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Replenishment model for deteriorating items with price dependent demand and complete backlogging in two storage facilities under the effect of inflation

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Abstract

This model is based on the study of a market situation where a retailer is interested in bulk purchasing. The retailer buys excess units of product then the available storage capacity. So he needs to get some extra space in the form of rented warehouse. It is assumed that because of suitable storage conditions in the rented warehouse (RW), there is no deterioration of items. In owned warehouse (OW), constant deterioration of items takes place. Demand is taken as a function of selling price. Demand and selling price are inversely related. With the increase in selling price, demand of the product decreases and vice versa. This model is constructed considering complete backlogging of demand during period of shortages under the effect of inflation. Numerical example is given to discuss the approach of study. The effect of variation of various parameters is presented and shown graphically.

Keywords

Two warehouse, Inflation, Finite planning horizon, Deterioration, Price dependent demand, Complete backlogging.

AMS Subject Classification

26A33, 30E25, 34A12, 34A34, 34A37, 37C25, 45J05.

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1. Introduction

Inventory is the assets to a company. It can be in the form of raw material, work-in-progress or finished goods. It is surplus quantity of manufactured products or raw material after deducting consumed goods. It is always difficult to decide the optimal level of inventory to be stocked. Stocking more quantity for long periods may results in more storage costs. Quality of items can also deteriorate or they can become obsolete with the passage of time. Rate of deterioration is much higher in case of perishable goods like fruits, vegetables, milk products, items that come with an expiry date, etc. Fashion goods like clothes, mobiles, jewelry, handbags, sun shades and shoes are in demand for very short period as trend of fashion goods changes with the introduction of new designs. On contrary, keeping less inventory is also non profitable for suppliers/retailers due to shortage costs. Brand value of a product may also damage due to non availability of the product. Availability of the substitutes in the market also hinders the market performance of a product. So it is big challenge for suppliers/retailers to maintain optimum inventory levels. Various researchers have tried to solve this problem of producers, suppliers and retailers. Banerjee[1] has given a model to decide the economic lot size for purchasers and vendors. Bonney[3] discussed the concept of trends in inventory management. Levin et al[12] has given model for the management of operation systems. Gupta and Vrat[8], Mandal and Phaujdar[13], Pando[17] has taken stock dependent demand rate. Goh[7] studied general demand with various holding cost functions. Bose et al[4], Lee and Ying[10] studied inventory model with time dependent consumption rate whereas Chowdhury et al[5] has taken time-quadratic demand function. Teng and Chang[23] has taken demand due to price of product and available stock.Sana and Chaudhuri[19] discusses the demand dependent on advertising. Chung et al[6] discussed inventory model with remanufacturing. Rastogi et al[18] assumed demand to be price dependent.First step of inventory management is controlling and overseeing purchases. The definition of inventory management changes depending upon the type of product to be sold and the channel through which it is sold. While deciding units to be ordered or produced a firm has to consider various factors like cost of ordering, transport cost, per unit purchase cost, storage space available and cost of storing goods, etc. Some suppliers offer heavy discounts on bulk purchasing. It becomes economical for retailers to purchase such large quantities as it lowers their per unit purchasing costs. And for the production units/suppliers it is profitable as products are sold before being deteriorated or becoming obsolete. But a big question arises for a retailer, where to store these goods. If a retailer has sufficient storage space with good storage conditions then there is no problem. Otherwise there is no use of buying such large quantity of goods. As on one side if a retailer sees profit on purchasing goods at lower price, on the other hand he has to face huge losses if the goods get damaged due to non availability of favorable storage facilities. To overcome these losses, retailer decides to take an extra space for storing goods with favorable storing conditions. This space is known as Rented Warehouse. It will be profitable only if the cost of rented warehouse is less than the profit earned by lowering of per unit cost due to bulk purchasing. Sarma[21], Sarma and Sastry[20], Benkherouf[2], Lee and Ying[10], Lee and Hsu[11], Rastogi et al[18] studied the inventory model based on two storage facilities. Yang[24] also studied a model for two storage facilities considering shortages and inflation. Palanivel and Uthayakumar[15] gives the effect of inflation and partial backlogging in two warehouse system for deteriorating items. For managing inventory it is very important to know what on hand stock a retailer has, its availability in warehouses and the way its moving in and out. Hou[9], Shah and Vaghela[22] studied inventory model for deteriorating under the effect of inflation and time value of money. Pal[14] et al explained EPQ model considering ramp type demand under the influence of inflation and shortages. Pandey et al[16] explains the strategy of quantity incentive. He also explains the idea of partial backlogging. As the economy is affected by inflation. Inflation effects the purchasing power of a customer. Therefore, it is an important parameter while calculating various costs.

