On generalized $b$ star - closed set in Topological Spaces

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Abstract

In this paper, we introduce a new class of sets called generalized $b$ star - closed sets in topological spaces (briefly $gb^*$-closed set). Also we discuss some of their properties and investigate the relations between the associated topology.

Keywords: $gb^*$-closed set, $b$-closed set, $gb$ closed set.

2010 MSC: 54A05.

1 Introduction

In 1970, Levine introduced the concept of generalized closed set and discussed the properties of sets, closed and open maps, compactness, normal and separation axioms. Later in 1996 Andrijevic gave a new type of generalized closed set in topological space called $b$ closed sets. The investigation on generalization of closed set has lead to significant contribution to the theory of separation axiom, generalization of continuity and covering properties. A.A.Omari and M.S.M. Noorani made an analytical study and gave the concepts of generalized $b$ closed sets in topological spaces.

In this paper, a new class of closed set called generalized $b$ star - closed set is introduced to prove that the class forms a topology. The notion of generalized $b$ star - closed set and its different characterizations are given in this paper. Throughout this paper $(X, \tau)$ and $(Y, \sigma)$ represent the non - empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned.

Let $A \subseteq X$, the closure of $A$ and interior of $A$ will be denoted by $cl(A)$ and $int(A)$ respectively, union of all $b$ - open sets $X$ contained in $A$ is called $b$ - interior of $A$ and it is denoted by $bint(A)$, the intersection of all $b$ - closed sets of $X$ containing $A$ is called $b$ - closure of $A$ and it is denoted by $bcl(A)$.

2 Preliminaries

Definition 2.1. Let $A$ subset $A$ of a topological space $(X, \tau)$, is called

1) a pre-open set [13] if $A \subseteq int(cl(A)).$

2) a semi-open set [?] if $A \subseteq cl(int(A)).$

3) a $\alpha$ -open set [9] if $A \subseteq int(cl(int(A))).$

4) a $b$ -open set [2] if $A \subseteq cl(int(A)) \cup int(cl(A)).$

5) a generalized $*$ -closed set (briefly $g^*$-closed) [8] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $g$ open in $X$.

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Proof. Let $X$ be a closed set in $X$. Therefore $bcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $X$.

The converse of above theorem need not be true as seen from the following example.

Example 3.1. Let $X = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$. The set $\{b\}$ is gb*-closed set but not a closed set.

Theorem 3.2. Every pre-closed set is gb*-closed.

Proof. Let $A$ be any pre-closed set in $X$ such that $A \subseteq U$ where $U$ is $g^*$-open. Since $A$ is pre closed $bcl(A) \subseteq pcl(A) \subseteq A$. Therefore $bcl(A) \subseteq U$. Hence $A$ is gb*-closed.

Example 3.2. Let $X = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$. The set $\{a, c\}$ is gb*-closed set but not a pre-closed set.

Theorem 3.3. Every semi-closed set is gb*-closed.

Proof. Let $A$ be any semi-closed set in $X$ such that $A \subseteq U$ where $U$ is $g^*$-open. Since $A$ is semi closed, $bcl(A) \subseteq scl(A) \subseteq U$. Therefore $bcl(A) \subseteq U$. Hence $A$ is gb*-closed.

Example 3.3. Let $X = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$. The set $\{a, c\}$ is gb*-closed set but not a semi-closed set.

Theorem 3.4. Every ag*-closed set is gb*-closed.

Proof. Let $A$ be any ag*-closed set in $X$ such that $A \subseteq U$ where $U$ is $g^*$-open. Since $A$ is ag*-closed set, $bcl(A) \subseteq acl(A) \subseteq U$. Therefore $bcl(A) \subseteq U$. Hence $A$ is gb*-closed.

Example 3.4. Let $X = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$. The set $\{b\}$ is gb*-closed set but not a ag*-closed set.

Theorem 3.5. Every $b$-closed set is gb*-closed.

Proof. Let $A$ be any $b$-closed set in $X$ such that $A \subseteq U$ where $U$ is $g^*$-open. Since $A$ is $b$-closed, $bcl(A) = A$. Therefore $bcl(A) \subseteq U$. Hence $A$ is gb*-closed.

Example 3.5. Let $X = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$. The set $\{a, c\}$ is gb*-closed set but not a $b$-closed set.

