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On totally $\pi g^{\mu}r$-continuous function in supra topological spaces

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Abstract

The focus of this paper is to use $\pi g^{\mu}r$-closed set to define and investigate a new class of function called totally $\pi g^{\mu}r$-continuous functions in supra topological spaces and to obtain some of its characteristics and properties.

Keywords: $\pi g^{\mu}r$-closed set, $\pi g^{\mu}r$-continuous functions, $\pi g^{\mu}r$-irresolute function and Totally $\pi g^{\mu}r$-continuous functions.

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1 Introduction

The first step of generalized closed sets was initiated by Levine [6]. Zaitsev[11] introduced the concept of π-closed sets and defined a class of topological spaces called quasi normal spaces. Palaniappan[8] studied the concept of regular generalized closed set in topological spaces. In 1980, R.C. Jain [2] defined totally continuous functions in topological spaces.

In 1983, Mashhour et al [7] introduced the supra topological spaces. In 2010, Sayed and Noiri et al [9] introduced supra b-open sets and supra b-continuity in supra topological spaces. The notions of $\pi\Omega$-closed and $\pi\Omega s$-closed sets in supra topological spaces was introduced by Arockiarani et al[1]. The concepts of totally supra b-continuous functions and slightly supra b-continuous functions are introduced and studied by Jamal M. Mustafa[3].

Let (X, μ), (Y, λ) be the supra topological spaces denoted by X, Y respectively throughout this paper.

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2 Preliminaries

Definition 2.1. *Let X be a non-empty set. The sub family $\mu \subseteq P(X)$ where P(X) is the power set of X is said to be a supra topology on X if $\phi, X \in U$ and U is closed under arbitrary unions. The pair (X, μ) is called a supra topological space. The elements of μ are said to be supra open in (X, μ). The complement of supra open set is called supra closed set.*

Definition 2.2. *A subset A of X is called*

1. *supra semi-open [1,7] if $A \subseteq cl^{\mu}(int^{\mu}(A))$.*
2. *supra regular open[1] if $A = int^{\mu}(cl^{\mu}(A))$ and regular closed if $A = cl^{\mu}(int^{\mu}(A))$*
3. *supra π-open [1] if A is the finite union of regular open sets.*

The complements of the above mentioned open sets are called their respective closed sets.

Definition 2.3. *A map $f : (X, \mu) \to (Y, \lambda)$ is called supra continuous[7] if the inverse image of each open set of Y is supra open in X.*

Definition 2.4 (4). *A subset A of a supra topological space X is said to be supra πgr-closed set [$\pi g^{\mu} r$-closed] if $rcl^{\mu}(A) \subset U$ whenever $A \subset U$ and U is supra π-open. The collection of all $\pi g^{\mu} r$-closed set is denoted by $\pi G^{\mu} RC(X)$.*

Definition 2.5. *A space X is called*

(i) $\pi g^{\mu} r - T_{1/2}$-space[4] if every $\pi g^{\mu} r$ -closed set is supra regular closed.

(ii) $\pi g^{\mu} r$-locally indiscrete[4] if every $\pi g^{\mu} r$-open subset of X is supra closed in X.

Definition 2.6 (4). *Let (X, τ) and (Y, σ) be two topological spaces with $\tau \subset \mu$ and $\sigma \subset \lambda$. A map $f : (X, \mu) \to (Y, \lambda)$ is called*

(i) $\pi g^{\mu} r$-continuous if the inverse image of each supra open set of Y is $\pi g^{\mu} r$-open in X.

(ii) $\pi g^{\mu} r$ -irresolute if the inverse image of each $\pi g^{\mu} r$-open set of Y is $\pi g^{\mu} r$-open in X.

Definition 2.7. *A map $f : (X, \mu) \to (Y, \lambda)$ is said to be*

(i) a $\pi g^{\mu} r$-closed map[5] if $f(U)$ is $\pi g^{\mu} r$-closed in Y for every supra closed set U in X.

(ii) a strongly $\pi g^{\mu} r$-closed map [5]if $f(U)$ is $\pi g^{\mu} r$-closed in Y for every $\pi g^{\mu} r$-closed set U in X.

