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Analysis of Poisson repeated queue general service with set up time and different vacation policies

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Abstract

This paper deals with unreliable M/G/1 queueing system with different vacation policies, non-persistent customers and repeated attempts. The jobs arrive in Poisson fashion. On finding the server busy, under setup, under repair or on vacation, the jobs either join to the orbit or balk from the system. The jobs in the orbit repeat their request for service after some random time. The inter retrial time of each job is general distributed. The jobs are served according to FCFS discipline. If the primary call, on arrival finds the server busy, it becomes impatient and leaves the system with probability $(1-\alpha)$ and with probability α, it enters into an orbit. The server provides preliminary first essential service (FES) and followed by second essential service (SES) to primary arriving calls or calls from the retrial group. On completion of SES the server may go for $i^{th}(i=1,2,3\cdots,M)$ type of vacation with probability $\beta_i(i=1,2,3,...,M)$ or may remain in the system to serve the next call, if any, with probability β_0 where $\sum_{i=0}^{M}\beta_i=0$.

Keywords: Repeated Queue with Non-Obstinate Customers, Two Phase, Set up time and Different Vacation Policies.

2010 MSC: 54A10, 54A20.

1 Introduction

In many waiting line systems, the role of server is played by mechanical/electronic device, such as computer, pallets, ATM, traffic light, etc., which is subject to accidental random failures.

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Queuing networks have great popularity as models of computer systems since 1971 because they allow modeling of multiple independent resources. Queueing systems are powerful tool for modeling communication networks, transportation networks, production lines, operating systems, etc. In recent years, computer networks and data communication systems are the fastest growing technologies, which lead to glorious development in many applications. For example, the swift advance in Internet, audio data traffic, video data traffic,etc.

D. Arivudainambi et.al has studied a Batch Arrival Retrial Queue Retrial queueing system are characterized by the fact that arriving customer who finds the server busy is to leave the service area and repeat his demand after some time called retrial time. The server works continuously as long as there is at least one customer in the system. When the server finishes serving a customer and finds the system empty, it goes away for a length of time called a vacation. For example, maintenance activities, telecommunication networks, customized manufacturing, production systems, etc.

When no customers are found in the orbit, the server goes on a first essential vacation. After first essential vacation, the server may either wait idle for customers or the server may take one of Type k (k = 1, 2, . . .K) vacations. At an optional vacation completion epoch, the server waits for the customers, if any in the orbit or for new customers to arrive.

Accidental (operational) breakdown of the server is also considered. There is a provision of k-phase repairs to restore the server to the state as before failure. First phase repair is essential whereas other (K-1) phases are optional. Madhu Jain et.al [2] focus on the Unreliable Server M/G/1 Queueing System. The repairman, who restores the server, requires some time to start the first phase of repair; this time is called as setup time. The service time, setup time and repair time of each phase are independent and general distributed. The probability generating function of steady state queue size at random epoch is obtained using supplementary variable technique.

We assume that only the job at the head of the queue is allowed to occupy the server if it is free. After some random time the blocked jobs return to repeat their request. Several studies on retrial queues have been made by many researchers working in the area of applied probability theory, from time to time. In almost all models of retrial queues, the time between retrials for any job is assumed to be exponentially distributed. The general retrial time policy arises naturally in many congestion problems related to many service systems where, after each service completion, the server spends a random amount of time in order to find the next job to be processed. In recent years there has been an increasing interest in the study of retrial queue with general retrial time.

Queueing systems with vacations have also found wide applicability as realized in retrial and feedback models for the analysis of computer and communication network and several other engineering systems. Vacation models are explained by their scheduling disciplines, according to which when a service stops, a vacation starts. A single server retrial queueing system with server vacations, no waiting space and finite population of the customers. M/G/1 queueing system with multiple types of feedback, gated vacations and obtained joint probability generating function of the system size at steady state.

Retrial queues have been widely used to model many problems arising in telephone switching systems, telecommunication networks, computer networks and computer systems. Retrial queueing models are characterized by the feature that arriving calls which find a server busy, do not line up or leave the system immediately forever, but go to some virtual place called as orbit and try their luck again after some random time. During the last two decades considerable attention has been paid to the analysis of queueing system with repeated calls (also called retrial queues or queues with returning customers.

In this paper, we examine the steady state behavior of an M/G/1 retrial queueing system with two phases of service, the first service being an essential service provided to all arriving customers and the second service is an optional service provided to some of the customers. A customer who finds the server busy either leaves the system with probability$(1 - \alpha)$ or joins the orbit with a probability α. Upon completion of each service, the server can remain in the system with a probability β_0 or may leave for the i^{th} type vacation with a probability $\beta_i(1 \leq i \leq M)$ and $\sum_{i=0}^{M} \beta_i = 1$.

