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On Some Decompositions of Continuity via $\delta-$Local Function in Ideal Topological Spaces

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Abstract

We introduce the notions of $\delta^* - pre - continuity$, $\delta^* - B_t - continuity$, and $\delta^* - \beta - continuity$, $\delta^* - B_\beta - continuity$ and to obtain some decompositions of continuity via $\delta - local$ function in ideal topological spaces.

Keywords: $\delta - pre-$open set, $\delta - \beta-$open set, $\beta-$open set, $\beta - I-$open set, $\delta - \alpha^*-$open set, $\delta^* - \alpha-$open set, decomposition of continuity.

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1 Introduction and Preliminaries

Ideals in topological spaces have been considered since 1930.This topic has won its importance by Vaidyanathaswamy [13]. Janković and Hamlett investigated further properties of ideal topological space [7]. Recently, in [3] Hatir et al. have introduced and studied $\delta-$local function in ideal topological space. In this paper, we have obtained decompositions of continuity using $\delta-$ local functions in ideal topological spaces.

Throughout this paper, spaces (X, τ) and (Y, τ) (or simply X and Y), always mean topological spaces on which no separation axiom is assumed. For a subset A of a topological space (X, τ), $Cl(A)$ and $Int(A)$ will denote the closure and interior of A in (X, τ), respectively.

A subset A of a topological space (X, τ) is said to be regular open (resp. regular closed) [12] if $A = Int(Cl(A))$ (resp. $A = Cl(Int(A))$). A is called $\delta - open$ [12] if for each $x \epsilon A$, there exists a regular open set G such that $x \in G \subset A$. The complement of a $\delta - open$ set is called $\delta - closed$. A point $x \in X$ is called a $\delta - cluster$ point of A if $Int(Cl(U)) \cap A \neq \phi$ for each open set V containing x. The set of all $\delta - cluster$ points of A is called the $\delta - closure$ of A and is denoted by $\delta Cl(A)$. The $\delta - interior$ of A is the union of all regular open sets of X contained in A and it is denoted by $\delta Int(A)$. A is $\delta - open$ if $\delta Int(A) = A$. $\delta - open$ sets forms a topology τ^δ. τ^δ is the same as the collection of all $\delta - open$ sets of (X, τ) and is denoted by $\delta O(X)$.

An ideal on a topological space (X, τ) is a nonempty collection of subsets of X which is satisfies (i) $A \in I$ and $B \subset A$ implies $B \in I$, (ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$. An ideal topological space is a topological space (X, τ) with an ideal I on X and if $P(X)$ is the set of all subsets of X, a set operator $(.)^*$: $P(X) \to P(X)$ called a local function [7, 8] of A with respect to τ and I is defined as follows: for $A \subset X$, $A^*(I, \tau) = \{x \in X : U \cap A \notin I \; for \; every \; U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau : x \in U\}$, simply write A^* instead of $A^*(I, \tau)$. For every ideal topological space, there exists a topology $\tau^*(I)$ or briefly τ^*, finer than τ, generated by $\beta(I, \tau) = \{U - W : U \in \tau \text{ and } W \in I\}$, but in general $\beta(I, \tau)$ is not always a topology [7]. Also $Cl^*(A) = A \cup A^*$ defines a Kuratowski closure operator for $\tau^*(I)$. If $A \in \tau^*$, $Int^*(A) = A$ and $Int^*(A)$ will denote the τ^* interior of A. If I is an ideal on X then (X, τ, I) is called an ideal topological space.

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Recently, Hatir et al. [3] introduced $\delta - local$ function in ideal topological spaces in the following manner. Let (X, τ, I) be an ideal topological space and A be a subset of X. Then $A^{\delta_*}(I, \tau) = \{x \in X : U \cap A \notin I \; for \; every \; U \in \delta O(X, x)\}$ is called the $\delta - local$ function of I on X with respect to I and τ. We denote simply A^{δ_*} for $A^{\delta_*}(I, \tau)$. Furthermore, $Cl^{\delta_*}(A) = A \cup A^{\delta_*}$ defines a Kuratowski closure operator for $\tau^{\delta_*}(I)$. We will denote τ^{δ_*} the topology generated by Cl^{δ_*}, that is, $\tau^{\delta_*} = \{U \subset X : Cl^{\delta_*}(X - U) = X - U\}$. Therefore, the topology τ^{δ_*} finer than τ^{δ} and also the topology τ^* finer than τ^{δ_*}.