Definition 2.8 (10). *A function $f : X \to Y$ is said to be supra totally continuous function if the inverse image of every supra open subset of Y is $cl^{\mu} open^{\mu}$ in X and is denoted by $totally^{\mu}$ continuous function.*

Definition 2.9 (7,10). *A supra topological space is said to be*

1 . $cl^{\mu}open^{\mu} - T_1$ *-space if for each pair of distinct points x and y in X, there exist* $cl^{\mu}open^{\mu}$ *sets U and V containing x and y respectively such that* $x \in U, y \notin U$ *and* $x \notin V, y \in V$ *containing one point but not other.*

2 . $Ultra^{\mu}$ *Hausdorff or* $Ultra^{\mu} - T_2$*-space if every two distinct points of X can be separated by disjoint* $cl^{\mu}open^{\mu}$ *sets.*

3. *supra normal if each pair of non-empty distinct supra closed sets can be separated by disjoint supra open sets.*

4 . $Ultra^{\mu}$ *normal if each pair of non-empty distinct supra closed sets separated by disjoint* $cl^{\mu}open^{\mu}$ *sets.*

5. $cl^{\mu}open^{\mu}$ *-normal if for each pair of disjoint* $cl^{\mu}open^{\mu}$ *sets U and V of X, there exists two disjoint supra open sets G and H such that* $U \subset G$ *and* $V \subset H$.

6. *supra regular if each closed set F of X and each* $x \notin F$*, there exists disjoint supra open sets* U *and* V *such that* $F \subset U$ *and* $x \in V$.

7. $Ultra^{\mu}$ *regular if each supra closed set F of X and each* $x \notin F$*, there exist disjoint* $cl^{\mu}open^{\mu}$ *sets U and V such that* $F \subset U$ *and* $x \in V$.

8. $cl^{\mu}open^{\mu}$ *-regular if for each* $cl^{\mu}open^{\mu}$ *set F of X and for each* $x \notin F$*, there exists two disjoint supra open sets U and V such that* $F \subset U$ *and* $x \in V$.

3 Totally $\pi g^{\mu}r$continuous functions.

Definition 3.1. *A function* $f : X \to Y$ *is said to be supra totally* πgr*-continuous function (totally* $\pi g^{\mu}r$*-continuous) if the inverse image of every* $\pi g^{\mu}r$*-open subset of* Y *is* $cl^{\mu}open^{\mu}$ *in X.*

Proposition 3.1. *(i) Every* $totally^{\mu}$ *continuous function is* $\pi g^{\mu}r$*-continuous.*

(ii) Every totally $\pi g^{\mu}r$*-continuous function is* $\pi g^{\mu}r$*-irresolute.*

Proof. (i) Let $f : X \to Y$ be a $totally^{\mu}$ continuous function. Then $f^{-1}(V)$ is $cl^{\mu}open^{\mu}$ for every open set V of Y. Hence $f^{-1}(V)$ is $\pi g^{\mu}r$-open in X. Therefore $f^{-1}(V)$ is $\pi g^{\mu}r$-continuous in X.

(ii) Let V be a $\pi g^{\mu}r$-open set in Y. Since f is totally $\pi g^{\mu}r$-continuous, then $f^{-1}(V)$ is $cl^{\mu}open^{\mu}$ in X and hence $f^{-1}(V)$ is $\pi g^{\mu}r$-open in X. Therefore f is $\pi g^{\mu}r$-irresolute.

□

Remark 3.1. *The converse of the above need not be true as shown in the following example.*

Example 3.1. *(i) Let $X = \{a, b, c, d\} = Y$, $\mu = \{\phi, X, \{a\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ and $\lambda = \{\phi, Y, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Let $f : X \to Y$ be an identity map. Then f is $\pi g^{\mu} r$-continuous but not totally$^{\mu}$ continuous.*

(ii) Let $X = \{a, b, c, d\} = Y$, $\mu = \{\phi, X, \{a\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$ and $\lambda = \{\phi, Y, \{c\}, \{c, d\}, \{b, d\}, \{b, c, d\}, \{a, c, d\}\}$. Let $f : X \to Y$ be defined by $f(a) = b, f(b) = c, f(c) = a$ and $f(d) = d$. Then f is $\pi g^{\mu} r$-irresolute but not totally $\pi g^{\mu} r$-continuous.