Theorem 3.6. Every $g^*$-closed set is gb*-closed.

Proof. Let $A$ be any $g^*$-closed set in $X$ such that $A \subseteq U$ where $U$ is $g^*$-open. Since $A$ is $g^*$-closed, $bcl(A) \subseteq cl(A) \subseteq U$. Therefore $bcl(A) \subseteq U$. Hence $A$ is gb*-closed.

3 Generalized $b$ star - closed set

In this section, we introduce generalized $b$ star - closed set and investigate some of their properties.

Definition 3.2. A subset $A$ of a topological space $(X, \tau)$, is called generalized $b$ star - closed set (briefly gb*-closed set) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $g^*$-open in $X$.

Theorem 3.1. Every closed set is gb*-closed.

Proof. Let $A$ be any closed set in $X$ such that $A \subseteq U$, where $U$ is $g^*$ open. Since $bcl(A) \subseteq cl(A) = A$. Therefore $bcl(A) \subseteq U$. Hence $A$ is gb*-closed set in $X$. 

The converse of above theorem need not be true as seen from the following example.

Example 3.6. Let $X = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$. The set $\{b\}$ is gb*-closed set but not a closed set.

Theorem 3.2. Every pre-closed set is gb*-closed.

Proof. Let $A$ be any pre-closed set in $X$ such that $A \subseteq U$ where $U$ is $g^*$-open. Since $A$ is pre closed, $bcl(A) \subseteq pcl(A) \subseteq A$. Therefore $bcl(A) \subseteq U$. Hence $A$ is gb*-closed set.

The converse of above theorem need not be true as seen from the following example.

Example 3.3. Let $X = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$. The set $\{a, c\}$ is gb*-closed set but not a pre-closed set.

Theorem 3.3. Every semi-closed set is gb*-closed.

Proof. Let $A$ be any semi-closed set in $X$ such that $A \subseteq U$ where $U$ is $g^*$-open. Since $A$ is semi closed, $bcl(A) \subseteq scl(A) \subseteq U$. Therefore $bcl(A) \subseteq U$. Hence $A$ is gb*-closed set.

The converse of above theorem need not be true as seen from the following example.

Example 3.4. Let $X = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$. The set $\{a, c\}$ is gb*-closed set but not a semi-closed set.

Theorem 3.4. Every ag*-closed set is gb*-closed.

Proof. Let $A$ be any ag*-closed set in $X$ such that $A \subseteq U$ where $U$ is $g^*$-open. Since $A$ is ag*-closed, $bcl(A) \subseteq acl(A) \subseteq U$. Therefore $bcl(A) \subseteq U$. Hence $A$ is gb*-closed set.

The converse of above theorem need not be true as seen from the following example.

Example 3.5. Let $X = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$. The set $\{b\}$ is gb*-closed set but not a ag*-closed set.

Theorem 3.5. Every $b$-closed set is gb*-closed.

Proof. Let $A$ be any $b$-closed set in $X$ such that $A \subseteq U$ where $U$ is $g^*$-open. Since $A$ is $b$-closed, $bcl(A) = A$. Therefore $bcl(A) \subseteq U$. Hence $A$ is gb*-closed set.

The converse of above theorem need not be true as seen from the following example.

Example 3.6. Let $X = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$. The set $\{a, c\}$ is gb*-closed set but not a $b$-closed set.

Theorem 3.6. Every $g^*$-closed set is gb*-closed.

Proof. Let $A$ be any $g^*$-closed set in $X$ such that $A \subseteq U$ where $U$ is $g^*$-open. Since $A$ is $g^*$-closed, $bcl(A) \subseteq cl(A) \subseteq U$. Therefore $bcl(A) \subseteq U$. Hence $A$ is gb*-closed set.
The converse of above theorem need not be true as seen from the following example.

**Example 3.6.** Let \( X = \{a, b, c\} \) with \( \tau = \{X, \emptyset, \{a, c\}\} \). The set \( \{a, b\} \) is gb\(^*\)-closed set but not a g\(^*\)-closed set.

**Theorem 3.7.** Every g\(^*\)-closed set is gb\(^*\)-closed set.

*Proof.* Let \( A \) be g\(^*\)-closed set in \( X \) such that \( A \subseteq U \) where \( U \) is g\(^*\) open. Since \( A \) is g\(^*\)-closed set, \( \text{bcl}(A) \subseteq \text{scl}(A) \subseteq U \). Hence \( A \) is gb\(^*\)-closed set.