Lemma 1.1. *[3] Let (X, τ, I) be an ideal topological space and A, B subsets of X. Then*
1) If $A \subset B$, then $Cl^{\delta_*}(A) \subset Cl^{\delta_*}(B)$
2) $Cl^{\delta_*}(A \cap B) \subset Cl^{\delta_*}(A) \cap Cl^{\delta_*}(B)$
3) If $U \in \tau^{\delta}$, then $U \cap Cl^{\delta_*}(A) \subset Cl^{\delta_*}(U \cap A)$
4) $Cl^{\delta_*}(\cup_i(A_i)) = \cup_i(Cl^{\delta_*}(A_i))$
5) If $I \subset J$, then $Cl^{J\delta_*}(A) \subset Cl^{I\delta_*}(A)$, ($J$ is ideal)

First we shall recall some definitions used in the sequel.

Definition 1.1. *A subset A of an ideal topological space (X, τ, I) is said to be*
1) $\alpha - I - open$ [4] if $A \subset Int(Cl^*(Int(A)))$,
2) $pre - I - open$ [2] if $A \subset Int(Cl^*(A))$,
3) $\beta - I - open$ [4] if $A \subset Cl(Int(Cl^*(A)))$,
4) $\delta^* - \alpha - open$ [6] if $A \subset Int(Cl^{\delta_*}(Int^*(A)))$,
5) $\delta - \alpha^* - open$ [6] if $A \subset Int(\delta Cl(Int^*(A)))$.

Definition 1.2. *A subset A of a topological space (X, τ) is said to be*
1) $\alpha - open$ [10] if $A \subset Int(Cl(Int(A)))$,
2) $pre - open$ [9] if $A \subset Int(Cl(A))$,
3) $\beta - open$ [1] if $A \subset Cl(Int(Cl(A)))$,
4) $\delta - pre - open$ [11] if $A \subset Int(\delta Cl(A))$,
5) $\delta - \beta - open$ [5] if $A \subset Cl(Int(\delta Cl(A)))$.

Definition 1.3. *Let $f : (X, \tau, I) \longrightarrow (Y, \sigma)$ be a function. If for each $V \in \sigma$, $f^{-1}(V)$ is a $\alpha - I - open$ (resp. $pre - I - open$, $\beta - I - open$, $\delta^* - \alpha - open$, $\delta - \alpha^* - open$), then f is said to be $\alpha - I - continuous$ [4] (resp. $pre - I - continuous$ [2], $\beta - I - continuous$ [4], $\delta^* - \alpha - continuous$ [6], $\delta - \alpha^* - continuous$ [6]).*

Definition 1.4. *Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a function. If for each $V \in \sigma$, $f^{-1}(V)$ is an $\alpha - open$ (resp. $pre - open$, $\beta - open$, $\delta - pre - open$, $\delta - \beta - open$), then f is said to be $\alpha - continuous$ [10] (resp. $pre - continuous$ [9], $\beta - continuous$ [1], $\delta - pre - continuous$ [11], $\delta - \beta - continuous$ [5]).*

2 $\delta^* - pre - open$ set and $\delta^* - \beta - open$ set

We give the following generalized open sets to obtain new decompositions of continuity.

Definition 2.5. *A subset A of an ideal topological space (X, τ, I) is said to be*
1) $\delta^* - pre - open$ if $A \subset Int(Cl^{\delta_*}(A))$,
2) $\delta^* - \beta - open$ if $A \subset Cl(Int(Cl^{\delta_*}(A)))$.