Theorem 3.1. *A bijection $f : X \to Y$ is totally $\pi g^{\mu} r$-continuous iff the inverse image of every $\pi g^{\mu} r$-closed subset of Y is $cl^{\mu} open^{\mu}$ in X.*

Proof. Let F be any $\pi g^{\mu} r$-closed set in Y. Then $Y - F$ is $\pi g^{\mu} r$-open set in Y. By definition, $f^{-1}(Y - F)$ is $cl^{\mu} open^{\mu}$ in X. Then
$f^{-1}(Y - F) = f^{-1}(Y) - f^{-1}(F) = X - f^{-1}(F)$ is $cl^{\mu} open^{\mu}$ in X. Hence $f^{-1}(F)$ is $cl^{\mu} open^{\mu}$ in X.
Conversely, let V be $\pi g^{\mu} r$-open in Y. By assumption, $f^{-1}(Y - V)$ is $cl^{\mu} open^{\mu}$ in Y. The above implies $X - f^{-1}(V)$ is $cl^{\mu} open^{|mu}$ in X and hence $f^{-1}(V)$ is $cl^{\mu} open^{\mu}$ in X. Therefore, f is totally $\pi g^{\mu} r$-continuous. □

Theorem 3.2. *Let $f : X \to Y$ be a function, where X and Y are supra topological spaces. The following are equivalent.*

(i) f is totally $\pi g^{\mu} r$ -continuous.

(ii) for each $x \in X$ and each $\pi g^{\mu} r$-open set V in Y with $f(x) \in V$, there is a $cl^{\mu} open^{\mu}$ set U in X such that $x \in U$ and $f(U) \subset V$.

Proof. $(i) \Rightarrow (ii)$: Let V be a $\pi g^{\mu} r$-open set in Y containing f(x), so that $x \in f^{-1}(V)$. Since f is totally $\pi g^{\mu} r$-continuous, $f^{-1}(V)$ is $cl^{\mu} open^{\mu}$ in X. Let $U = f^{-1}(V)$. Then U is $cl^{\mu} open^{\mu}$ in X and $x \in U$. Therefore $f(U) = f(f^{-1}(V)) = V$ and hence $f(U) \subset V$.

$(ii) \Rightarrow (i)$: Let V be $\pi g^{\mu} r$-open in Y. Let $x \in f^{-1}(V)$ be any arbitrary point. Then $f(x) \in V$. Therefore, by (ii), there is a $cl^{\mu} open^{\mu}$ set $f(G) \subset X$ containing x such that $f(G) \subset V$.
Hence $G \subset f^{-1}(V)$ is a $cl^{\mu} open^{\mu}$ neighborhood of x. Since x is arbitrary, $f^{-1}(V)$ is $cl^{\mu} open^{\mu}$ neighborhood of each of its points. Therefore $f^{-1}(V)$ is $cl^{\mu} open^{\mu}$ in X and hence f is totally $\pi g^{\mu} r$-continuous. □

Theorem 3.3. *For a function $f : X \to Y$, the following properties hold:*

(i) If f is supra continuous and X is locally$^{\mu}$ indiscrete, then f is totally$^{\mu}$ continuous.

(ii) If f is totally$^{\mu}$ continuous and Y is $\pi g^{\mu} r - T_{1/2}$- space, then f is totally $\pi g^{\mu} r$-continuous.

Proof. (i) Let V be a supra open set in Y. Since f is supra continuous, $f^{-1}(V)$ is supra open in X. Again since X is $locally^{\mu}$ indiscrete, $f^{-1}(V)$ is supra closed in X.
Therefore $f^{-1}(V)$ is $cl^{\mu}open^{\mu}$ in X and hence f is $totally^{\mu}$ continuous.

(ii) Let V be $\pi g^{\mu} r$-open in Y. Then $Y - V$ is $\pi g^{\mu} r$-closed in Y. Since Y is $\pi g^{\mu} r - T_{1/2}$-space, $Y - V$ is supra regular closed in Y. Thus V is supra open in Y. Since f is $totally^{\mu}$ continuous, $f^{-1}(V)$ is $cl^{\mu}open^{\mu}$ in X.
Therefore f is totally $\pi g^{\mu} r$-continuous.

