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Strong total domination and weak total domination in Mycielski's graphs

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Abstract
Let $G=(V,E)$ be a graph. A set $S\subseteq V$ is called a *weak total dominating set* (WTD-set) if each vertex $v\in V-S$ is adjacent to a vertex $u\in S$ with $\deg(v)>\deg(u)$ and every vertex in S adjacent to a vertex in S. The *weak total domination number*, denoted by $\gamma_{wt}(G)$, is minimum cardinality of a weak total dominating set. Anologuosly, a dominating set $S\subseteq V$ is called a *strong total dominating set* (STD-set) if each vertex $v\in V-S$ is dominated by some vertices $u\in S$ with $\deg(v)<\deg(u)$ and each vertex in S adjacent to a vertex in S. The *strong total domination number*, denoted by $\gamma_{st}(G)$, is minimum cardinality of a strong dominating set. Weak total and strong total domination parameters were introduced by Chellali et al. and Akbari and Jafari Rad, respectively.
In this paper, we consider weak total and strong total domination of Mycielski's Graph, denoted by $\mu(G)$. We also provide some upper and lower bound about weak total domination of Mycielski's graph related with minimum and maximum degree number of a graph. In addition, the inequality about relationship between strong total domination of Mycieski's graph $\mu(G)$ and underlying graph G, $\gamma_{st}(G)+1\leq\gamma_{st}(\mu(G))\leq\gamma_{st}(G)+2$, is obtained. Among other results, we characterize graphs G achieving the lower bound $\gamma_{st}(G)+1=\gamma_{st}(\mu(G))$.

Keywords
Graph Theory, Strong Total Domination, Weak Total Domination, Mycielski's Graph.

AMS Subject Classification
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Contents

1. Introduction

Let G be n order connected simple graphs. $V(G)$ and $E(G)$ are vertex and edge set of G, respectively. The *open neighborhood of* $v\in V$ is $N_G(v)=\{u\in V:\ uv\in E(G)\}$ and *closed neighborhood* of $v\in V$ is $N_G[v]=N_G(v)\cup\{v\}$. A vertex $w\in V(G)$ is an *S-private neighbor* of $u\in S$ if $N[w]\cap S=\{u\}$, while the *S-private neighbor set of* u, denoted by $pn[u,S]$, is the set of all *S-private neighbors of* u. If v is a vertex of $V(G)$, then the *degree* of v denoted by $\deg_G(v)$, is the cardinality of its open neighborhood. The *maximum* and *minimum degree* of a graph G is denoted by $\Delta(G)=\Delta$ and $\delta(G)=\delta$, respectively.

A subset $S\subseteq V$ is a *dominating set of* G is every vertex in $V-S$ has a neighbor in S and the *domination number of* G, denoted by $\gamma(G)$ is the minimum cardinality of a dominating set. For detailed information about domination parameters readers are referred to books [6, 7]. A dominating set that is independent is called an *independent dominating set of* G. The minimum cardinality of an independent dominating set is called *independent domination number*, $i(G)$. A dominating set that is connected is called a *connected dominating set of* G. The minimum cardinality of an connected dominating set is called *connected domination number,* $\gamma_c(G)$. A *total dominating set*, denoted by TD-set of G with no isolated vertex is a set S of vertices of G and total domination number that is the minimum cardinality of a total dominating set denoted by $\gamma_t(G)$. Every graph without isolated vertices has

TD-set. Total domination was introduced by Cockayne et al. [4]. A dominating set $S \subseteq V$ is called a *weak dominating set* (WD-set) if each vertex $v \in V - S$ is dominated by some vertices $u \in S$ with $\deg(v) > \deg(u)$. The *weak domination number*, denoted by $\gamma_w(G)$, is minimum cardinality of a weak dominating set. Similarly, a dominating set $S \subseteq V$ is called a *strong dominating set* (SD-set) if each vertex $v \in V - S$ is dominated by some vertices $u \in S$ with $\deg(v) < \deg(u)$. The *strong domination number*, denoted by $\gamma_s(G)$, is minimum cardinality of a strong dominating set. The concept weak and strong domination number introduced by Sampathkumar and Pushpa Latha in [11]. A weak dominating set $S \subseteq V$ induces a subgraph with no isolated vertex is called *weak total dominating set* (WTD-set). The *weak total domination number,* $\gamma_{wt}(G)$ of G is minimum cardinality of WTD-set ($\gamma_{wt} - set$). Chellali et al. have introduced the parameter weak total domination number [3]. Analogously, the parameter *strong total domination number , denoted by* $\gamma_{st}(G)$, have been defined as minimum cardinality of *strong total dominating set* (γ_{st}-set) that is a strong dominating set $S \subseteq V$ induces a subgraph with no isolated vertex [1]. Also, in [1] Akbari and Jafari Rad have obtained Nordhaus-Gaddum bounds for weak and strong total domination number and in [8] complexity of strong and weak total dominations have been considered for some graphs.