2. Notations and Assumptions

2.1 Notations

The following notations are used for the formulation of this model:

A The ordering cost per order.
C_{hr} The holding cost per item in RW.
C_{ho} The holding cost per item in OW, $C_{hr} > C_{ho}$.
C_d The deterioration cost per unit per cycle.
C_s The shortage cost for backlogged items per unit per cycle.
C_p The purchasing cost per unit.
p The selling price per unit.
PC Purchasing cost of the inventory for the first replenishment cycle.
SC Shortage cost of the inventory for the first replenishment cycle.
HC_{rw} The holding Cost of the inventory in the rented warehouse.
HC_{ow} The holding Cost of the inventory in the owned warehouse.
DC_{ow} The deterioration Cost of the inventory in the owned warehouse.
TC_{FC} The total cost of inventory in first replenishment cycle.
TC The total cost of the complete planning horizon.
λ The life time of the items in OW.
θ The deterioration rate in OW,$0 \leq \theta \leq 1$.
T The length of the order cycle.
H The planning horizon.
n The total number of replenishments during the finite planning horizon, $n = \frac{H}{T}$.
Q_1 The capacity of the OW.
Q_2 The maximum inventory level in RW.
S The maximum inventory level per cycle.
I The replenishment to be made in each cycle.
I_b The maximum amount of shortages to be backlogged.
I_m Maximum inventory to be stored in both warehouses.
$I_r(t)$ The inventory level in RW at time t.
$I_o(t)$ The inventory level in OW at time t.
$I_s(t)$ The negative inventory level at time t.
Q The 2nd , 3rd,. . . ., mth order size.
r The discount rate represents the time value of money.
f The inflation rate.
R The net discount rate of inflation i.e.,$R = r - f$, is constant.

2.2 Assumptions

1. A single item is considered over the prescribed period of planning horizon.
2. There is no replacement or repair of deteriorated items.
3. The lead time is zero.
4. The deterioration takes place after the life time of items.
5. The rate of replenishment is infinite.
6. There is effect of inflation and time value of money.

7. The demand rate is considered to be a function of selling price per unit. There is inverse relationship between selling price and demand of a product. i.e.$D = \frac{\alpha}{p^{\beta}}$ where α, β are positive demand coefficients and $\beta > 1$.

8. In RW due to good preserving conditions, there is no deterioration of items.

9. During stock out situation, shortages are allowed and completely backlogged.

10. The OW has limited capacity of storing units of item but the capacity of RW is assumed to be unlimited. The items of RW are consumed prior to the items of OW to save additional money spent on RW.

3. Formulation of the model

This paper is based on the assumption of a planning horizon sub-divided into a n number of equal interval i.e. each interval is of length $T = \frac{H}{n}$. Hence, each consecutive replenishment will be made at $T_i = iT \ (i = 0,1,2,3,.....,n)$. Therefore, every time interval $[iT,(i+1)T]$ can be separated into two sections:- one with positive inventory level (i.e. no shortages) and other with negative inventory level (i.e. shortages that are completely backlogged). In other words, we can say that time period of positive inventory is k^{th} fraction of $[iT,(i+1)T]$ and is equal to $kT,(0 < k < 1)$. Shortages start occurring at time $t_i = (k+i-1)T; (i = 1,2,3,...,n)$ and are accumulated upto $t = iT; (i = 1,2,3,...,n)$ before next replenishment order is made.

The first replenishment is made at T=0 and the replenishment lot size is S. Q_1 units are stored in OW and the rest is stored in RW. The items of RW are consumed in preference to items of OW. Various costs are calculated considering the effect of inflation and time value of money.

As it is assumed that there is no deterioration in RW due to availability of good preservation conditions, therefore level of inventory in RW is decreasing due to demand only. The mathematical equation representing above relation is as below:-

$$\frac{dI_r(t)}{dt} = -\frac{\alpha}{p^{\beta}}, \quad 0 \le t \le t_r$$

Solving above equation and using condition, $I_r(t_r) = 0$, we get

$$I_r(t = \frac{\alpha}{p^{\beta}}(t_r - t), \quad 0 \le t \le t_r \tag{3.1}$$

In RW, the maximum inventory level Q_2 will be at t=0, therefore

$$Q_2 = \frac{\alpha}{p^{\beta}} t_r \tag{3.2}$$

The holding cost for rented warehouse (HC_{rw}) for first replenishment cycle are calculated as

$$HC_{rw} = c_{hr}\int_0^{t_r} I_r(t)e^{-Rt}dt = c_{hr}\int_0^{t_r} \frac{\alpha}{p^{\beta}}(t_r - t)e^{-Rt}dt$$
$$= \frac{\alpha c_{hr}}{p^{\beta}R^2}(e^{-Rt_r} - 1 + Rt_r) \tag{3.3}$$