□

Proposition 3.2. *The composition of two totally $\pi g^{\mu} r$-continuous function is totally $\pi g^{\mu} r$-continuous.*

Proof. Obvious. □

Proposition 3.3. *(i) If $f : X \to Y$ is totally $\pi g^{\mu} r$-continuous and $g : Y \to Z$ is $\pi g^{\mu} r$-irresolute, then $g \circ f : X \to Z$ is totally $\pi g^{\mu} r$ -continuous.*

(ii) If $f : X \to Y$ is totally $\pi g^{\mu} r$-continuous and $g : Y \to Z$ is $\pi g^{\mu} r$-continuous, then $g \circ f : X \to Z$ is $totally^{\mu}$ continuous.

Proof. (i) Let V be a $\pi g^{\mu} r$-open set in Z. Then $\pi g^{\mu} r$-irresoluteness of g implies $g^{-1}(V)$ is $\pi g^{\mu} r$-open in Y. Since f is the totally $\pi g^{\mu} r$- continuous, $f^{-1}(g^{-1}(V))$ is $cl^{\mu}open^{\mu}$ in Z.
Hence $g \circ f$ is totally $\pi g^{\mu} r$-continuous.

(ii) Similar to that of (i).

□

Theorem 3.4. *(i) Let $f : X \to Y$ be a $\pi g^{\mu} r$-open map and $g : Y \to Z$ be any function. If $g \circ f : X \to Z$ is totally $\pi g^{\mu} r$-continuous, then g is $\pi g^{\mu} r$-irresolute.*

(ii) Let $f : X \to Y$ be a strongly $\pi g^{\mu} r$-open map and $g : Y \to Z$ be any function. If $g \circ f : X \to Z$ is totally $\pi g^{\mu} r$-continuous, then g is $\pi g^{\mu} r$-irresolute.

Proof. (i) Let $g \circ f : X \to Z$ be a totally $\pi g^{\mu} r$-continuous function and let V be a $\pi g^{\mu} r$-open set in Z. Then $(g \circ f)^{-1}(V)$ is cl^{μ} $open^{\mu}$ in X. Hence $f^{-1}(g^{-1}(V))$ is cl^{μ} $open^{\mu}$ in X. Therefore $f^{-1}(g^{-1}(V))$ is supra open in X. Since f is $\pi g^{\mu} r$-open, $f(f^{-1}(g^{-1}(V)) = g^{-1}(V)$ is $\pi g^{\mu} r$-open in Y.
Hence g is $\pi g^{\mu} r$- irresolute.

(ii) Similar to that of the above.

□

4 Applications

Definition 4.1. *A supra topological space is said to be*

1. $\pi g^{\mu} r - T_0$*-space if for each distinct points in* X*, there exists a* $\pi g^{\mu} r$*-open set containing one point but not the other.*

2. $\pi g^{\mu} r - T_1$*-space if for each pair of distinct points* x *and* y *in* X*, there exist* $\pi g^{\mu} r$*-open sets* U *and* V *containing* x *and* y *respectively such that* $x \in U$*,* $y \notin U$ *and* $x \notin V$ *,*$y \in V$ *containing one point but not other.*

3. $\pi g^{\mu} r - T_2$*-space if every two distinct points of* X *can be separated by disjoint* $\pi g^{\mu} r$*-open sets.*

4. $\pi g^{\mu} r$*-normal if for each pair of disjoint closed sets* U *and* V *of* X*, there exist two disjoint* $\pi g^{\mu} r$*-open sets G and H such that* $U \subset G$ *and* $V \subset H$*.*

5. $\pi g^{\mu} r$*-regular if for each* $\pi g^{\mu} r$*-closed set F of* X *and each* $x \notin F$*, there exist two disjoint* $\pi g^{\mu} r$*-open sets* U *and* V *such that* $F \subset U$ *and* $x \in V$*.*

6. $\pi g^{\mu} r$*-connected if X is not the union of two disjoint non-empty* $\pi g^{\mu} r$*-open subsets of* X*.*

Theorem 4.1. *If* $f : X \to Y$ *is totally* $\pi g^{\mu} r$*-continuous injection and* Y *is* $\pi g^{\mu} r - T_0$*, then* X *is* $Ultra^{\mu}$ *Hausdorff.*

