number

Definition 4.17. *The bondage number b(G) of a fuzzy graph $G(V,E,\sigma,\mu))$ is minimum number of fuzzy arcs among all sets of arcs X = (x,y) sub set of E such that*
$CONN_{G-(x,y)}(u,v) < CONN_G(u,v)$ for all $u\epsilon V-\gamma(G)$ and a $v\ \epsilon\gamma(G)$. Here $\gamma(G)$ represent minimum dominate set

Definition 4.18. *The non-bondage number $b_n(G)$ of a fuzzy graph $G(V,E,\sigma,\mu))$ is maximum number of fuzzy arcs among all sets of arcs X = (x,y) sub set of E such that*
$CONN_{G-(x,y)}(u,v) = CONN_G(u,v)$ for all $u\epsilon V-\gamma(G)$ and a $v\ \epsilon\gamma(G)$.Here $\gamma(G)$ represent minimum dominate set

Definition 4.19. *The total bondage number $b_t(G)$ of a fuzzy graph $G(V,E,\sigma,\mu))$ is minimum number of fuzzy arcs among all sets of arcs X = (x,y) sub set of E such that*
$CONN_{G-(x,y)}(u,v) < CONN_G(u,v)$ for all $u\epsilon V-\gamma_t(G)$ and a $v\ \epsilon\gamma_t(G)$.Here $\gamma_t(G)$ represent minimum dominate set

Definition 4.20. *The total non-bondage number $b_{tn}(G)$of a fuzzy graph $G(V,E,\sigma,\mu))$ is maximum number of fuzzy arcs among all sets of arcs X = (x,y) sub set of E such that*
$CONN_{G-(x,y)}(u,v) = CONN_G(u,v)$ for all $u\epsilon V-\gamma_t(G)$ and a $v\ \epsilon\gamma_t(G)$.Here $\gamma_t(G)$ represent minimum dominate set

Definition 4.21. *The efficient bondage number $b_e(G)$ of a fuzzy graph $G(V,E,\sigma,\mu))$ is minimum number of fuzzy arcs among all sets of arcs X = (x,y) sub set of E such that*
$CONN_{G-(x,y)}(u,v) < CONN_G(u,v)$ for all $u\epsilon V-\gamma_e(G)$ and a $v\ \epsilon\gamma_e(G)$.Here $\gamma_e(G)$ represent minimum dominate set

Definition 4.22. *The efficient non-bondage number $b_{en}(G)$of a fuzzy graph $G(V,E,\sigma,\mu))$ is maximum number of fuzzy arcs among all sets of arcs X = (x,y) sub set of E such that*
$CONN_{G-(x,y)}(u,v) = CONN_G(u,v)$ for all $u\epsilon V-\gamma_e(G)$ and a $v\ \epsilon\gamma_e(G)$.Here $\gamma_e(G)$ represent minimum dominate set

Definition 4.23. *The co - bondage number b_c (G) of fuzzy graph G $(V,E,\sigma,\mu))$ is minimum number of fuzzy arcs required to add graph G such that $\gamma(G+e) < \gamma(G)$ and*
$\mu(e) = Max\{\mu(u,w),\mu(u,x)\}$ or Max $\{\mu(w,v),\mu(x,v)\}$, for any u, v in V.

5 Co-bondage Number

Theorem 5.1. *For any graph G,*

$$b_c \leq p-1-\Delta_n(G) \tag{5.1}$$

where $\Delta_n(G)$ is the total number of strong arcs in Δ of G .

Proof. We know that any graph G has p nodes then p-1 arcs are sufficient to connected all other nodes, but in case G has maximum number strong arcs$\Delta_n(G)$, so we add some arcs to G for connected to all other nodes is called co-bondage and that remain number is $p-1-\Delta_n(G)$. This proves. □

Theorem 5.2. *For any graph G,*

$$b_c(\bar{G}) \leq \delta_n(G)) \tag{5.2}$$

where $\bar{G}$ and $\delta_n(G)$ are the complement and the total number of strong arcs in $\delta$of G

Proof. A node has $q - \delta_n(G)$ strong arcs in complement of any graph, so we need to add $\delta_n(G)$ arcs for connected to all other nodes. Hence proves. □