Proposition 2.1. *1) Every $\alpha - I - open$ set is $\delta^* - \alpha - open$,*
2) Every $\delta^* - \alpha - open$ set is $\delta - \alpha^* - open$,
3) Every $\delta - \alpha^* - open$ set is $\delta - pre - open$,
4) Every $\delta^* - \alpha - open$ set is $\delta^* - pre - open$,
5) Every $\delta^* - pre - open$ set is $\delta - pre - open$,
6) Every $\delta^* - pre - open$ set is $\delta^* - \beta - open$,
7. Every $\delta^* - \beta - open$ set is $\delta - \beta - open$.

Proof. Straightforward from the definitions of the topologies τ^*, τ^{δ} and τ^{δ_*} and [6]. □

Remark 2.1. *None of them in the proposition 1 is reversible as shown by examples below. Also* $\alpha-open$ *set and* $\delta^*-\alpha-open$ *[6],* $pre-open$ *set and* $\delta^*-pre-open$, $\beta-open$ *set and* $\delta^*-\beta-open$ *are independent notions.*

Example 2.1. *Let* $X=\{a,b,c,d\}$, $\tau=\{\phi,X,\{a\},\{b,d\},\{a,c\},\{a,b,d\}\}$ *and* $I=\{\phi,\{a\}\}$. *Then* $\tau^*=\{\phi,X,\{a\},\{c\},\{b,d\},\{a,c\},\{a,b,d\},\{b,c,d\}\}$ *and* $\tau^\delta=\{\phi,X,\{b,d\},\{a,c\}\}$. *Take* $A=\{b,c,d\}$. *Therefore, A is a* $\delta^*-pre-open$ *set and* $\delta^*-\beta-open$ *set, but neither* $pre-open$ *nor* $\beta-open$ *and not* $pre-I-open$ *set.*

Example 2.2. *Let* $X=\{a,b,c,d,e\}$, $\tau=\{\phi,X,\{a\},\{c\},\{a,c\},\{a,b\},\{a,b,c\}\}$ *and* $I=\{\phi,\{a\}\}$. *Then* $\tau^\delta=\{\phi,X,\{c\},\{a,b\},\{a,b,c\}\}$. *Take* $A=\{a,c,d\}$. *Therefore, since* $Int(Cl^{\delta_*}(A))=\{a,c\}$ *and* $Int(Cl(A))=X$, *A is* $pre-open$ *set and* $\alpha-open$ *set,* $\delta-pre-open$ *set and also* $\delta^*-\beta-open$ *set, but neither* $\delta^*-pre-open$ *nor* $\delta^*-\alpha-open$.

Example 2.3. *Let* $X=\{a,b,c,d,e\}$, $\tau=\{\phi,X,\{a\},\{c,e\},\{a,c,e\},\{a,b\},\{a,b,c,e\}\}$ *and* $I=\{\phi,\{e\}\}$. *Then* $\tau^\delta=\{\phi,X,\{c,e\},\{a,b\},\{a,b,c,e\}\}$. *Take* $A=\{a,e\}$. *Therefore, A is* $\beta-open$ *set and also* $\delta-\beta-open$ *set, but not* $\delta^*-\beta-open$ *since* $\{a,e\}\nsubseteq Cl(Int(Cl^{\delta_*}(A)))=\{a,b,d\}$.

Proposition 2.2. *The arbitrary union of* $\delta^*-pre-open$ *sets* ($\delta^*-\beta-open$ *sets) are* $\delta^*-pre-open$ *set* ($\delta^*-\beta-open$ *set).*

Proof. Let A_i be $\delta^*-pre-open$ sets for every i. Then, $A_i\subset Int(Cl^{\delta_*}(A_i))$ for every i. Hence, $\cup_iA_i\subset\cup_i(Int(Cl^{\delta_*}(A_i)))\subset Int(Cl^{\delta_*}(\cup_iA_i))$ by Lemma 1.1(4). Consequently, $\cup_iA_i$ is $\delta^*-pre-open$ set. For $\delta^*-\beta-open$ set, the proof is similar. □