Proof. Let x and y be any pair of distinct points of X. Then $f(x) \neq f(y)$ in Y. Since Y is $\pi g^{\mu} r - T_0$, there exists a $\pi g^{\mu} r$-open set U containing $f(x)$ but not $f(y)$. Then $x \in f^{-1}(U)$ and $y \notin f^{-1}(U)$. Since f is totally $\pi g^{\mu} r$-continuous, $f^{-1}(U)$ is cl^{μ} $open^{\mu}$ in X. Also $x \in f^{-1}(U), y \notin f^{-1}(U)$.
(i.e) $y \in X - f^{-1}(U)$. This implies every pair of distinct points of X can be separated by disjoint cl^{μ} $open^{\mu}$ sets in X. Therefore, X is $Ultra^{\mu}$ Hausdorff. □

Theorem 4.2. *If* $f : X \to Y$ *is totally* $\pi g^{\mu} r$*-continuous injection and* Y *is* $\pi g^{\mu} r - T_1$*, then* X *is* cl^{μ} $open^{\mu} - T_1$*.*

Proof. Let x and y be any two distinct points of X. Then $f(x) \neq f(y)$. As Y is $\pi g^{\mu} r - T_1$, there exists $\pi g^{\mu} r$-open sets U and V of Y such that $f(x) \in U$, $f(y) \notin U$ and $f(y) \in V$, $f(x) \notin V$. Therefore, we have $x \in f^{-1}(U), y \notin f^{-1}(U)$ and $y \in f^{-1}(V)$, $x \notin f^{-1}(V)$. Since f is totally $\pi g^{\mu} r$-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are cl^{μ} $open^{\mu}$ in X. Hence X is $cl^{\mu} open^{\mu} - T_1$. □

Theorem 4.3. *If* $f : X \to Y$ *is totally* $\pi g^{\mu} r$*-continuous injection and* Y *is* $\pi g^{\mu} r - T_2$*, then* X *is* $Ultra^{\mu}$ *Hausdorff.*

Proof. Let x and y be any pair of distinct points of X. Then $f(x) \neq f(y)$ in Y. Since Y is $\pi g^{\mu}r - T_2$, there exist disjoint $\pi g^{\mu}r$-open sets U and V such that $f(x) \in U$ and $f(y) \in V$. Since f is totally $\pi g^{\mu}r$-continuous, $f^{-1}(U)$ is cl^{μ} $open^{\mu}$ in X. Also $x \in f^{-1}(U)$ and $y \notin f^{-1}(U)$. (i.e) $y \in X - f^{-1}(U)$. This implies every pair of distinct points of X can be separated by disjoint $cl^{\mu}open^{\mu}$ sets in X. Therefore, X is $ultra^{\mu}$ Hausdorff. □

Theorem 4.4. *If $f : X \to Y$ is totally $\pi g^{\mu}r$-continuous, supra closed injection and Y is $\pi g^{\mu}r$-normal, then X is $Ultra^{\mu}$ normal.*

Proof. Let F_1 and F_2 be disjoint supra closed subsets of X. Since f is supra closed, $f(F_1)$ and $f(F_2)$ are disjoint supra closed in Y. Since Y is $\pi g^{\mu}r$-normal, $f(F_1)$ and $f(F_2)$ are separated by disjoint $\pi g^{\mu}r$-open subsets G_1 and G_2 of Y. Then $F_1 \subset f^{-1}(G_1)$ and $F_2 \subset f^{-1}(G_2)$. As f is totally $\pi g^{\mu}r$-continuous, $f^{-1}(G_1)$ and $f^{-1}(G_2)$ are cl^{μ} $open^{\mu}$ sets in X. Also $f^{-1}(G_1) \cap f^{-1}(G_2) = f^{-1}(G_1 \cap G_2) = \phi$. Thus each pair of non-empty supra closed sets in X can be separated by disjoint cl^{μ} $open^{\mu}$ sets in X. Hence X is $Ultra^{\mu}$ normal. □

Theorem 4.5. *If $f : X \to Y$ is totally $\pi g^{\mu}r$-continuous, surjection and X is supra connected, then Y is $\pi g^{\mu}r$-connected.*

Proof. Suppose Y is not $\pi g^{\mu}r$-connected. Let A and B form disconnection in Y. Then A and B are $\pi g^{\mu}r$-open sets in Y and $Y = A \cup B$, where $A \cap B = \phi$. Also $X = f^{-1}(Y) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are non-empty $cl^{\mu}open^{\mu}$ sets and hence supra open in X, as f is totally $\pi g^{\mu}r$-continuous. Further $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = \phi$. Then X is not supra connected, which is a contradiction. Therefore Y is $\pi g^{\mu}r$-connected. □