Let G be n order graph and $\mu(G)$ is Mycielski's graph of G with$V(\mu(G)) = V(G) \cup V(G') \cup \{z\}$ where $V(G) = \{v_i : 1 \leq i \leq n\}$ is vertex set of G and $V(G') = \{v'_i : 1 \leq i \leq n\}$ is copy of the vertex set $V(G)$ and $E(\mu(G)) = E(G) \cup \{v_i v'_j \,:\, v_i v_j \in E(G)\} \cup \{v'_j z :\ \forall v'_j \in V(G')\}$. In addition, $\deg_{\mu(G)}(v_i) = 2\deg_G(v_i), \deg_{\mu(G)}(v'_i) = \deg_G(v_i) + 1$ and $\deg_{\mu(G)}(z) = n$. Mycielski's Graph of P_5 is given in Figure1 .

Some results have been obtained about domination parameters with respect to Mycielski's graph [2, 5, 9].

In this paper, we consider weak total and strong total domination of Mycielski's Graph. Also, we analysis graphs to get upper and lower bounds related with invariants of G such as minimum and the maximum degree of a graph. In the next section, we provide some useful results about parameters related with Mycielski's graph. In section 3, boundaries about weak total domination of Mycielski's graph is obtained. In Section 4, we obtain the inequality $\gamma_{st}(G) + 1 \leq \gamma_{st}(\mu(G)) \leq \gamma_{st}(G) + 2$ that is about relationship between strong total domination of Mycieski's graph and original graph. Also, we characterize graphs G achieving the lower bound $\gamma_{st}(G) + 1 = \gamma_{st}(\mu(G))$.

2. Known Results

In this section, we provide some known results about weak total and strong total domination in graphs and also some domination parameters related to Mycieski's graphs. Let begin with following illustration about strong total and weak total domination set

Example 2.1. *Let G be a connected graph with* 6 *vertices as in Figure 2* $S_1 = \{v_2, v_3, v_5, v_6\}$ *and* $S_2 = \{v_1, v_2, v_4, v_6\}$ *are some WTD-set of G and* $S = \{v_1, v_2\}$ *is a STD-set of G. Besides these sets, we can also generate some other WDT and STD-sets. In addition,* $\gamma_{wt}(G) = 4$ *and any* $\gamma_{wt} - set$ *of G must contain* v_2 *and* v_6 *of which degree smaller than their neighbors in the graph,* $\gamma_{st}(G) = 2$ *and any* $\gamma_{st} - set$ *of G must contain* v_1 *with degree larger than their neighbors in the graph.*

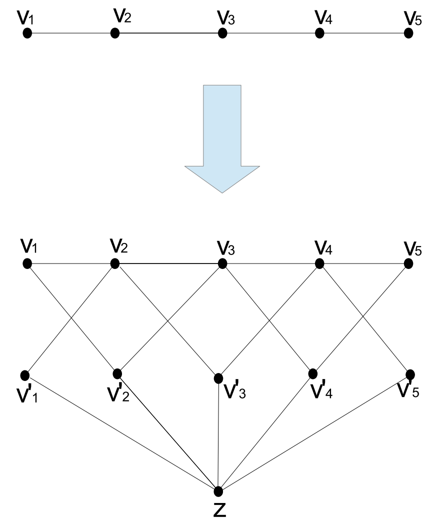

Figure 1. P_5 and Mycielski Graph of P_5

Theorem 2.2. *[5] For any graph G,* $\gamma(\mu(G)) = \gamma(G) + 1$.

Theorem 2.3. *[2] For any graph G,* $\gamma_s(\mu(G)) = \gamma_s(G) + 1$.