Proposition 5.1. *For any cycle C_p (all arcs are bondage arc) with $p \geq 4$ nodes.*

$$b_c(C_p) = \begin{cases} 1, & if p = 1(mod 3) \\ 2, & if p = 2(mod 3) \\ 3, & if p = 3 otherwise \end{cases} \tag{5.3}$$

Proof. Let $C_p : v_1v_2...v_pv_1$ denote a cycle on p $\geq$ 4 nodes. We consider the following case.
Case 1: if $p \equiv$ 1(mod 3), then by joining the node $v_{p-1} to v_1$,we obtain a graph G which is a cycle C_{p-1} : $v_1v_2?v_{p-1}v_1$together with a path $v_{p-1}v_pv_1$. This implies that,$\gamma(G) = \gamma(C_{(p-1)}) \leq \gamma((C_p)$.This proves (case 1).
Case 2: if p$\equiv$ 2 (mod 3), then by joining the node $v_{p-1} and v_{p-2} to v_1$,we obtain a graph G which is a cycle $C_{p-2} : v_1v_2...v_{p-2}v_1$ together with a path $v_{p-2}v_{p-1}v_pv_1$. This implies that, $\gamma(G) = \gamma(C_{(p-2)}) \leq \gamma((C_p)$.This proves (case 2)
Case 3: if $p \equiv$ 3 (mod 3), then by joining the node $v_{p-1}, v_{p-2}$and v_{p-3} to v_1 ,we obtain a graph G which is a cycle $C_{p-3} : v_1v_2...v_{p-3}v_1$ together with a path $v_{p-3}v_{p-2}v_{p-1}v_pv_1$. This implies that, $\gamma(G) = \gamma(C_{p-3}) \leq \gamma(C_p)$.This proves (case 3). □

Proposition 5.2. *For any path P_p with $p \leq 4$ nodes.*

$$b_c(P_p) = \begin{cases} 1, & if p = 1(mod 3) \\ 2 & if p = 2(mod 3) \\ 3, & if p = 3 otherwise. \end{cases} \tag{5.4}$$

.

Theorem 5.3. *Let T be a tree with at least two cut nodes such that each cut node is adjacent to an end node then*

$$b_c(T) = r, \tag{5.5}$$

where r is the minimum number of end node adjacent to a cut node

Proof. Let S be the set of all cut node of T. then S is aγ- set for T. Let uϵS be a cut node which is adjacent to minimum number of end nodes $u_1, u_2, ..u_r$. Since there exist a cut node $v \epsilon S$such that v is adjacent to u by joining $u_1, u_2, ..u_r$to v the graph obtain has S-u as a γ- set. This proves. □

Theorem 5.4. *For any graph G,*

$$b_c(G) \leq \Delta_n(G) + 1. \tag{5.6}$$

Furthermore the bound is attained if and only if every γ- set D of G satisfying the following conditions:

- *D is independent*
- *Every node in D is of maximum degree:*
- *Every node in V-D is adjacent to exactly one node in D*

Proof. Let D be a γ- set of G . We consider the following case.
Case 1. Suppose D is not independent. Then there exit two adjacent node u,v ϵD.Let S $\subset$ V-D such that for each node w ϵ S, $N(w) \cap D = \{v\}$. Then by joining each node in S to u , we see that D-$\{$v$\}$ is a γ- set of the resulting graph. Thus, $b_c(G) \leq |S| \leq \Delta_n(G) - 1$.
Case 2 . Suppose D is independent Then each vertex v ϵ D is an isolated node in$\langle D \rangle$. Let S be a set define in case 1.Since D has at least two node by joining each in $S \cup \{v\}$ to some node w ϵ D-$\{$v$\}$,we obtain a graph which has D-$\{$V$\}$ as a γ- set. Hence, $b_c(G) \leq |S \cup \{v\}| \leq \Delta_n(G) + 1$. Other parts of theorem are directly from above case. □

Corollary 5.1. *Fo any graph G*

$$bc(G) \leq min\{p - \Delta_n(G) - 1, \Delta_n(G) + 1\} \tag{5.7}$$

.