Now, in OW, initially there is no change in inventory level during $0 \le t \le \lambda$. After it, items start deteriorating at a constant rate,$\theta(\lambda \le t \le t_r)$. At time t_r inventory level in RW reduces to zero and consumption of items is started from OW. So, during $t_r \le t \le t_1$ inventory is changing due to demand and deterioration both. The mathematical equations representing the above relations are as:- As rate of change of inventory is zero, during $0 \le t \le \lambda$

$$i.e. \frac{dI_0(t)}{dt} = 0$$

Therefore,

$$I_0(t) = Q_1, 0 \le t \le \lambda \tag{3.4}$$

(Because at t=0, Q_1 units of the inventory are stored in OW.) During the time interval $[\lambda, t_r]$ inventory decreases due to constant deterioration only. The differential equation is as below:-

$$\frac{dI_0(t)}{dt} = -\theta I_0(t), \lambda \le t \le t_r$$

Using condition $I_0(\lambda) = Q_1$ and solving above equation, we get

$$I_o(t) = Q_1 e^{\theta(\lambda - t)}, \lambda \le t \le t_r \tag{3.5}$$

Behaviour of inventory during time interval $[t_r, t_1]$ is shown below:-

$$\frac{dI_0(t)}{dt} = -\theta I_0(t) - \frac{\alpha}{p^{\beta}}, t_r \le t \le t_1$$

Solving and using condition,$I_0(t_1) = 0$, we get

$$I_0(t) = \frac{\alpha}{p^{\beta}\theta}(e^{\theta(t_1 - t)} - 1), t_r \le t \le t_1 \tag{3.6}$$

As inventory function is a continuous function of time, therefore applying condition of continuity at $t = t_r$, we get

$$t_r = t_1 - \frac{Q_1}{(\frac{\alpha}{p^{\beta}})} e^{\theta\lambda} \tag{3.7}$$

The holding cost in OW for the first replenishment cycle is calculated as

$$HC_{OW} = c_{ho}\int_0^{t_1} I_0(t)e^{-Rt}dt$$
$$= c_{ho}[\int_0^{\lambda} I_0(t)e^{-Rt}dt + \int_{\lambda}^{t_r} I_0(t)e^{-Rt}dt + \int_{t_r}^{t_1} I_0(t)e^{-Rt}dt]$$
$$= c_{ho}[\int_0^{\lambda} Q_1e^{-Rt}dt + \int_{\lambda}^{t_r} Q_1e^{\theta(\lambda-t)}e^{-Rt}dt + \int_{t_r}^{t_1}\frac{\alpha}{p^{\beta}\theta}(e^{\theta(t_1-t)}-1)e^{-Rt}dt]$$
$$= c_{ho}[\frac{Q_1}{R}(1-e^{-R\lambda}) + \frac{Q_1}{\theta+R}(e^{-R\lambda}-e^{\theta\lambda-(\theta+R)t_r}) + \frac{\alpha}{p^{\beta}\theta}(\frac{e^{\theta t_1-(\theta+R)t_r}-e^{-Rt_r}}{\theta+R}+\frac{e^{-Rt_1}-e^{-Rt_r}}{R})] \tag{3.8}$$

The deterioration cost in OW under the effect of inflation and time value of money for the first replenishment cycle is as below:-

$$DC_{OW} = \theta c_d\int_{\lambda}^{t_1} I_0(t)e^{-Rt}dt$$
$$= \theta c_d[\int_{\lambda}^{t_r} I_0(t)e^{-Rt}dt + \int_{t_r}^{t_1} I_0(t)e^{-Rt}dt]$$
$$= \theta c_d[\int_{\lambda}^{t_r} Q_1e^{\theta(\lambda-t)}e^{-Rt}dt + \int_{t_r}^{t_1}\frac{\alpha}{p^{\beta}\theta}(e^{\theta(t_1-t)}-1)e^{-Rt}dt]$$
$$= \theta c_d[\frac{Q_1}{\theta+R}(e^{-R\lambda}-e^{\theta\lambda-(\theta+R)t_r}) + \frac{\alpha}{p^{\beta}\theta}(\frac{e^{\theta t_1-(\theta+R)t_r}-e^{-Rt_r}}{\theta+R}+\frac{e^{-Rt_1}-e^{-Rt_r}}{R})] \tag{3.9}$$