In this paper, we consider an M/G/1 retrial queue with a second optional phase of service. Our main motivation is coming from some applications to telephone systems. We pretend to model stochastically those situations arising in daily life when a person making a phone call experiments that his service time consists in one preliminary service phase maybe followed by a second service phase.

2 Mathematical Modeling

In this section, a single server retrial queueing system is considered. The primary customers arrive according to a Poisson process with rate λ. If a primary customer, on arrival finds the server busy, he becomes non-persistent and leaves the system with probability $(1-\alpha)$ or with probability α, he enters into an orbit. The retrial times of the individual customers are assumed to be i.i.d random variables with a distribution function $1 - e^{-\mu x}$. The server provides a first essential service (FES) to all arriving customers. As soon as the FES of a customer is completed, then the customer may leave the system with probability 1-r or may opt for the second optional service (SOS) with probability r. The service times S1 of the FES and S2 of the SOS are independent random variables having general distributions with distribution functions $B1(.), B2(.)$, Laplace Stieljes transforms (LST) $\beta_1^*(.), \beta_2^*(.)$

respectively. The total service time of a customer in the system is therefore given by

$$S = \begin{cases} S_1 + S_2 & \text{with probability r} \\ S_2 & \text{with probability 1-r} \end{cases}$$

The total service time S has a distribution function B(.) and has LST

$$B^*(s) = (1-r)B_1^*(s) + rB_1^*(s)B_2^*(s).$$

Let b_1, b_2 denote the expected values of the FES and SOS respectively. Then

$$E(S) = b_1 + rb_2$$
$$E(S^2) = E(S_1^2) + E(S_2^2) + 2rb_1b_2.$$

Let v(x) denote the hazard function of the service time,i.e $v(x) = \frac{B'(x)}{1-B(x)}$ As soon as the service is completed, the server may go for the $i^{th}(i = 1, 2\cdots, M)$ type of vacation with probability β_i,or may remain in the system to serve the next customer, if any, with probability β_0 where $\sum_{i=0}^{M} \beta_i = 1$. Let V_k be the duration of the server vacation time in the k^{th} vacation scheme and let $V_k(x)$ and $v_k(x)$ denote the distribution function and the hazard rate function respectively of the random variable $V_k(1 \le k \le M)$. We assume that the interarrival times, retrial times, vacation times and service times are mutually independent of each other.

As soon as the service of the jobs is completed, the server deactivates and takes at most J vacations repeatedly until at least one job recorded in the orbit upon returning from a vacation. If one or more jobs are present in the orbit in case when the server returns from a vacation, the server reactivates otherwise goes back for subsequent vacation. If no job is present in the orbit at the end of J^{th} vacation, the server remains dormant until at least one job arrives in the orbit. We assume that j^{th} phase $(j = 1, 2\cdots, J)$ vacation time follows general distribution law.

The repairman, who restores the server, requires some time to start the first phase of repair; this time is called as setup time R(t). The service time, setup time and repair time of each phase are independent and general distributed. Let N(t) denote the number of customers in the orbit at time t. The server state is denoted by,

$$c(t) = \begin{cases} 0 & \text{if the server is idle} \\ 1 & \text{if the server is doing FES or SOS} \\ 2 & \text{if the server is on vacation} \end{cases}$$

We also note that the state of the system at time can be described by means of the process $Y(t) = (C(t), N(t), \xi(t))$, where C(t) is equal to 0,1 or 2 according to whether the server is idle, busy with FES or busy with SOS at time t,N(t) denotes the number of customers in orbit at time t. If $c(t) \in \{1, 2\}$ then $\xi(t)$ represents the corresponding elapsed time of the service in progress

3 Steady State System Size Distributions

We assume that the steady state condition $p/q < 1$ is fulfilled, so that we can set

$$P_0 = lim_{t\to\infty} P_0(t)$$

$$P_n(x) = lim_{t\to\infty} P_0(t,x)$$

for $x \geq 0$ and $n \geq 1$

$$Q_n(x) = lim_{t\to\infty} P_0(t,x)$$

for $x \geq 0$ and $n \geq 1$.