Theorem 5.5. *For any non trivial tree T,*

$$b(t) \leq 2 \tag{5.8}$$

.

Theorem 5.6. *Let T be a tree with diam(T) = 5 and has exactly two cut nodes which are adjacent to end nodes and further they have sane degree then,*

$$b_c(T) \geq b(T) + 1 \tag{5.9}$$

, where dia(T) is the diameter of T.

Theorem 5.7. *For any tree T,*

$$b_c(T) \leq 1 + min\{degu\} \tag{5.10}$$

, where u is a cut node adjacent to an end node.

Proof. Since there exist a γ- set containing u and take v other than u , then $N(u) \cap V - \gamma = deg\{u\}$ and add to v with u then $\gamma - u$ as γ -set of G and bc is minimum of such above set .hence proves. □

Theorem 5.8. *let D be a γ -set of G . If there exist a node v in D which is adjacent to every other node in D,then,*

$$b_c \leq p - \gamma(G) - 1 \tag{5.11}$$

.

Proof. we know that $\Delta_n(G) \geq deg(?v) \geq \gamma(G)$ Hence proves. □

6 Non bondage number:

Theorem 6.9. *For any graph G without isolated nodes and $\Delta_n(G) = p - 1$*

$$b_{tn}(G) = q - p + 1. \tag{6.12}$$

Proof. let G be a graph with a node u such that deg u= $\Delta_n(G) = p - 1$.then there exit a node such that e= uv ϵE. Thus u,v is a total dominating set of G. let X be the set of arcs which not strong of u. Then clearly $b_{tn} = |X| = q - \Delta_n(G) = q - p + 1$. □

Theorem 6.10. *For any graph G without isolated nodes,*

$$b_{tn} \leq q - \Delta_n(G). \tag{6.13}$$

Proof. This follow from (12) and that fact that $\Delta_n(G) \leq p - 1$. □

Theorem 6.11. *Let G be a graph without isolated nodes. If H is a sub graph of G , then*

$$b_{tn}(H) \leq b_{tn}(G). \tag{6.14}$$

Proof. Every total non bondage set of H is a total non-bondage set of G. Thus(14)holds. □

Theorem 6.12. *For a complete graph K_p , with p≥ 3 nodes,*

$$b_{tn}(K_P) = \frac{(p-1)(p-2)}{2}. \tag{6.15}$$

Proof. Let K_p be a complete graph with p≥ 3 nodes. The degree of every node of K_p is p-1 by theorem 6.9
$b_{tn}(K_P) = q - p + 1$
$= \frac{(p-1)(p)}{2} - (p-1)$
$= \frac{(p-1)(p-2)}{2}$. □

Theorem 6.13. *For a wheel W_p with $p \geq 4$ nodes,*

$$b_{tn}(W_p) = p - 1. \tag{6.16}$$

Proof. Let W_p be a wheel with $p \geq 4$ nodes. The W_p has a node such that deg u = p-1. By theorem 6.9, $b_{tn}(W_p) = p - 1$. □

Theorem 6.14. *For a complete bipartite graph $K_{m,n}$, 2≤ $m \leq n$, $b_{tn}(K_{mn})$ = mn-m-n-1.*

Theorem 6.15. *For any graph G,*

$$b_{tn}(\bar{G}) + b_{tn}(G) \leq \frac{((p-1)(p-2))}{2} \tag{6.17}$$

Proof. By Theorem 6.10 $b_{tn} \leq q - \Delta_n$,then
$b_{tn}(\bar{G}) + b_{tn}(G) \leq \bar{q} + q - (\Delta_n + \delta_n)$
$= \frac{(p(p-1))}{2} - (\Delta_n + \delta_n)$
$\leq \frac{(p(p-1))}{2} - (p-1)$
$\leq \frac{((p-2)(p-1))}{2}$. □

Theorem 6.16. *For any graph G without isolate nodes,*

$$b_t(G) \leq b_{tn}(G) + 1. \tag{6.18}$$

Proof. Let X be a b_{tn} ? set of G . then for an edge $e\epsilon G - X, X \bigcup \{e\}$is a total bondage set of G hence (18) holds. □