Remark 2.2. *The intersection of two* $\delta^*-pre-open$ *sets* ($\delta^*-\beta-open$ *sets) need not be a* $\delta^*-pre-open$ *set* ($\delta^*-\beta-open$ *set) as in the following example.*

Example 2.4. *Let* $X=\{a,b,c,d\}$, $\tau=\{\phi,X,\{a\},\{a,c\},\{a,d\},\{a,c,d\}\}$ *and* $I=\{\phi,\{b\}\}$.*Then* $\tau^\delta=\{\phi,X\}$ *and* $\tau^*=\tau$. *Take* $A=\{b,c\}$ *and* $B=\{a,b\}$ *are* $\delta^*-pre-open$ *set and* $\delta^*-\beta-open$ *set , but* $A\cap B=\{b\}$ *is neither* $\delta^*-pre-open$ *set nor* $\delta^*-\beta-open$ *set since* $Cl(Int(Cl^{\delta_*}(\{b\})))=\phi$.

Corollary 2.1. *[3] Let* (X,τ,I) *be an ideal topological space and* $A\subset X$.
1) If $A\subset A^{\delta_*}$, *then* $\delta Cl(A)=Cl^{\delta_*}(A)$
2) If $I=\{\phi\}$, *then* $\delta Cl(A)=Cl^{\delta_*}(A)$.

Proposition 2.3. *Let* (X,τ,I) *be an ideal topological space and* $A\subset X$. *If* $A\subset A^{\delta_*}$ *(If* $I=\{\phi\}$*), then*
1) $\delta-pre-open$ *set and* $\delta^*-pre-open$ *set are equivalent*
2) $\delta-\beta-open$ *set and* $\delta^*-\beta-open$ *set are equivalent.*

Proof. By Corollary 2.1, if $A\subset X$, then it $\delta Cl(A)=Cl^{\delta_*}(A)$. Thus we get the result. □

Proposition 2.4. *Let* (X,τ,I) *be an ideal topological space and* $A,B\subset X$. *Then the following statements hold:*
1) If $A\in\tau^\delta$ *and B is* $\delta^*-pre-open$ *set, then* $A\cap B$ *is* $\delta^*-pre-open$ *set,*
2) If $A\in\tau^\delta$ *and B is* $\delta^*-\beta-open$ *set, then* $A\cap B$ *is* $\delta^*-\beta-open$ *set.*

Proof. 1) Let $A\in\tau^\delta$ and B is $\delta^*-pre-open$ set. Then,

$$\begin{aligned}A\cap B&\subset\delta Int(A)\cap Int(Cl^{\delta_*}(B))=\delta Int(\delta Int(A))\cap Int(Cl^{\delta_*}(B))\\&\subset Int(\delta Int(A)\cap Int(Cl^{\delta_*}(B)))=Int(\delta Int(A)\cap Cl^{\delta_*}(B))\\&\subset Int(Cl^{\delta_*}(A\cap B))\quad\text{(by Lemma 1.1.)}\end{aligned}$$

The proof of (2) are same with the proof of (1). □

Proposition 2.5. *Let I and J be any two ideals on a topological space* (X,τ) *with* $I\subset J$. *If a subset A of X is* $\delta^*-pre-(J)open$ *set* ($\delta^*-\beta-(J)open$ *set), then it is* $\delta^*-pre-(I)open$ *set* ($\delta^*-\beta-(I)open$ *set).*

Proof. Follows from directly Lemma 1.1(5). □

The above discussions are summarized in the following diagram.