By the method of supplementary variables, we obtain

$$\{\lambda + n\mu\}P_{0,n} = \beta_0 \int_0^\infty P_{1,n}(x)v(x)dx + \sum_{k=1}^{M} \int_0^\infty P_{2,n}^k(x)v_k(x)dx + \lambda C(x) \tag{3.1}$$

$$\frac{d}{dx(P_{1,n(x)})} = -\{\lambda\alpha + v(x)\}P_{1,n}(x) + (1-\delta_{n,0})\lambda\alpha P_{1,n-1}(x) \tag{3.2}$$

$$\frac{d}{dx(P_{2,n}^k(x))} = -\{\lambda\alpha + v_k(x)\}P_{2,n}^k(x) + (1-\delta_{n,0})\lambda\alpha P_{2,n-1}^k(x) \tag{3.3}$$

$$P_{1,n}(0) = \lambda P_{0,n} + (n+1)\mu P_{0,n+1} \tag{3.4}$$

$$P_{2,n}^k(0) = \beta_k \int_0^\infty P_{1,n}(x)v(x)dx \tag{3.5}$$

$$R_n(0) = -\{\lambda\alpha + R(x)\}P_{0,x} + \beta_k \int_0^\infty P_{2n}C(x)dx \tag{3.6}$$

Now we define the following probability generating functions for $|z| \leq 1$ and $x \geq 0$

$$P_0(z) = \sum_{n=0}^{\infty} P_{0,n} z^n$$

$$P_1(x,z) = \sum_{n=0}^{\infty} P_{1,n} z^n$$

$$P_2^k(x,z) = \sum_{n=0}^{\infty} P_{2,n}^k(x) z^n$$

$$R_0(z) = \sum_{n=0}^{\infty} R_{0,n}(x) z^n$$

The above functions along with (3.1) to (3.6) give us the following result.

$$\lambda P_0(z) + z\mu P_0(z) = \beta \int_0^\infty p_1(x,z)v(x)dx + \sum_{k=1}^{M} \int_0^\infty P_2^k(x,z)v_k(x)dx + \lambda R_n(x) \tag{3.7}$$

$$\frac{\partial}{\partial x(P_1(x,z))} = -\{\lambda\alpha + v(x)\}P_1(x,z) + \lambda\alpha z P_1(x,z) \tag{3.8}$$

$$\frac{\partial}{\partial x(P_2^k(x,z))} = -\{\lambda\alpha + v_k(x)\}P_2^k(x,z) + \lambda\alpha z P_2^k(x,z) \qquad \text{for k=1,2,...,M} \tag{3.9}$$

$$P_1(0,z) = \lambda P_0(z) + \mu P_0^1(z) \tag{3.10}$$

$$P_2^k(0,z) = \beta \int_0^k P_1(x,z)v(x)dx \quad fork = 1,2,...,M \tag{3.11}$$

$$R_n(0,z) = -\{\lambda\alpha + R(x)\}p_0 + \beta_k \int_0^\infty p_2(x,z)C(x)dx \tag{3.12}$$

From (3.8)and (3.9)

$$P_1(x,z) = P_1(0,Z)e^{-\lambda\alpha(1-z)x}(1 - B(x)) \tag{3.13}$$

$$P_2^k(x,z) = P_2^k(0,z)e^{-\lambda\alpha(1-z)x}(1 - v_k(x)) \qquad \text{for k=1,2,...,M} \tag{3.14}$$

Define

$$P_1(z) = \int_0^\infty P_1(x,z)dx \tag{3.15}$$

$$P_2^k(z) = \int_0^\infty P_2^k(x,z)dx \tag{3.16}$$

$$R_0(z) = \int_0^\infty R_0(x,z)dx \tag{3.17}$$

Therefore

$$P_1(z) = \frac{1 - \beta^*(\lambda\alpha(1-z))}{\lambda\alpha(1-z)}\{\lambda P_0(z) + \mu P_0(z)\} \tag{3.18}$$

$$P_2^k(z) = \frac{1 - v_k^*(\lambda\alpha(1-z))}{\lambda\alpha(1-z)}\{\lambda P_0(z) + \mu P_0(z)\}\beta_k B^*(\lambda\alpha(1-z)) \qquad \text{for k=1,2,...,M} \tag{3.19}$$

From(3.7)

$$P_0(z) = P_0(1)exp\left\{\frac{\lambda}{\mu}\int_1^z \frac{1 - S(u)}{S(u) - u}du\right\}\left\{\beta_0 + \sum_{k=1}^{M}\beta_k v^*(\lambda\alpha(1-z))\right\} \tag{3.20}$$

where $S(z) = B^*(\lambda\alpha(1-z))$ and $B^*(\lambda\alpha(1-z)) = \{(1-r) + rB_2(\lambda\alpha - \lambda\alpha z)\}B_1^*(\lambda\alpha - \lambda\alpha z)$