Theorem 2.4. *[2]For any graph G,* $\gamma_w(G) + 1 \leq \gamma_w(\mu(G)) \leq 2\gamma_w(G)$.

Theorem 2.5. *[5] Let G be a graph. Then* $\gamma(\mu(G)) = \gamma(G) + 1$, $\gamma_t(\mu(G)) = \gamma(G) + 1$.

Theorem 2.6. *[9]For any graph G,* $i(\mu(G)) = i(G) + 1$.

Theorem 2.7. *[9] If* $\gamma_c(G) \geq 3$ *then,* $\gamma_c(\mu(G)) \leq \gamma_c(G) + 1$.

Proposition 2.8. *[3] For any graph G with no isolated vertices,* $\gamma_{wt}(G) \leq n + 1 - \delta$.

Proposition 2.9. *[1]For paths and cycles,* $\gamma_{st}(P_n) = \gamma_{st}(C_n) = \gamma_t(P_n) = \gamma_t(C_n)$

Proposition 2.10. *[1]For any graph G of order n, maximum degree* Δ *and with no isolated vertices,* $\gamma_{st}(G) \leq n + 1 - \Delta$.

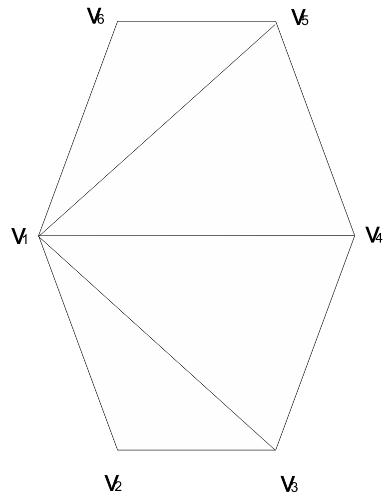

Figure 2. Any connected graph with 6 vertices

3. Weak Total Domination of Mycielski's Graph

In this section, we investigate some results about weak total domination number of Mycielski's Graph. We initiate with special graph families and also, we characterize graph attaining some bounds about weak total domination of Mycielski's graph.

Proposition 3.1. *For special graphs:*

a. $\gamma_{wt}(\mu(P_n)) = n+3,\ n \geq 5$

b. $\gamma_{wt}(\mu(C_n)) = n+1,\ n \geq 4$

c. $\gamma_{wt}(\mu(K_n)) = 3,\ n \geq 3$

d. $\gamma_{wt}(\mu(K_{1,n-1})) = 2n+1,\ n \geq 3$

e. $\gamma_{wt}(\mu(W_{1,n-1})) = n+1,\ n \geq 4$

f. $\gamma_{wt}(\mu(K_{m,n})) = m+n+1$.

Proof. a. Let $n \geq 5$ and $A = \{v_1, v_n, v'_1, v'_n\}$. Let D be $\gamma_{wt}-set$ of $\mu(P_n)$. Due to the degree of vertices in A, it must be $A \subseteq D$. However, A only weakly dominates vertices v_2 ,$v'_{2,} v_{n-1}, v'_{n-1}$ in $\mu(P_n)$. Also, $\forall v_i$ and v'_i in $\mu(P_n)$, $\deg(v_i) > \deg(v'_i)$ for i$\in \{2,...,n-1\}$ and all vertices in $V(G')$ are disjoint. The vertices $v'_{2,} v'_{n-1}$ are weakly dominated by A, remaining disjoint vertices v'_i, $i \in \{3,...,n-2\}$, is not dominated by A. In order to weakly dominates vertices of P'_n in $\mu(P_n)$, it is needed to add $n-4$ vertices from $V(G')$ to A. Thus, $|A| = n-4+4 = n$. All vertices in $\mu(P_n)$ are weakly dominated by A. To obtain $\gamma_{wt}-$set, it implies that $A \cup \{v_2,\ v_{n-1},\ z\}$ or $A \cup \{v'_2,\ v'_{n-1},\ z\}$. Hence, we get $|D| = n+3$ as desired.

b. $\forall$ v_i and v'_i in $\mu(C_n)$, $\deg(v'_i) = 3$ and $\deg(v_i) = 4$, $i \in \{1,...,n\}$. According to form of Mycielski's construction, vertices in $V(G')$ are disjoint. Let D be γ_{wt}-set of $\mu(C_n)$. It is obvious that $D = V(G') \cup \{z\}$. Thus, $\gamma_{wt}(\mu(C_n)) = n+1$.