Diagram 1

$$\begin{array}{ccccccccccccc}
\tau & \to & \alpha IO & & & -\to & PIO & & & --\to & \beta IO & & \\
 & & | & \searrow & & & | & \searrow & & & | & \searrow & \\
 & & | & & \alpha O & --\to & & & PO & --\to & & & \beta O \\
 & & \downarrow & & | & & \downarrow & & | & & \downarrow & & | \\
 & & \delta^*\alpha O & & & -\to & \delta^*PO & & & --\to & \delta^*\beta O & & | \\
 & & & \searrow & \downarrow & & & \searrow & \downarrow & & & \searrow & \downarrow \\
 & & & & \delta\alpha^*O & --\to & & & \delta PO & --\to & & & \delta\beta O
\end{array}$$

By αIO,(resp. PIO, βIO, αO, PO, βO, $\delta^*\alpha O$, δ^*PO, $\delta^*\beta O$, $\delta\alpha^*O$, δPO, $\delta\beta O$) in diagram, we denote the family of all $\alpha - I - open$ sets (resp. $pre - I - open$, $\beta - I - open$, $\alpha - open$, $pre - open$, $\beta - open$, $\delta^* - \alpha - open$, $\delta^* - pre - open$, $\delta^* - \beta - open$, $\delta - \alpha^* - open$, $\delta - pre - open$, $\delta - \beta - open$) of a space (X, τ) and (X, τ, I).

Definition 2.6. *A subset A of an ideal topological space (X, τ, I) is called*
1) A $\delta^* - t - set$ if $Int(A) = Int(Cl^{\delta_*}(A))$,
2) A $\delta^* - \beta - set$ if $Int(A) = Cl(Int(Cl^{\delta_*}(A)))$,
3) A $\delta - \alpha^* - set$ [6] if $Int(A) = Int(\delta Cl(Int^*(A)))$,
4) A $\delta^* - \alpha - set$ [6] if $Int(A) = Int(Cl^{\delta_*}(Int^*(A)))$.

Proposition 2.6. *Let A be a subset of an ideal topological space (X, τ, I). The following properties hold:*
1). Every $\delta^* - t - set$ is $\delta^* - \alpha - set$,
2) Every $\delta^* - \beta - set$ is $\delta^* - t - set$,
3) Every $\delta - \alpha^* - set$ is $\delta^* - \alpha - set$ [6].

Proof. Straightforward from the definitions of the topologies τ^δ and τ^{δ_*} and [6]. □

Remark 2.3. *None of them in Proposition 2.5 is reversible as shown by examples below. Also the notions of $\delta^* - t - set$ and $\delta - \alpha^* - set$ are independent notions [6].*

Example 2.5. *Let $X = \{a, b, c, d, e\}$, $\tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{a, b, c\}\}$ and $I = \{\phi, \{c\}\}$. Then $\tau^\delta = \{\phi, X, \{c\}, \{a, b\}, \{a, b, c\}\}$ and $\tau^* = \{\phi, X, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{a, b, c\}, \{a, b, d, e\}\}$. Take $A = \{b, c\}$. Therefore A is $\delta^* - \alpha - set$ and $\delta - \alpha^* - set$, but not $\delta^* - \beta - set$ and not $\delta^* - t - set$ since $Cl(Int(Cl^{\delta_*}(\{b, c\}))) = X \neq Int(\{b, c\})$ and $\{c\} = Int(\{b, c\}) = Int(Cl^{\delta_*}(Int^*(\{b, c\}))) = Int(\delta Cl(Int^*(\{b, c\}))) = \{c\}$.*

In this example if we take $A = \{c, d\}$, we obtain that A is $\delta^* - t - set$, but not $\delta^* - \beta - set$ since $Int(\{c, d\}) \neq Cl(Int(Cl^{\delta_*}(\{c, d\}))) = \{c, d, e\}$ and $Int(\{c, d\}) = Int(Cl^{\delta_*}(\{c, d\})) = \{c\}$.