The converse of above theorem need not be true as seen from the following example.

**Example 3.7.** Let \( X = \{a, b, c\} \) with \( \tau = \{X, \emptyset, \{a\}, \{a, b\}\} \). The set \( \{a, c\} \) is gb\(^*\) - closed set but not a g\(^*\)-closed set.

**Theorem 3.8.** Every gb\(^*\) - closed set is rgb\(^*\) - closed set.

*Proof.* Let \( A \) be any gb\(^*\) - closed set in \( X \) such that \( A \subseteq U \) where \( U \) is g\(^*\) open. Since \( A \) is gb\(^*\) closed, \( \text{bcl}(A) \subseteq \text{pcl}(A) \subseteq U \). Hence \( A \) is rgb\(^*\) - closed set.

The converse of above theorem need not be true as seen from the following example.

**Example 3.8.** Let \( X = \{a, b, c\} \) with \( \tau = \{X, \emptyset, \{a\}\} \). The set \( \{a, b\} \) is rgb\(^*\)-closed set but not a gb\(^*\)-closed set.

## 4 Characteristics of gb\(^*\)-closed set

**Theorem 4.9.** If \( A \) and \( B \) are gb\(^*\)-closed sets in \( X \) then \( A \cup B \) is gb\(^*\)-closed set in \( X \).

*Proof.* Let \( A \) and \( B \) are gb\(^*\)-closed sets in \( X \) and \( U \) be any g\(^*\) open set containing \( A \) and \( B \). Therefore \( \text{bcl}(A) \subseteq U \), \( \text{bcl}(B) \subseteq U \). Since \( A \subseteq U \), \( B \subseteq U \) then \( A \cup B \subseteq U \). Hence \( \text{bcl}(A \cup B) = \text{bcl}(A) \cup \text{bcl}(B) \subseteq U \). Therefore \( A \cup B \) is gb\(^*\)-closed set in \( X \).

**Theorem 4.10.** If a set \( A \) is gb\(^*\)-closed set if and only if \( \text{bcl}(A) - A \) contains no non empty g\(^*\)-closed set.

*Proof.* Necessary: Let \( F \) be a g\(^*\) closed set in \( X \) such that \( F \subseteq \text{bcl}(A) - A \). Then \( A \subseteq XF \). Since \( A \) is gb\(^*\) closed set and \( X - F \) is g\(^*\) open then \( \text{bcl}(A) \subseteq X - F \). (i.e.) \( F \subseteq X - \text{bcl}(A) \). So \( F \subseteq (X - \text{bcl}(A)) \cap (\text{bcl}(A) - A) \). Therefore \( F = \emptyset \).

Sufficiency: Let us assume that \( \text{bcl}(A) - A \) contains no non empty g\(^*\)-closed set. Let \( A \subseteq U \), \( U \) is g\(^*\) open. Suppose that \( \text{bcl}(A) \) is not contained in \( U \), \( \text{bcl}(A) \cap U \) is a non-empty g\(^*\)-closed set of \( \text{bcl}(A) - A \) which is contradiction. Therefore \( \text{bcl}(A) \subseteq U \). Hence \( A \) is gb\(^*\)-closed.

**Theorem 4.11.** If \( A \) is gb\(^*\)-closed set in \( X \) and \( A \subseteq B \subseteq \text{bcl}(A) \), Then \( B \) is gb\(^*\)-closed set in \( X \).

*Proof.* Since \( B \subseteq \text{bcl}(A) \), we have \( \text{bcl}(B) \subseteq \text{bcl}(A) \) then \( \text{bcl}(B) - B \subseteq \text{bcl}(A) - A \). By theorem 4.10, \( \text{bcl}(A) - A \) contains no non empty g\(^*\)-closed set. Hence \( \text{bcl}(B) - B \) contains no non empty g\(^*\)-closed set. Therefore \( B \) is gb\(^*\)-closed set in \( X \).

**Theorem 4.12.** If \( A \subseteq Y \subseteq X \) and suppose that \( A \) is gb\(^*\) closed set in \( X \) then \( A \) is gb\(^*\) - closed set relative to \( Y \).