Theorem 6.17. *For any graph G,*

$$b_t(\bar{G}) + b_t(G) \leq \frac{((p-1)(p-2))}{2} + 2 \tag{6.19}$$

Proof. by Theorem 6.16 $b_t(\bar{G}) + b_t(G) \leq b_{tn}(\bar{G}) + b_{tn}(G) + 2 \leq ((p-1)(P-2))/2 + 2$ □

Theorem 6.18. *For any fuzzy graph G,*

$$b_{en}(G) = q - p + \gamma_e(G) \tag{6.20}$$

, where q is total number of fuzzy arcs and p is total number of node.

Proof. Let D be a minimal dominated set of G and its denote by $\gamma(G)$. For each node $v\epsilon V \setminus D$ choose exactly one strong arc which is incident to node v and to a node in D. Let E_1 be the set of all such arcs. The clearly E-E_1 is a $b_{tn}(G)$? set of G. $b_{en}(G) = q - (p - \gamma_e(G))$
$b_{en}(G) = q - p + \gamma_e(G)$ □

Corollary 6.2. *For any graph G without isolate nodes,*

$$b_{en(G)} \leq q - \Delta_n. \tag{6.21}$$

Theorem 6.19. *If K_p is a complete graph with $p \geq 3$ nodes, then*

$$b_{en}(K_p) = \frac{(p-1)(p-2}{2}. \tag{6.22}$$

Proposition 6.3. *If W_p is a wheel with $p \geq 4$ nodes ,then*

$$b_{en}(W_p) = p - 1. \tag{6.23}$$

Proof. Let W_p be a wheel with p $\geq$ 4 nodes then W_p has a node v such that deg v = p-1 $b_{en}(W_p) = p - 1$. □

Proposition 6.4. *If $K_{1,p}$ is a star with p, 1 nodes then*

$$b_{en}(K_{1,p}) = 0. \tag{6.24}$$

Proposition 6.5. *If $K_{m,n}$ is a complete bipartite graph with $2 \leq m \leq n$, then*

$$b_{en}(K_{m,n}) = mn - m - n + 2 \tag{6.25}$$

Proof. Let $K_{m,n}$ be a complete bipartite graph with $2 \leq m \leq n$ then q=mn,p=m+n and $\gamma_e(G)$=2 hence by theorem 6.18, the result □

Theorem 6.20. *For graph G and $\bar{G}$ with no isolated nodes,*

$$b_{en}(\bar{G}) + b_{en}(G) \leq \frac{((p-1)(p-2))}{2} \tag{6.26}$$

7 Relationships between b_n(G) and b

Theorem 7.21. *Let T$\neq$ P_4 be a tree with at least two cut nodes .then*

$$b_n(G) \geq b(T). \tag{7.27}$$

Theorem 7.22. *For any fuzzy graph*

$$b(G) \leq b_n(G) + 1 \tag{7.28}$$

Theorem 7.23. *If G be a cycle graph then*

$$b_n(\bar{G}) + b_n(G) \leq \frac{p(p-3)}{2}, \tag{7.29}$$

$p \geq 4$

Proof. by theorem 6 $b_n(G) = q - p + \gamma(G)$
$b_n(\bar{G}) = \bar{q} - p + \gamma(\bar{G})$
$b_n(\bar{G}) + b_n(G) = q - p + \gamma(G) + \bar{q} - p + \gamma(\bar{G})$
$= q + \bar{q} - 2p + ?(\bar{G}) + \gamma(G)$
$= \frac{(p(p-1))}{2} - 2p + \gamma(\bar{G}) + \gamma(G)$
$\leq \frac{(p(p-1))}{2} - 2p + p$
$\leq \frac{(p(p-3))}{2}$. □

Theorem 7.24. *For any graph G,*

$$b_n(\bar{G}) + b_n(G) \leq \frac{((p-1)(p-2))}{2} \tag{7.30}$$

Proof. By Theorem 7 $b_n \leq q - \Delta_n$,then
$b_n(\bar{G}) + b_n(G) \leq \bar{q} + q - (\Delta_n + \delta_n)$
$= \frac{(p(p-1))}{2} - (\Delta_n + \delta_n)$
$\leq \frac{(p(p-1))}{2} - (p-1)$
$\leq \frac{((p-2)(p-1))}{2}$. □