c.For $G = K_n$, it is easy to see that $\gamma_{wt}(\mu(K_2)) = 2$. If $n \geq 3$, $\deg(z) = \deg(v'_i) = n$. Let D be γ_{wt}-set of $\mu(K_n)$. Since all vertices in $V(G')$ can be weakly dominated by z, then $z \in D$. In order to weakly total dominate all vertices in $V(G)$, it is needed to choose two vertices from $V(G')$. Assume that these vertices are v'_i and v'_j. Therefore, $D = \{v'_i\ ,\ v'_j,\ z\}$. Hence, $\gamma_{wt}(\mu(K_n)) = 3$.

d. For $G = K_{1,\ n-1} (n \geq 3)$; there are disjoint $2n-2$ vertices in $\mu(K_{1,n-1})$ which have minimum degree also these vertices are adjacent to the vertex v_i that has maximum degree. Thus, $\gamma_{wt}(\mu(K_{1,n-1})) = 2n-1$.

e. For $G = W_{1,\ n-1}$ $(n \geq 4)$; there are disjoint $n-1$ vertices in $V(G')$ which have minimum degree and also these vertices adjacent to z. Hence, $\mu(W_{1,n-1})$ weakly dominated by these n vertices. Thus, $\gamma_{wt}(\mu(W_{1,n-1})) = n$.

f. For $G = K_{m,n}$. It is easy to see that there are disjoint $m+n$ vertices which have $\min\{m,n\}$ and $\min\{m,n\}$ +1 degree in $V(G')$. Hence, for a γ_{wt}-set of $\mu(K_{m,n})$ contains at least $m+n+1$ vertices. Thus, $\gamma_{wt}(\mu(K_{m,n})) = m+n+1$. □

Theorem 3.2. *Let G be n order graph with* $\delta \geq 2$ *then* $3 \leq \gamma_{wt}(\mu(G)) \leq n+1$.

Proof. For lower bound it is not possible to weakly total dominate $\mu(G)$ by two vertices. For upper bound, always possible to obtain a WTD-set such as $V(G') \cup \{z\}$ provided that $\delta \geq 2$.

□

Observation 1: Let $|\delta|$ be number of minimum degree vertices. According to definition of weak domination $|\delta| \leq \gamma_w(G) \leq \gamma_{wt}(G)$.

Proposition 3.3. *Let G be n order graph* $n \geq 3$, $\delta = 1$ *then*

$$n+|\delta| \leq \gamma_{wt}(\mu(G)) \leq n+1+|\delta|$$

where $|\delta|$ *is number of minimum degree vertices.*

Proof. Let D be WTD-set of $\mu(G)$. D must be included all pendant vertices in G denoted by v_{δ_i} for $1 \leq i \leq |\delta|$. All support vertices in G has degree more than two in $\mu(G)$. According to form of $\mu(G)$, all vertices in $V(G')$ are disjoint vertices and also has degree less than all neighbors except v_{δ_i}. Thus, $V(G') \cup \{v_{\delta_i}\}$ weakly dominates $G \cup G'$. In order to obtain a weakly total dominating set, D can be included $\{z\}$. Thus D may be $V(G') \cup \{z\} \cup \{v_{\delta_i}\}$ for all i. Hence, $\gamma_{wt}(\mu(G)) \leq n+1+|\delta|$.

From the Observation 1, the vertices in $\{v_{\delta_i}\}$ $i = 1,2,...,n$ and their copies in G' may weakly dominate all vertices in $\mu(G)$. For a weakly total dominating set , D must include $n-|\delta|$ vertices in G'(or G). Hence, $2|\delta| + n - |\delta| \leq \gamma_{wt}(\mu(G))$. □

Proposition 3.4. *Let G be n order graph $\Delta = n-1,\ \delta \geq 2$*

$$n+1-|\Delta| \leq \gamma_{wt}(\mu(G)) \leq n+2-|\Delta|$$

where $|\Delta|$ is number of maximum degree vertices.

Proof. Let D be a WTD-set of $\mu(G)$.