*Proof.* Given that \( A \subseteq Y \subseteq X \) and \( A \) is gb\(^*\) closed set in \( X \). To prove that \( A \) is gb\(^*\) - closed set relative to \( Y \). Let us assume that \( A \subseteq Y \cap U \), where \( U \) is g\(^*\) open in \( X \). Since \( A \) is gb\(^*\) - closed set, \( A \subseteq U \) implies \( \text{bcl}(A) \subseteq U \). It follows that \( Y \cap \text{bcl}(A) \subseteq Y \cap U \). That is \( A \) is gb\(^*\) - closed set relative to \( Y \).

**Theorem 4.13.** If \( A \) is both g\(^*\) open and gb\(^*\) - closed set in \( X \), then \( A \) is g\(^*\) closed set.

*Proof.* Since \( A \) is g\(^*\) open and gb\(^*\) closed in \( X \), \( \text{bcl}(A) \subseteq U \). But \( A \subseteq \text{bcl}(A) \). Therefore \( A = \text{bcl}(A) \). Hence \( A \) is g\(^*\) closed set.

**Theorem 4.14.** For \( x \in X \), then the set \( X - \{x\} \) is a gb\(^*\)-closed set or g\(^*\) - open.

*Proof.* Suppose that \( X - \{x\} \) is not g\(^*\) open, then \( X \) is the only g\(^*\) open set containing \( X - \{x\} \). (i.e.) \( \text{bcl}(X - \{x\}) \subseteq X \). Then \( X - \{x\} \) is gb\(^*\) - closed in \( X \).
5 Generalized $b$ star - open set and generalized $b$ star - neighbourhoods

In this section, we introduce generalized $b$ star - open sets (briefly $gb^*$ - open) and generalized $b$ star - neighbourhoods (briefly $gb^*$ - neighbourhood) in topological spaces by using the notions of $gb^*$ - open set and study some of their properties.

**Definition 5.3.** A subset $A$ of a topological space $(X, \tau)$, is called semi generalized $b^*$ - open set (briefly $gb^*$ - open set) if $A^c$ is $gb^*$ - closed in $X$. We denote the family of all $gb^*$ - open sets in $X$ by $gb^* - O(X)$.

**Theorem 5.18.** If $A$ and $B$ are $gb^*$ - open sets in a space $X$. Then $A \cap B$ is also $gb^*$ - open set in $X$.

**Proof.** If $A$ and $B$ are $gb^*$ - open sets in a space $X$. Then $A^c$ and $B^c$ are $gb^*$ - closed sets in a space $X$. By Theorem 4.13 $A^c \cup B^c$ is also $gb^*$ - closed set in $X$. (i.e.) $A^c \cup B^c = (A \cap B)^c$ is a $gb^*$ - closed set in $X$. Therefore $A \cap B$ is $gb^*$ - open set in $X$. \qed

**Remark 5.1.** The union of two $gb^*$-open sets in $X$ is generally not a $gb^*$-open set in $X$.

**Example 5.9.** Let $X = \{a, b, c\}$ with topology $\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. If $A = \{b\}, B = \{c\}$ are p$gb^*$-open sets in $X$, then $A \cup B = \{b, c\}$ is not $gb^*$-open set in $X$.

**Theorem 5.16.** If $\text{int}(B) \subseteq B \subseteq A$ and if $A$ is $gb^*$-open in $X$, then $B$ is $gb^*$-open in $X$.

**Proof.** Suppose that $\text{int}(B) \subseteq B \subseteq A$ and $A$ is $gb^*$-open in $X$ then $A^c \subseteq B^c \subseteq \text{cl}(A^c)$. Since $A^c$ is $gb^*$-closed in $X$, by Theorem 5.15 $B$ is $gb^*$-open in $X$. \qed

**Definition 5.4.** Let $x$ be a point in a topological space $X$ and let $x \in X$. A subset $N$ of $X$ is said to be a $gb^*$-neighbourhood of $x$ iff there exists a $gb^*$-open set $G$ such that $x \in G \subseteq N$.

**Definition 5.5.** A subset $N$ of $X$ is called a $gb^*$-neighbourhood of $A \subseteq X$ iff there exists a $gb^*$-open set $G$ such that $A \subseteq G \subseteq N$.