Theorem 7.25. *If G be a cycle graph then*

$$b(\bar{G}) + b(G) \leq \frac{(p(p-3))}{2} + 2 \tag{7.31}$$

Proof. by Theorem 9
$b(G) \leq b_n(G) + 1$
$b(\bar{G}) + b(G) \leq b_n(\bar{G}) + b_n(G) + 2$
by Theorem 10
$b(\bar{G}) + b(G) \leq \frac{(p(p-3))}{2} + 2$ □

Theorem 7.26. *Any graph G,*

$$b(\bar{G}) + b(G) \leq \frac{((p-1)(p-2))}{2} + 2 \tag{7.32}$$

Theorem 7.27. *If G be a tree then*

$$b_n(\bar{G}) + b_n(G) \geq \gamma(\bar{G}) + \gamma(G) - 2, \tag{7.33}$$

if $p \geq 4$

Proof. $b_n(G) \geq \gamma(G) - 1$, then
$b_n(\bar{G}) + b_n(G) \geq \gamma(\bar{G}) + \gamma(G) - 2.$ □

8 Block

Definition 8.24. *A connected fuzzy graph is called block if all nodes are satisfies the condition* $CONN_{G-v}(u, v) = CONN_G(u, v)$ *for every u, v in G..*

9 Strong line graph

Definition 9.25. *Given a fuzzy graph G, its strong line graph* $L_s(G)$ *is a fuzzy graph,* $L_s(G)$ *is a graph G such that Each node of* $L_s(G)$ *represents an arc of G; and ? Two nodes of* $L_s(G)$ *are adjacent if and only if their corresponding arcs are strong and share a common end point in G.*

Definition 9.26. *Given a fuzzy graph G, its double strong line graph* $L_s^*(G)$ *is a fuzzy graph,* $L_s^*(G)$ *is a graph G such that*
Each node of $L_s^*(G)$ *represents a strong arc of G; and Two nodes of* $L_s^*(G)$ *are adjacent if and only if their corresponding arcs are strong and share a common end point in G.*

Theorem 9.28. *For any cycle fuzzy graph, then Ls(G) has one isolate node iff G has a non ? bondage arc.*

Proof. Let $L_s(G)$ has one isolated node, so G has one weakest arc (x, y) by theorem (x, y) non bondage. Conversely, let G has a non-bondage arc (x, y), clearly (x, y) weakest arc and node x and y does not common node for two strong arcs. So corresponding vertex of arc (x, y) in $L_s(G)$ is isolated. □

Theorem 9.29. *Let* $L_s(G)$ *be fuzzy black graph such that* $L_s(G)$ *is a tree. Then a node in* $L_s(G)$ *iff G has non-bondage or a arc adjacent with a non-bondage arc.*

Proof. Given $L_s(G)$ be fuzzy block then $CONN_{G-v}(u, w) = CONNG(u, w)$by definition here node of $L_s(G)$ is a arc of G, so we clearly that $CONN_{G-(x,y)}(u, w) = CONNG(u, w)$, converse true trivially. □

Theorem 9.30. *A complete fuzzy graph of*$L_s(G)$ *is not a complete.*

Proof. Given that G is compete fuzzy graph so there exist a cycle in G and every node of even pair does not adjacent with odd pair so corresponding nodes not adjacent in $L_s(G)$. □

Theorem 9.31. *Let G be a path graph n nodes then* $L_s^*(G)$ *has n-2 arcs.*

Proof. Given G be a path with n nodes then n-1 arcs so $L_s^*(G)$ has a path with n-1 nodes so it has n-2 arcs. □

Theorem 9.32. *Let G be complete graph with n nodes then* $L_s^*(G)$ *is 2(n-2) regular graph.*

Proof. Given G be complete graph so every node has n-1 strong arc then every arc adjacent with 2n-4, so $L_s^*(G)$ has 2(n-2) regular graph. □