Now the probability generating function of the orbit size is

$$P(z) = P_0(z) + P_1(z) + \sum_{k=1}^{M} P_2^k(z) + R_0(z) = \left\{\frac{\alpha(R(z) - z) + 1 - R(z)}{\alpha(R(z) - z)}\right\}P_0(z) \tag{3.21}$$

From the normalizing condition,$lim_{z\to 1}P(z) = 1$we get

$$P_0(1) = \frac{1 - \rho\alpha}{1 + \rho(1-\alpha)} \tag{3.22}$$

where $\rho = \lambda\{rb_2 + b_1 + \sum_{k=1}^{M} \beta_k E[v_k]\}$ Hence

$$P(z) = \left\{ \frac{\alpha(S(z)-z) + (1-s(z))(1-R(z))}{\alpha(S(z)-z)} \right\} \frac{1-\rho\alpha}{1+\rho(1-\alpha)} e^{\frac{\lambda}{\mu}\int_1^z \frac{1-S(u)}{S(u)-u} du} \tag{3.23}$$

Let K(z) be the PGF of the system size distribution. Then

$$K(z) = \left\{ \frac{(\alpha-1)(S(z)-z) + (1-z)B^*(\lambda\alpha(1-z)) + R^*(1-z)}{\alpha(S(z)-z)} \right\} P_0(z) \tag{3.24}$$

4 Stochastic Decomposition

Let K_∞ be the random variable which denotes the system size at time t in the steady state in the classical M/G/1 queue with non-obstinate customers and different vacation policies. It can be verified that the probability generating function of K_∞ is given by

$$K_\infty(z) = \frac{(\alpha-1)[S(z)-z] + (1-z)B^*(\lambda\alpha(1-z)) + R^*(1-z)}{\alpha(S(z)-z)} \frac{1-\rho\alpha}{1+\rho(1-\alpha)} \tag{4.1}$$

Now we introduce a random variable S_μ with the generating function

$$E(z^s\mu) = exp\left\{ \frac{\lambda}{\mu} \int_1^z \frac{1-S_\mu}{S(u)-u} du \right\} \tag{4.2}$$

The right hand side of the above equation is $\frac{P_0(z)}{P_0(1)}$. The distribution of the random variable S_μ coincides with the distribution of the number of sources of repeated calls given that the server is free. From (24), (25) and (26), we see that

$$K(z) = K_\infty(z)E(z^S\mu)$$

Thus the random variable S_μ explains the increase in the size of the system due to the presence of retrials.

5 Performance Measures

Some useful results of our model are listed below. (a) The mean number of customers in the orbit

$$L_q = P(1) = \frac{\lambda}{\mu(\frac{\rho\alpha}{1-\rho\alpha})} + \frac{\lambda^2\alpha\gamma*}{2(1-\rho\alpha)(1+\rho(1-\alpha))}$$

$$\text{where}\gamma* = E[S^2] + 2E[S]\sum_{k=1}^{M}\beta_k E[v_k] + \sum_{k=1}^{M}\beta_k E[V_k^2]$$

(b) The blocking probability that an arriving customers finds the server busy or away on vacation

$$b = 1 - P_0(1) = \frac{\rho}{1+\rho(1-\alpha)} \tag{5.1}$$

(c) The mean waiting time in the orbit

$$W_q = \frac{L_q}{\lambda\alpha} \tag{5.2}$$

(d) The mean number of customers in the system

$$L = K(1) = \frac{\lambda}{\mu}\frac{\rho\alpha}{(1-\rho\alpha)} + \frac{\lambda E(S)}{1+\rho(1-\alpha)} + \frac{\lambda^2\alpha\gamma*}{2(1-\rho\alpha)(1+\rho(1-\alpha))}$$

$$= \frac{\lambda}{\mu}\frac{\rho\alpha}{(1-\rho\alpha)} + E[v_\infty] \tag{5.3}$$

The expected increase in the congestion in the system due to the presence of retrials is therefore given by $\frac{\lambda}{\mu}\frac{\rho\alpha}{(1-\rho\alpha)}$

(e) The mean response time $W = \frac{L}{\lambda\alpha}$

(f) The steady state distribution of the server state is given by

$$\text{Prob}\{\text{server is idle}\} = P_0(1) = \frac{1-\rho\alpha}{1+\rho(1-\alpha)} \tag{5.4}$$

$$\text{Prob}\{\text{server is busy}\} = P_1(1) = \frac{\lambda(b_1+rb_2)}{1+\rho(1-\alpha)} \tag{5.5}$$

$$\text{Prob}\{\text{server is on vacation}\} = \sum_{k=1}^{M} P_2^k(1) = \frac{\lambda\sum_{k=1}^{M}\beta_k E[v_k]}{1+\rho(1-\alpha)}. \tag{5.6}$$

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