Theorem 4.6. *If $f : X \to Y$ is totally $\pi g^{\mu}r$-continuous and $\pi g^{\mu}r$-open injective map from a $cl^{\mu}open^{\mu}$ regular space X onto Y, then Y is $\pi g^{\mu}r$-regular.*

Proof. Let F be a $cl^{\mu}open^{\mu}$ set in Y. Then F is $\pi g^{\mu}r$-open and $\pi g^{\mu}r$-closed in Y. Let $y \notin F$. Take $y = f(x)$. Since f is totally $\pi g^{\mu}r$-continuous, $f^{-1}(F)$ is $cl^{\mu}open^{\mu}$ in X. Let $G = f^{-1}(F)$. Then we have $x \notin G$. Since X is $cl^{\mu}open^{\mu}$ regular, there exists disjoint supra open sets U and V such that $G \subset U$ and $x \in V$. This implies $F = f(G) \subset f(U)$ and $y = f(x) \in f(V)$. Further, since f is a $\pi g^{\mu}r$-open map, $f(U \cap V) = f(\phi) = \phi$ and $f(U)$ and $f(V)$ are $\pi g^{\mu}r$-open sets in Y. Thus for each supra closed set F in Y and each $y \notin F$, there exists disjoint $\pi g^{\mu}r$-open sets $f(U)$ and $f(V)$ in Y such that $F \subset f(U)$ and $y \in f(V)$. Therefore Y is $\pi g^{\mu}r$-regular. □

Theorem 4.7. *If $f : X \to Y$ is totally $\pi g^{\mu}r$-continuous and supra closed injective map. If Y is $\pi g^{\mu}r$- regular, then X is $Ultra^{\mu}$ regular.*

Proof. Let F be a supra closed set not containing x. Since f is supra closed, $f(F)$ is supra closed in Y not containing $f(x)$. Since Y is $\pi g^{\mu}r$-regular, there exist disjoint $\pi g^{\mu}r$-open sets A and B such that $f(x) \in A$ and $f(F) \subset B$. The above implies $x \in f^{-1}(A)$ and $F \subset f^{-1}(B)$. As f is totally $\pi g^{\mu}r$-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are $cl^{\mu}open^{\mu}$ in X and $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = f^{-1}(\phi) = \phi$. Thus for a point x and a supra closed set not containing x are separated by disjoint $cl^{\mu}open^{\mu}$ sets $f^{-1}(A)$ and $f^{-1}(B)$. Therefore X is $Ultra^{\mu}$ regular. □

Theorem 4.8. *If $f : X \to Y$ is totally $\pi g^{\mu}r$-continuous and $\pi g^{\mu}r$-closed injective map. If Y is $\pi g^{\mu}r$- regular, then X is $Ultra^{\mu}$ regular.*

Proof. Let F be a supra closed set not containing x. Since f is $\pi g^{\mu}r$-closed, $f(F)$ is $\pi g^{\mu}r$-closed in Y not containing $f(x)$. Since Y is $\pi g^{\mu}r$-regular, there exist disjoint $\pi g^{\mu}r$-open sets A and B such that $f(x) \in A$ and $f(F) \subset B$. The above implies $x \in f^{-1}(A)$ and $F \subset f^{-1}(B)$. As f is totally $\pi g^{\mu}r$-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are $cl^{\mu}open^{\mu}$ in X and $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = f^{-1}(\phi) = \phi$. Thus for a point x and a supra closed set F not containing x are separated by disjoint $cl^{\mu}open^{\mu}$ sets $f^{-1}(A)$ and $f^{-1}(B)$. Therefore X is $Ultra^{\mu}$ regular. □

Theorem 4.9. *If $f : X \to Y$ is $totally^{\mu}$ continuous and $\pi g^{\mu}r$-open injective map from a $cl^{\mu}open^{\mu}$ normal space X onto a space Y, then Y is $\pi g^{\mu}r$-normal.*