Example 2.6. *Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{d\}, \{b, d\}, \{a, d\}, \{a, b, d\}\}$ and $I = \{\phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$. Then $\tau^\delta = \{\phi, X, \{a\}, \{b, d\}, \{a, b, d\}\}$ and $\tau^* = \wp(X)$. if we take $A = \{b, c\}$, then A is $\delta^* - t - set$ and $\delta^* - \alpha - set$, but not $\delta - \alpha^* - set$ since $Int(\{b, c\}) = Int(Cl^{\delta_*}(\{b, c\})) = \phi$ and $Int(\{b, c\}) = \phi \neq Int(\delta Cl(Int^*(\{b, c\}))) = \{b, d\}$.*

Definition 2.7. *Let (X, τ, I) be an ideal topological space. A subset A in X is said to be a $\delta^* - B_t - set$ (resp. $\delta^* - B_\beta - set$, $\delta^* - B_\alpha - set$ [6], $\delta - B\alpha^* - set$ [6]) if there is a $U \in \tau$ and a $\delta^* - t - set$ (resp. $\delta^* - \beta - set$, $\delta - \alpha^* - set$, $\delta^* - \alpha - set$) V in X such that $A = U \cap V$.*

Proposition 2.7. *For a subset A of a space (X, τ, I), the following properties hold:*
1) Every $\delta^* - t - set$ is $\delta^* - B_t - set$,
2) Every $\delta^* - \beta - set$ is $\delta^* - B_\beta - set$,
3) Every $\delta - \alpha^* - set$ is $\delta - B\alpha^*$ $-set$ [6],
4) Every $\delta^* - \alpha - set$ is $\delta^* - B_\alpha - set$ [6],
5) Every open set is $\delta^* - B_t - set$ (resp. $\delta^* - B_\beta - set$, $\delta-$ $B\alpha^* - set$, $\delta^* - B_\alpha - set$).

Proof. Since $A = A \cap X$ and $X \in \tau$, we get 1-4, also if $A \in \tau$, we get 5. □

Remark 2.4. *None of them in Proposition 2.6 is reversible as shown by example below and [6].*

Example 2.7. *Let* $X = \{a,b,c,d,e\}$, $\tau = \{\phi, X, \{a\}, \{c\}, \{a,c\}, \{a,b\}, \{a,b,c\}\}$ *and* $I = \{\phi, \{c\}\}$ *and* $\tau^{\delta} = \{\phi, X, \{c\}, \{a,b\}, \{a,b,c\}\}$. *If we take* $A = \{a\}$, *then* A *is* $\delta^* - B_t - set$ $(\delta^* - B_\beta - set)$, *but not* $\delta^* - t - set$ $(\delta^* - \beta - set)$ *since* $\{a\} \in \tau$ *and* $\{a\} = \{a\} \cap X$ *also* $Int(Cl^{\delta_*}(\{a\})) = \{a,b\} \neq Int(\{a\})$. *In this example,* $\{c,d\}$ *is* $\delta^* - B_t - set$ *and* $\delta^* - B_\beta - set$, *but* $\{c,d\} \notin \tau$.

By Proposition 2.5, we have the following diagram

Diagram 2

$$\begin{array}{ccccc} \delta^* - B_\beta - set & \Longrightarrow & \delta^* - B_t - set & \Longrightarrow & \delta^* - B_\alpha - set \\ & & & & \Uparrow \\ & & & & \delta - B\alpha^* - set \end{array}$$

Theorem 2.1. *Let* A *be a subset of an ideal topological space* (X, τ, I). *Then the following statements are equivalent:*

1) A is open,

2) A is $\delta^* - pre - open$ *and* $\delta^* - B_t - set$,

3) A is $\delta^* - \beta - open$ *and* $\delta^* - B_\beta - set$,

4) A is $\delta^* - \alpha - open$ *and* $\delta^* - B_\alpha - set$ *[6],*

5) A is $\delta - \alpha^* - open$ *and* $\delta - B\alpha^* - set$ *[6].*

Proof. (1)$\Longrightarrow$(2). This is obvious from diagrams 1-2 and Proposition 2.6 (5).