- $|\Delta| = 1$; Let v' be copy of the vertex v that has degree Δ in G. D must include the vertices $V(G') - \{v'\}$. Therefore, $S = V(G') - \{v'\} \cup \{z\}$ is minimal WTD-set for $\mu(G)$. Thus, $\gamma_{wt}(\mu(G)) = n - |\Delta| + 1$.

- $|\Delta| \geq 2$; Let D be set included all vertices in $V(G')$ except copy of $\Delta-$degree vertices. There are two cases;

  Let, all vertices in $V(G)$ be weakly dominated by D. For totality, $\{z\}$ must be included by D. Thus, $|D| = n - |\Delta| + 1$.

  Let, all vertices in $V(G)$ not be weakly dominated by D. Let v' be a copy of $\Delta-$degree vertex. Then, $D \cup \{v'\}$ weakly dominates all vertices in $V(G)$. Hence, $D \cup \{v'\} \cup \{z\}$ is a WTD-set of $\mu(G)$. We have $|D| = n - |\Delta| + 2$.

  According to these two cases we obtain that $n+1-|\Delta| \leq \gamma_{wt}(\mu(G)) \leq n+2-|\Delta|$.

□

4. Strong Total Domination of Mycieski Graph

In this section, we begin with some basic results about strong total domination complete bipartite graph $K_{n_1,\ldots,n_p}$ and the characterization of graph that have $\gamma_{st}(G) = 2$. Also, we derive some results about strong total domination of Mycielski's graph. The bound $\gamma_{st}(G)+1 \leq \gamma_{st}(\mu(G)) \leq \gamma_{st}(G)+2$ has been presented. In addition, we characterize graphs G holding the lower bound $\gamma_{st}(G)+1 = \gamma_{st}(\mu(G))$.

Proposition 4.1. *Let G be n order graph such that at least one vertex has degree $(n-1)$. Then*

$$\gamma_{st}(G) = 2.$$

Proof. Let v be a $\Delta = n-1$ degree vertex of G. G can be strongly total dominates by v and its any neighbour. Thus $\gamma_{st}(G) = 2$. □

Theorem 4.2. *Let G be n order graph,*

$$\gamma_{st}(G) \geq \frac{n-|SS|}{\Delta-1}.$$

Proof. Let S be a $\gamma_{st}-$set of G. Let F be the set of all edges of G that have one end vertex in S the other in $V-S$. Also, a vertex in $SS(G)$ has degree at most Δ. Therefore, $|F| \leq |SS|(\Delta-1) + (|S|-|SS|)(\Delta-2)$. In addition, a vertex can be dominated by more than one vertex then $|F| \geq n-|S|$. Using the inequalities the it is obtained that $|S| \geq \frac{n-|SS|}{\Delta-1}$. □

Proposition 4.3. *Let $K_{m,n}$ be complete bipartite graph with $m+n$ vertices then* $\gamma_{st}(K_{m,n}) = \begin{cases} 2 & ,if\ m=n \\ \min\{m,n\}+1 & ,otherwise \end{cases}$.

Proof. Let $K_{m,n}$ be complete bipartite graph with partitions V_1 and V_2. If $m=n$ then degree of vertices are equal in each subset V_1 and V_2. Thus, a $\gamma_{st}-$set can be done as choosing a vertex from V_1 and V_2. Thus, $\gamma_{st}(K_{m,n}) = 2$. Without loss of generality, $m > n$ and $|V_1| = m$ and $|V_2| = n$. Therefore, degree of vertices in V_2 are equal and greater than degree of vertices in V_1. According to form of complete bipartite graph, all vertices in V_2 must be in γ_{st}-set of $K_{m,n}$. In order to totally strong dominates, a vertex from V_1 must be in $\gamma_{st}-set$. Hence, $\gamma_{st}(K_{m,n}) = n+1 = min\{m,n\}+1$. □

Proposition 4.4. *For any integers $n_p \geq \ldots \geq n_1 \geq 1$,* $\gamma_{st}(K_{n_1,n_2,\ldots,n_p}) = \min\{n_1,n_2,\ldots,n_p\}+1$.

Proof. Let V_i be vertex set that included n_i vertices. According to form of the graph there are n_1 vertex in V_1 that has maximum degree and adjacent to all other vertices in V_i, $i \neq 1$. Also, these disjoint vertices in V_1 must be included by any STD-set of $K_{n_1,n_2,\ldots,n_p}$. In order to totally dominates, it is needed a vertex in V_i, $i \neq 1$. Hence, $\gamma_{st}(K_{n_1,n_2,\ldots,n_p}) = n_1+1$. □

Observation 2:[1] Support vertices always in the STD-set of graph.