Proof. Let F_1 and F_2 be two disjoint supra closed sets in Y. Since f is $totally^{\mu}$ continuous, $f^{-1}(Y - F_1) = X - f^{-1}(F_1)$ and $f^{-1}(Y - F_2) = X - f^{-1}(F_2)$ are $cl^{\mu}open^{\mu}$ subsets of X. That is $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are $cl^{\mu}open^{\mu}$ in X. Take $U = f^{-1}(F_1)$ and $V = f^{-1}(F_2)$. Then $U \cap V = f^{-1}(F_1) \cap f^{-1}(F_2) = f^{-1}(F_1 \cap F_2) = f^{-1}(\phi) = \phi$. Since X is $cl^{\mu}open^{\mu}$-normal, there exist disjoint supra open sets A and B such that $U \subset A$ and $V \subset B$. This implies $F_1 = f(U) \subset f(A)$ and $F_2 = f(V) \subset f(B)$. Further, since f is $\pi g^{\mu}r$-open, $f(A)$ and $f(B)$ are disjoint $\pi g^{\mu}r$-open in Y. Thus each pair of disjoint supra closed sets in Y can be separated by disjoint $\pi g^{\mu}r$-open sets. Therefore Y is $\pi g^{\mu}r$-normal. □

Definition 4.2. *A supra topological space X is called a $\pi g^{\mu}rc$-normal if for each pair of disjoint $\pi g^{\mu}r$-closed sets U and V of X, there exist two disjoint $\pi g^{\mu}r$-open sets G and H such that $U \subset G$ and $V \subset H$.*

Theorem 4.10. *If $f : X \to Y$ is totally $\pi g^{\mu}r$-continuous and $\pi g^{\mu}r$-open injective map from a $cl^{\mu}open^{\mu}$ normal space X onto a space Y, then Y is $\pi g^{\mu}rc$ -normal.*

Proof. Let F_1 and F_2 be two disjoint $\pi g^{\mu}r$-closed sets in Y. Since f is totally $\pi g^{\mu}r$-continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are disjoint $cl^{\mu}open^{\mu}$ subsets of X. Take $U = f^{-1}(F_1)$ and $V = f^{-1}(F_2)$. Since f is injective, $U \cap V = f^{-1}(F_1) \cap f^{-1}(F_2) = f^{-1}(F_1 \cap F_2) = f^{-1}(\phi) = \phi$. Since X is $cl^{\mu}open^{\mu}$-normal, there exist disjoint supra open sets A and B such that $U \subset A$ and $V \subset B$. This implies $F_1 = f(U) \subset f(A)$ and $F_2 = f(V) \subset f(B)$. Further, since f is $\pi g^{\mu}r$-open, $f(A)$ and $f(B)$ are disjoint $\pi g^{\mu}r$-open sets in Y. Thus each pair of disjoint $\pi g^{\mu}r$-closed sets in Y can be separated by disjoint $\pi g^{\mu}r$-open sets. Therefore Y is $\pi g^{\mu}r$c-normal. □

Definition 4.3. *A function $f : X \to Y$ is said to be totally $\pi g^{\mu}r$-open if the image of every $\pi g^{\mu}r$-open set in X is $cl^{\mu}open^{\mu}$ in Y.*

Theorem 4.11. *If a bijective function $f : X \to Y$ is said to be totally $\pi g^{\mu}r$-open, then the image of every $\pi g^{\mu}r$-open set in X is $cl^{\mu}open^{\mu}$ in Y.*

Proof. Let F be a $\pi g^{\mu}r$-closed set in X. Then$X - F$ is $\pi g^{\mu}r$-open in X. Since f is totally $\pi g^{\mu}r$-open, $f(X - F) = Y - f(F)$ is $cl^{\mu}open^{\mu}$ in Y. Hence $f(F)$ is $cl^{\mu}open^{\mu}$ in Y. □

Theorem 4.12. *A surjective function $f : X \to Y$ is totally $\pi g^{\mu}r$-open iff for each subset B of Y and for each $\pi g^{\mu}r$-closed set U containing $f^{-1}(B)$, there is a $cl^{\mu}open^{\mu}$ set V of Y such that$B \subset V$ and $f^{-1}(V) \subset U$.*

Proof. Suppose $f : X \to Y$ be a surjective totally $\pi g^{\mu}r$-open function and $B \subset Y$. Let U be a $\pi g^{\mu}r$-closed set of X such that $f^{-1}(B) \subset U$. Then $V = Y - f(X - U)$ is $cl^{\mu}open^{\mu}$ subset of Y containing B such that $f^{-1}(V) \subset U$.