(2)$\Longrightarrow$(1). Since A is a $\delta^* - B_t - set$, we have $A = U \cap V$, where U is an open set and $Int(V) = Int(Cl^{\delta_*}(V))$. By the hypothesis, A is also $\delta^* - pre - open$, and we have

$$\begin{aligned} A \subset Int(Cl^{\delta_*}(A)) &= Int(Cl^{\delta_*}(U \cap V)) \subset Int(Cl^{\delta_*}(U) \cap Cl^{\delta_*}(V)) \\ &= Int(Cl^{\delta_*}(U)) \cap Int(Cl^{\delta_*}(V)) = Int(Cl^{\delta_*}(U)) \cap Int(V). \end{aligned}$$

Hence

$$\begin{aligned} A &= U \cap V = (U \cap V) \cap U \subset (Int(Cl^{\delta_*}(U)) \cap Int(V)) \cap U \\ &= (Int(Cl^{\delta_*}(U)) \cap U) \cap Int(V) = U \cap Int(V). \end{aligned}$$

Notice $A = U \cap V \supset U \cap Int(V)$. Therefore, we obtain $A = U \cap Int(V)$.

(1) $\Longleftrightarrow$ (3). The proof is same with (1) $\Longleftrightarrow$ (2). □

3 Decompositions of continuity

Definition 3.8. *Let* $f : (X, \tau, I) \longrightarrow (Y, \sigma)$ *be a function. If for each* $V \in \sigma$, $f^{-1}(V)$ *is a* $\delta^* - pre - open$ *set* ($\delta^* - \beta - open$ *set), then f is said to be* $\delta^* - pre - continuous$ ($\delta^* - \beta - continuous$).

Definition 3.9. *Let* $f : (X, \tau, I) \longrightarrow (Y, \sigma)$ *be a function. If for each* $V \in \sigma$, $f^{-1}(V)$ *is a* $\delta^* - B_t - set$ *(resp.* $\delta^* - B_\beta - set$, $\delta^* - B_\alpha - set$, $\delta - B\alpha^* - set$*), then f is said to be* $\delta^* - B_t - continuous$ *(resp.* $\delta^* - B_\beta - continuous$, $\delta^* - B_\alpha - continuous$ *[6],* $\delta - B\alpha^* - continuous$ *[6]).*

By Diagrams 1-2, we have the following proposition.

Proposition 3.8. *1) A* $\delta^* - B_\beta - continuous$ *function is* $\delta^* - B_t - continuous$,

2) A $\delta^* - B_t - continuous$ *function is* $\delta^* - B_\alpha - continuous$,

3) A $\delta - B\alpha^* -$ *continuous function is* $\delta^* - B_\alpha - continuous$,

4) A $\delta^* - \alpha - continuous$ *function is* $\delta^* - pre - continuous$,

5) A $\delta^* - pre - continuous$ *function is* $\delta^* - \beta - continuous$,

6) A $\delta^* - \alpha - continuous$ *function is* $\delta - \alpha^* -$ *continuous,*

7) A $\delta - \alpha^* -$ *continuous function is* $\delta - pre - continuous$,

8) A $\delta^* - pre - continuous$ *function is* $\delta - pre - continuous$.

By Theorem 2.1, we have the following main theorem.

Theorem 3.2. *For a function $f:(X,\tau,I)\longrightarrow(Y,\sigma)$, the following properties are equivalent:*
1) f is continuous,
2) f is $\delta^*-pre-continuous$ and $\delta^*-B_t-continuous$,
3) f is $\delta^*-\beta-continuous$ and $\delta^*-B_\beta-continuous$.

Remark 3.5. *1) $\delta^*-pre-continuous$ and $\delta^*-B_t-continuous$ are independent of each other,*
2) $\delta^*-\beta-continuous$ and $\delta^*-B_\beta-continuous$ are independent of each other.