Observation 3: Let G be n order graph. If $\Delta < n-1$ then the vertex z must be in the γ_{st}-set of $\mu(G)$.

Proposition 4.5. *Let D be γ_{st}-set of G in $\mu(G)$ then D strongly total dominates $V(G) \cup V(G')$ in $\mu(G)$.*

Proof. Let D be γ_{st}-set of G in $\mu(G)$. According to the form of $\mu(G)$, $N(D) = V(G) \cup V(G')$ where $N(D) = \bigcup_{v \in D} N(v)$. Since $\deg(v) \geq \deg(v')$, $v \in V(G)$ and $v' \in V(G')$, D strongly total dominates $V(G) \cup V(G')$. □

Proposition 4.6. *For special graphs*

a. $\gamma_{st}(\mu(P_n)) = \gamma_{st}(P_n)+1$

b. $\gamma_{st}(\mu(C_n)) = \begin{cases} \gamma_{st}(C_n)+2 & ,\text{if} n \equiv 0(\mod 4) \\ \gamma_{st}(C_n)+1 & ,otherwise \end{cases}$

c. $\gamma_{st}(\mu(K_n)) = 3$

d. $\gamma_{st}(\mu(K_{1,n-1})) = 3$

e. $\gamma_{st}(\mu(W_{1,n-1})) = 3$

f. $\gamma_{st}(\mu(K_{m,n})) = \begin{cases} 4 & ,\text{if } m=n \\ n+2 & ,otherwise \end{cases}$, for $m \geq n$.

Proof. a. Let D be γ_{st}-set of $\mu(P_n)$ and S be γ_{st}-set of P_n. From Observation 3, z must be included by D then $|S| < |D|$. From Proposition 4.5, vertices in G and G' are strongly total dominated by S. For totality, at least one vertex must be included by D. Let v' be copy of support vertex v of P_n.

Thus, $D = (S - \{v\}) \cup \{v'\} \cup \{z\}$ is γ_{st}-set for $\mu(P_n)$ then $\gamma_{st}(\mu(G)) = |D| = \gamma_{st}(P_n) + 1$.

b. Let S be γ_{st}-set of C_n. If $n \neq 0 (\mod 4)$ then there exist a vertex $v \in S$ such that $pn[v,S] = \emptyset$. Thus, $(S - \{v\}) \cup \{v'\} \cup \{z\}$ is γ_{st}-set of $\mu(G)$ where v' is copy of v in $\mu(G)$. Therefore, if $n \neq 0 (\mod 4)$, we have $\gamma_{st}(C_n) = \gamma_{st}(C_n) + 1$. If n$\equiv 0 (\mod 4)$ then for all $v \in S$, $pn[v,S] \neq \emptyset$. Due to the form of $\mu(G)$, $V(G)$ is not strongly dominated by a vertex $v' \in V(G')$. Thus, $S \cup \{v'\} \cup \{z\}$ is γ_{st}-set of $\mu(G)$.Therefore, if $n = 0 (\mod 4)$, we have $\gamma_{st}(C_n) = \gamma_{st}(C_n) + 2$.

c.d.e. Let D be γ_{st}-set of $\mu(G)$ and v be a vertex in G and $\deg v = n - 1$. From Observation 2 and 3, v and z are contained by D. All vertices in $\mu(G)$ are strongly dominated by $\{v\} \cup \{z\}$. For totality at least one vertex from G' must be included by D. Hence, $D = \{v\} \cup \{z\} \cup \{y_i\}$.Finally we have, $\gamma_{st}(\mu(G)) = 3$.

f. Let $m = n$, from Proposition 4.3 $\gamma_{st}(K_{m,n}) = 2$. Let S and D be a $\gamma_{st}-$set of $K_{m,n}$ and $\mu(K_{m,n})$, respectively. It is easily to see that $D = S \cup \{z\} \cup \{v'\}$. Hence, $\gamma_{st}(\mu(K_{m,n})) = 4$. Let $m > n$ then due to the degree of vertices, n disjoint vertices in $K_{m,n}$ must be included. For γ_{st}-set of $\mu(G)$ it is needed two more vertices such that $v' \in V'(G)$ and z. Therefore $\gamma_{st}(\mu(K_{m,n})) = n + 2$ for $m > n$. □

Theorem 4.7. *Let G be n order graph then* $\gamma_{st}(G) + 1 \leq \gamma_{st}(\mu(G)) \leq \gamma_{st}(G) + 2$.