Conversely, let F be a $\pi g^{\mu}r$-closed set of X. Then $f^{-1}(Y - f(F)) \subset X - F$ is $\pi g^{\mu}r$-open. By hypothesis, there exists a $cl^{\mu}open^{\mu}$ set V of Y such that $Y - f(F) \subset V$, which implies $f^{-1}(V) \subset X - F$. Therefore $F \subset X - f^{-1}(V)$.

Hence $Y - V \subset f(F) \subset f(X - f^{-1}(V)) \subset Y - V$, which is $cl^{\mu}open^{\mu}$ in Y. Thus the image of a $\pi g^{\mu}r$-open set in X is $cl^{\mu}open^{\mu}$ in Y. Therefore f is totally $\pi g^{\mu}r$-open function. □

Theorem 4.13. *For any bijective function $f : X \to Y$, the following statements are equivalent.*

(i) f^{-1} is totally $\pi g^{\mu}r$-continuous.

(ii) f is totally $\pi g^{\mu}r$-open.

Proof. $(i) \Rightarrow (ii)$: Let U be a $\pi g^{\mu}r$-open set of X. By assumption $(f^{-1})^{-1}(U) = f(U)$ is $cl^{\mu}open^{\mu}$ in Y. So, f is totally $\pi g^{\mu}r$-open.

$(ii) \Rightarrow (i)$: Let V be a $\pi g^{\mu}r$-open in X. Then by assumption, $f(V)$ is $cl^{\mu}open^{\mu}$ in Y. Thus $(f^{-1})^{-1}(V)$ is $cl^{\mu}open^{\mu}$ in Y and hence f^{-1} is totally $\pi g^{\mu}r$-continuous. □

Theorem 4.14. *The composition of two $\pi g^{\mu}r$-totally open function is totally $\pi g^{\mu}r$-open.*

Proof. Obvious. □

Theorem 4.15. *If $f : X \to Y$ is $\pi g^{\mu}r$-irresolute and $g : Y \to Z$ is totally $\pi g^{\mu}r$-continuous, then $g \circ f : X \to Z$ is $\pi g^{\mu}r$-irresolute.*

Proof. Let V be a $\pi g^{\mu} r$-open set in Z. Since g is totally $\pi g^{\mu} r$-continuous, $g^{-1}(V)$ is $cl^{\mu}open^{\mu}$ in Y. Then $g^{-1}(V)$ is $\pi g^{\mu} r$-open in Y. As f is $\pi g^{\mu} r$-irresolute, $f^{-1}(g^{-1}(V) = (g \circ f)^{-1}(V)$ is $\pi g^{\mu} r$-open in X. Hence $(g \circ f)$ is $\pi g^{\mu} r$-irresolute. □

Theorem 4.16. *Let $f : X \to Y$ and $g : Y \to Z$ be two functions such that $g \circ f : X \to Z$ is totally $\pi g^{\mu} r$-open function. Then*

(i) If f is $\pi g^{\mu} r$-irresolute and surjective, then g is totally $\pi g^{\mu} r$-open.

(ii) If g is totally$^{\mu}$ continuous and injective, then f is totally $\pi g^{\mu} r$-open.

Proof. (i) Let V be a $\pi g^{\mu} r$-open set in Y. By hypothesis, $f^{-1}(V)$ is $\pi g^{\mu} r$-open in X. Again, since $(g \circ f)$ is totally $\pi g^{\mu} r$-open,
$((g \circ f)(f^{-1}(V))) = g(V)$ is $cl^{\mu}open^{\mu}$ in Z. Hence g is totally $\pi g^{\mu} r$-open.

(ii) Let V be a $\pi g^{\mu} r$-open set in X. Since $(g \circ f)$ is totally $\pi g^{\mu} r$-open, $(g \circ f)(V)$ is $cl^{\mu}open^{\mu}$ in Z. Then $(g \circ f)(V)$ is supra regular open and hence $\pi g^{\mu} r$-open in Z. As g is $totally^{\mu}$ continuous, $g^{-1}((g \circ f)(V)) = f(V)$ is $cl^{\mu}open^{\mu}$ in Y. Hence f is totally $\pi g^{\mu} r$-open.

□

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