**Theorem 5.17.** Every neighbourhood $N$ of $x \in X$ is a $gb^*$-neighbourhood of $x$.

**Proof.** Let $N$ be a neighbourhood of point $x \in X$. To prove that $N$ is a $gb^*$-neighbourhood of $x$. By Definition of neighbourhood, there exists an open set $G$ such that $x \in G \subseteq N$. Hence $N$ is a $sg^*b$-neighbourhood of $x$. \qed

**Remark 5.2.** In general, a $gb^*$-neighbourhood of $x \in X$ need not be a neighbourhood of $x$ in $X$ as seen from the following example.

**Example 5.10.** Let $X = \{a, b, c\}$ with topology $\tau = \{X, \phi, \{c\}, \{a, c\}\}$. Then $gb^*$-$O(X) = \{X, \phi, \{c\}, \{a, c\}, \{b, c\}\}$. The set $\{b, c\}$ is $gb^*$-neighbourhood of point $c$, since the $gb^*$-open sets $\{c\}$ is such that $c \in \{c\} \subseteq \{b, c\}$. However, the set $\{b, c\}$ is not a neighbourhood of the point $c$, since no open set $G$ exists such that $c \in G \subseteq \{b, c\}$.

**Remark 5.3.** The $gb^*$-neighbourhood $N$ of $x \in X$ need not be a $gb^*$-open in $X$.

**Theorem 5.18.** If a subset $N$ of a space $X$ is $gb^*$-open, then $N$ is $gb^*$-neighbourhood of each of its points.

**Proof.** Suppose $N$ is $gb^*$-open. Let $x \in N$. We claim that $N$ is $gb^*$-neighbourhood of $x$. For $N$ is a $gb^*$-open set such that $x \in N \subseteq N$. Since $x$ is an arbitrary point of $N$, it follows that $N$ is a $gb^*$-neighbourhood of each of its points. \qed

**Remark 5.4.** In general, a $gb^*$-neighbourhood of $x \in X$ need not be a neighbourhood of $x$ in $X$ as seen from the following example.

**Example 5.11.** Let $X = \{a, b, c\}$ with topology $\tau = \{X, \phi, \{a, b\}\}$. Then $gb^*$-$O(X) = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. The set $\{b, c\}$ is $gb^*$-neighbourhood of point $b$, since the $gb^*$-open sets $\{b\}$ is such that $b \in \{b\} \subseteq \{b, c\}$. Also the set $\{b, c\}$ is $gb^*$-neighbourhood of point $\{b\}$. Since the $gb^*$-open set $\{a, b\}$ is such that $b \in \{b\} \subseteq \{a, b\}$. (i.e.) $\{b, c\}$ is $gb^*$-neighbourhood of each of its points. However, the set $\{b, c\}$ is not a $gb^*$-open set in $X$. 


Theorem 5.19. Let $X$ be a topological space. If $F$ is $gb^*$ - closed subset of $X$ and $x \in F^{c}$. Prove that there exists a $gb^*$ - neighbourhood $N$ of $x$ such that $N \cap F = \phi$.

Proof. Let $F$ be $gb^*$ - closed subset of $X$ and $x \in F^{c}$. Then $F^{c}$ is $gb^*$ - open set of $X$. So by Theorem 5.18 $F^{c}$ contains a $gb^*$ - neighbourhood of each of its points. Hence there exists a $gb^*$ - neighbourhood $N$ of $x$ such that $N \subset F^{c}$. (i.e.) $N \cap F = \phi$. 

Definition 5.6. Let $x$ be a point in a topological space $X$. The set of all $gb^*$ - neighbourhood of $x$ is called the $gb^*$ - neighbourhood system at $x$, and is denoted by $gb^* - N(x)$.

Theorem 5.20. Let a $gb^*$ - neighbourhood $N$ of $X$ be a topological space and each $x \in X$, Let $gb^* - N(X, \tau)$ be the collection of all $gb^*$ - neighbourhood of $x$. Then we have the following results.

(i) For all $x \in X$, $gb^* - N(x) \neq \phi$.

(ii) $N \in gb^* - N(x) \Rightarrow x \in N$.

(iii) $N \in gb^* - N(x), M \supset N \Rightarrow M \in gb^* - N(x)$.

(iv) $N \in gb^* - N(x), M \in gb^* - N(x) \Rightarrow N \cap M \in gb^* - N(x)$. if finite intersection of $gb^*$ open set is $gb^*$ open.