Proof. Let S be a γ_{st}-set of G. From Proposition 4.5, vertices in G and G' are strongly total dominated by S. All vertices in $\mu(G)$ is also strongly dominated by $S \cup \{z\}$. It is known that for any graph G; $\gamma_s(G) \leq \gamma_{st}(G)$. The strong total domination number of $\mu(G)$ is at least $\gamma_{st}(G) + 1$. Therefore, $\gamma_{st}(G) + 1 \leq \gamma_{st}(\mu(G))$. Let y_i be a vertex in G'. It is obvious that $S \cup \{y_i\} \cup \{z\}$ is a STD-set of $\mu(G)$. Thus, $\gamma_{st}(\mu(G)) \leq \gamma_{st}(G) + 2$. □

Theorem 4.8. *Let G be n order graph, $\Delta < n - 1$ and $\delta = 1$. Then*

$$\gamma_{st}(\mu(G)) = \gamma_{st}(G) + 1.$$

Proof. Let S and D be a γ_{st}-set of G and $\mu(G)$, respectively and x_i be support vertex then from Observation 2, x_i is contained by S. Also, from Observation 3, z is included by D. Any vertex from G' ensures totality. Therefore, $S \cup \{x_i'\} \cup \{z\}$ is any STD-set of $\mu(G)$, where x_i' be copy of x_i.It is obvious that degree of the pendant vertex, denoted by u, less than degree of x_i' in Mycielski's Graph. Since $N_{\mu(G)}(u) = \{x_i, x_i'\}$, D is not included both x_i and x_i'. Thus, the lower bound of Theorem 4.7 can be obtained as $D = (S - \{x_i\}) \cup \{x_i'\} \cup \{z\}$. Hence,

$$\gamma_{st}(\mu(G)) = \gamma_{st}(G) + 1.$$

□

Proposition 4.9. *Let G be n order graph such that at least one vertex has degree $(n - 1)$. Then*

$$\gamma_{st}(\mu(G)) = 3.$$

Proof. Similar proof can be done as shown in case c, d, e of Proposition 4.6. □

Remark 4.10. *It can be said from Propositions 4.9, if graph contain at least one vertex of degree $n - 1$ then* $\gamma_{st}(\mu(G)) = \gamma_{st}(G) + 1$.

Theorem 4.11. *Let G be a graph and S be a γ_{st}-set of G. If at least one vertex $u \in S$ such that $pn[u,S] = \emptyset$ then*

$$\gamma_{st}(\mu(G)) = \gamma_{st}(G) + 1.$$

Proof. Let u' be a vertex in G' that is copy of u. If $pn[u,S] = \emptyset$, then this means that u is not private neighborhood of any vertex in G. The vertex u is contained by S due to totality. From Proposition 4.5, $(S - \{u\}) \cup \{u'\}$ strongly total dominates $V(G) \cup V(G')$. Therefore, $(S - \{u\}) \cup \{u'\} \cup \{z\}$ is a $\gamma_{st} - set$ of $\mu(G)$. Hence, $\gamma_{st}(\mu(G)) = \gamma_{st}(G) + 1$. □

Let $SS(G)$ be the set of all vertices $v \in V(G)$ such that $\deg(v) > \deg(u)$ for every $u \in N(v)$. $SS(G)$ may be empty. If $SS(G) \neq \emptyset$ then it is included by every STD set of G.

5. Conclusion

Mycielski present a construction that increase chromatic number [10]. This construction has been taken attention and beside chromatic number, there have been many results about various parameters of Mycieski's graphs in literature. However, there exist considerably fewer research on strong total and weak total domination numbers. In this paper, strong total domination and weak total domination of Mycielski's graph was investigated and also the strong and weak total domination numbers of Mycielski's graph, $\mu(G)$, associated with strong and weak total domination of underlying graph, G. This provides a good starting point for discussion and further research. Future research on strong total and weak total domination number might extend the explanations of some graph operations.

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