Example 3.8. *Let $X=Y=\{a,b,c,d,e\}$, $\tau_1=\{\phi,X,\{a\},\{c\},\{a,c\},\{a,b\},\{a,b,c\}\}$, $I=\{\phi,\{a\}\}$ and then $\tau^\delta=\{\phi,X,\{c\},\{a,b\},\{a,b,c\}\}$ and also $\tau_2=\{\phi,Y,\{a,b\}\}$. Define a function $f:(X,\tau_1,I)\longrightarrow(Y,\tau_2)$ as follows: $f(a)=f(c)=a, f(b)=c, f(d)=b, f(e)=d$. Then f is $\delta^*-B_t-continuous$, but not $\delta^*-pre-continuous$ since $f^{-1}(\{a,b\})=\{a,c,d\}$ and $\{a,c\}=Int\{a,c,d\}=Int(Cl^{\delta_*}(\{a,c,d\}))=\{a,c\}$, thus $\{a,c,d\}$ is δ^*-B_t-set, but not $\delta^*-pre-open$.*

Example 3.9. *Let $X=Y=\{a,b,c,d\}$, $\tau_1=\{\phi,X,\{a\},\{c\},\{a,c\},\{a,d\},\{a,c,d\}\}$, $I=\{\phi,\{b\}\}$ and then $\tau^\delta=\{\phi,X\}$, $\tau^*=\tau$ and also $\tau_2=\{\phi,Y,\{b\}\}$. Define an identity function $f:(X,\tau_1,I)\longrightarrow(Y,\tau_2)$. Then f is $\delta^*-B_\beta-continuous$, but not $\delta^*-\beta-continuous$, since $f^{-1}(\{b\})=\{b\}$ and $Int(\{b\})=\phi=Cl(Int(Cl^{\delta_*}(\{b\})))$, thus $\{b\}$ is $\delta^*-B_\beta-set$, but not $\delta^*-\beta-open$ set.*

Example 3.10. *Let $X=Y=\{a,b,c,d,e\}$, $\tau_1=\{\phi,X,\{a\},\{c\},\{a,c\},\{a,b\},\{a,b,c\}\}$, $I=\{\phi,\{a\}\}$ and then $\tau^\delta=\{\phi,X,\{c\},\{a,b\},\{a,b,c\}\}$ and $\tau_1^*=\{\phi,X,\{a\},\{c\},\{a,c\},\{a,b\},\{a,b,c\},\{a,b,d,e\}\}$. Let $\tau_2=\{\phi,Y,\{a,b\}\}$. Define a function $f:(X,\tau_1,I)\longrightarrow(Y,\tau_2)$ as follows: $f(a)=c$, $f(b)=a$, $f(c)=b$, $f(d)=d$, $f(e)=d$. Then f is $\delta^*-pre-continuous$, but not $\delta^*-B_t-continuous$ $f^{-1}(\{a,b\})=\{b,c\}$ and $\{c\}=Int(\{b,c\})\neq Int(Cl^{\delta_*}(\{b,c\}))=X$, thus $\{b,c\}$ is $\delta^*-pre-open$, but not δ^*-B_t-set. In this example, if we take same function, Then f is $\delta^*-\beta-continuous$, but not $\delta^*-B_\beta-continuous$ since $f^{-1}(\{a,b\})=\{b,c\}$ and $\{c\}=Int(\{b,c\})\neq Cl(Int(Cl^{\delta_*}(\{b,c\})))=X$.*

Corollary 3.2. *For a function $f:(X,\tau)\longrightarrow(Y,\sigma)$, the following properties are equivalent:*
1) f is continuous,
2) f is $\delta-pre-continuous$ and $\delta-B-continuous$ [4].
3) f is $pre-continuous$ and $B-continuous$ [2].

Corollary 3.3. *For a function $f:(X,\tau,I)\longrightarrow(Y,\sigma)$, the following properties are equivalent:*
1) f is continuous,
2. f is $\alpha-I-continuous$ and $C_I-continuous$ [4],
3. f is $pre-I-continuous$ and $B-I-continuous$ [2],
4. f is $\delta^*-\alpha-continuous$ and $\delta^*-B_\alpha-continuous$ [6],
5. f is $\delta-\alpha^*-continuous$ and $\delta-B\alpha^*-continuous$ [6].

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