(v) $N \in gb^* - N(x) \Rightarrow$ there exists $M \in gb^* - N(x)$ such that $M \subset N$ and $M \in gb^* - N(y)$ for every $y \in M$.

Proof. 1. Since $X$ is $gb^*$ - open set, it is a $gb^*$ - neighbourhood of every $x \in X$. Hence there exists at least one $gb^*$ - neighbourhood (namely $X$) for each $x \in X$. Therefore $gb^* - N(x) \neq \phi$ for every $x \in X$.

2. If $N \in gb^* - N(x)$, then $N$ is $gb^*$ - neighbourhood of $x$. By Definition of $gb^*$ - neighbourhood, $x \in N$.

3. Let $N \in gb^* - N(x)$ and $M \supset N$. Then there is a $gb^*$ - open set $G$ such that $x \in G \subset N$. Since $N \subset M$, $x \in G \subset M$ and so $M$ is $gb^*$ - neighbourhood of $x$. Hence $M \in gb^* - N(x)$.

4. Let $N \in gb^* - N(x)$, $M \in gb^* - N(x)$. Then by Definition of $gb^*$ - neighbourhood, there exists $gb^*$ - open sets $G_1$ and $G_2$ such that $x \in G_1 \subset N$ and $x \in G_2 \subset M$. Hence

$$x \in G_1 \cap G_2 \subset N \cap M \quad (5.1)$$

Since $G_1 \cap G_2$ is a $gb^*$ - open set, it follows from (5.1) that $N \cap M$ is a $gb^*$ - neighbourhood of $x$.

Hence $N \cap M \in gb^* - N(x)$.

5. Let $N \in gb^* - N(x)$, Then there is a $gb^*$ - open set $M$ such that $x \in M \subset N$. Since $M$ is $gb^*$ - open set, it is $gb^*$ - neighbourhood of each of its points.

Therefore $M \in gb^* - N(y)$ for every $y \in M$. 

Theorem 5.21. Let $X$ be a nonempty set, and for each $x \in X$, let $gb^* - N(x)$ be a nonempty collection of subsets of $X$ satisfying following conditions.

(i) $N \in gb^* - N(x) \Rightarrow x \in N$.

(ii) $N \in gb^* - N(x), M \in gb^* - N(x) \Rightarrow N \cap M \in gb^* - N(x)$.

Let $\tau$ consists of the empty set and all those non-empty subsets of $G$ of $X$ having the property that $x \in G$ implies that there exists an $N \in gb^* - N(x)$ such that $x \in N \subset G$. Then $\tau$ is a topology for $X$.

Proof. 1. $\phi \in \tau$ By definition. We have to show that $x \in \tau$. Let $x$ be any arbitrary element of $X$. Since $gb^* - N(x)$ is non-empty, there is an $N \in gb^* - N(x)$ and so $x \in N$ by (i). Since $N$ is a subset of $X$, we have $x \in N \subset X$. Hence $x \in \tau$.

2. Let $G_1 \in \tau$ and $G_2 \in \tau$. If $x \in G_1 \cap G_2$ then $x \in G_1$ and $x \in G_2$. Since $G_1 \in \tau$ and $G_2 \in \tau$ there exists $N \in gb^* - N(x)$ and $M \in gb^* - N(x)$, such that $x \in N \subset G_1$ and $x \in M \subset G_2$. Then $x \in N \cap M \subset G_1 \cap G_2$.

But $N \cap M \in gb^* - N(x)$ by (2). Hence $G_1 \cap G_2 \in \tau$. 

□
6 Conclusion

The classes of generalized b star -closed sets defined using $g^*$ open sets form a topology. The $gb^*$-closed sets can be used to derive a new decomposition of continuity, closed maps and open maps, contra continuous function, almost contra continuous function, closure and interior. This idea can be extended to fuzzy topological space and fuzzy intuistic topological spaces.

7 Acknowledgment

The authors gratefully acknowledge the Dr. G. Balaji, Professor of Mathematics & Head, Department of Science & Humanities, AI-Ameen Engineering College, Erode - 638 104, for encouragement and support. The authors also heartfelt thank to Dr. M. Vijayarakavan, Associate Professor, Department of Mathematics, VMKV Engineering College, Salem 636 308, Tamil Nadu, India, for his kind help and suggestions.

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Received: January 19, 2017; Accepted: March 05, 2017

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