10 Neighbourhood Extension

Definition 10.27. *Let G be graph and$S_i \subseteq V$, each S_i is collection of each nodes in G. If $G_E(\tau, \rho)$ said to be Neighbourhood extension, then satisfied following condition.*

- *Each node of G_E represents an strong neighbourhood set of G*
- *Two nodes of G_E are adjacent iff their correspond neighbour set have at least one common node where$\rho(S_i, S_j) = min\{\mu(x, v_i), \mu(v_j, x) / x \epsilon S_i \cap S_j\}$*

Definition 10.28. *Let G^* be Connected graph,if $G_E \cong G^*$ then G said be C- Neighbour Extendable graph also called strong Neighbourhood Extendable otherwise weak Neighbourhood Extendable*

Definition 10.29. *Let G^* be tree graph,if $G_E \cong G^*$ then G said be t- Neighbour Extendable graph also called semi strong Neighbourhood Extendable*

Theorem 10.33. *Let G(not distinct label) be cycle graph then G_E is complement of G.*

Proof. We know that G is connected graph and each node adjacent two nodes since G is cycle,so strong neighbour of each node of G is two.Clearly, if v_i, v_j, are adjacent nodes then $N_s(v_i) \cap N_s(v_j) = \emptyset$ but alternative nodes have some same nodes, so make arcs between them it will be form complement of G. □

Corollary 10.3. *Let G be cycle graph with n nodes then G is strong neighbour extendable if n is odd, otherwise weak neighbour extendable*

Proof. Case 1: If n is odd, by using theorem 10.33 there exist two paths in G_E ie one is $v_1, v_3, ...v_n, v_2$ say P_1 another path say P_2 is $v_2, v_4, ...v_{n-1}, v_1$ so P_1 and P_2 has same nodes say $v_1 and v_2$ clearly G_E is connected graph and G is strong neighbour extendable Case 2: If n is even, by using theorem 10.33 there exist two paths in G_E ie one is $v_1, v_3, ...v_{2n-1}v_1$ say P_1 another path say P_2 is $v_2, v_4, ...v_{2n}, v_2$ so P_1 and P_2 does not have same nodes.Clearly G_E is disconnected graph and G weak neighbour extendable □

Theorem 10.34. *Let G be complete graph with n nodes, then G is strong neighbour extendable*

Proof. We know that G complete graph then every node has n-1 strong arcs so strong neighbour set of every node has n-1 nodes $N_S(v_i) = \{v_1, v_2, ...v_{i-1}, v_{i+1}, ..v_n\} \forall v_i \epsilon G$ then
$N_S(v_i) \bigcap N_s(v_j) = \{v_1, v_2, ...v_{i-1}, v_{i+1}, ..v_{j-1}, v_{j+1}, ..v_n\} \forall v_i, v_j \epsilon G$,this implies that G_E is complete graph,so G is strong neighbour extendable □

Theorem 10.35. *Let P_n be path with n nodes then G is not neighbour extendable.*

Proof. LetP_n be path with n nodes then there exist unique path , so every arcs are strong arc and two nodes have one strong neighbour other nodes have two strong neighbours, but their neighbour sets are distinct, so G_E is null graph therefore G is not neighbour extendable. □

Theorem 10.36. *Let G be complete bipartite graph then G is not strong neighbour extendable*

Proof. Given G is complete bipartite graph so node set is partition of two set say V_1 and V_2 then each node of V_1 is strong neighbour of every node in V_2 there two distinct path form in G_E ,One path connect every node in v_1 another path connect every node in V_2 so G_E is disconnect graph therefore G is not strong neighbour extendable. □

Definition 10.30. *(Deficiency Number) Let G be fuzzy graph but G is not strong neighbour extendable then the deficiency number is required number of arcs to make G is strong neighbour extendable .*

11 Conclusion

Above non bondage value ($\neq 0$) is not true for all graphs because $K_{1,n}$ or star graph and P_3 non-bondage value is 0 and also bondage number is equal to 1 for such above graphs and co-bondage of complete graph is not determine,$L^*(G)$ is not tree for all arc are strong or not distinct label .

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