When inventory level in both the warehouses reaches zero, shortages start accumulating. In this model, complete backlogging of demand is considered. Therefore in time interval $[t_1, T_1]$ there is complete backlogging of demand. The differential equation for above is as:-

$$\frac{dI_s(t)}{dt} = -\frac{\alpha}{p^{\beta}}, t_1 \le t \le T_1$$

Solving above equation and applying boundary condition $I_s(t_1) = 0$, we get

$$I_s(t) = \frac{\alpha}{p^{\beta}}(t_1 - t), t_1 \le t \le T_1 \tag{3.10}$$

The present value of shortage cost for the first replenishment cycle is:-

$$SC = -c_s\int_{t_1}^{T_1}\frac{\alpha}{p^{\beta}}(t_1-t)e^{-Rt}dt = \frac{\alpha c_s}{p^{\beta}R^2}[R(t_1-T_1)+e^{-R(t_1-T_1)}-1]e^{-RT_1} \tag{3.11}$$

Replenishment for the consecutive cycles is to be made at t=0 and T_i. For making replenishment total demand and shortages for the previous cycles are to be considered. Hence, Maximum inventory available in two warehouses is

$$I_m = Q_1 + Q_2 = Q_1 + \frac{\alpha}{p^{\beta}}t_r \tag{3.12}$$

And maximum level of shortages or accumulated demand to be satisfied in next cycle is

$$I_b = \frac{\alpha}{p^{\beta}}(T_1 - t_1) \tag{3.13}$$

Now the present value of making replenishment at various cycles is

$$PC = p(I_m + I_be^{-RT}) = c_p(Q_1 + \frac{\alpha}{p^{\beta}}t_r) + \frac{c_p\alpha}{p^{\beta}}(T_1-t_1)e^{-RT_1} \tag{3.14}$$

Taking into consideration all the above costs, the total cost for a single replenishment cycle becomes:

$$TC_{FC} = A + HC_{rw} + HC_{ow} + DC_{ow} + PC + SC$$
$$= A + \frac{\alpha c_{hr}}{p^{\beta}R^2}(e^{-Rt_r}-1+Rt_r) + c_{ho}\frac{Q_1}{R}(1-e^{-R\lambda}) + \frac{Q_1(c_{ho}+\theta c_d)}{\theta+R}(e^{-R\lambda}-e^{\theta\lambda-(\theta+R)t_r})$$
$$+ (\frac{c_{ho}}{\theta}+c_d)\frac{\alpha}{p^{\beta}}(\frac{e^{\theta t_1-(\theta+R)t_r}-e^{-Rt_1}}{\theta+R}+\frac{e^{-Rt_1}-e^{-Rt_r}}{R}) + c_p(Q_1+\frac{\alpha}{p^{\beta}}t_r) + \frac{c_p\alpha}{p^{\beta}}(T_1-t_1)e^{-RT_1}$$
$$+ \frac{c_s\alpha}{R^2p^{\beta}}[R(t_1-T_1)+e^{-R(t_1-T_1)}-1]e^{-RT_1} \tag{3.15}$$

Hence for the finite planning horizon H, the total cost is calculated as below:

$$TC = \sum_{i=0}^{n-1} TC_{FC}e^{-RiT} + Ae^{-RH} = TC_{FC}(\frac{1-e^{-RH}}{1-e^{\frac{-RH}{n}}}) + Ae^{-RH}$$

Where $W = (\frac{1-e^{-RH}}{1-e^{\frac{-RH}{n}}})$ i.e.

$$TC = Ae^{-RH} + W[A + \frac{\alpha c_{hr}}{p^{\beta}R^2}(e^{-Rt_r}-1+Rt_r) + c_{ho}\frac{Q_1}{R}(1-e^{-R\lambda}) + \frac{Q_1(c_{ho}+\theta c_d)}{\theta+R}(e^{-R\lambda}-e^{\theta\lambda-(\theta+R)t_r})$$
$$+ (\frac{c_{ho}}{\theta}+c_d)\frac{\alpha}{p^{\beta}}(\frac{e^{\theta t_1-(\theta+R)t_r}-e^{-Rt_1}}{\theta+R}+\frac{e^{-Rt_1}-e^{-Rt_r}}{R}) + c_p(Q_1+\frac{\alpha}{p^{\beta}}t_r) + \frac{c_p\alpha}{p^{\beta}}(T_1-t_1)e^{-RT_1}$$
$$+ \frac{c_s\alpha}{R^2p^{\beta}}(R(t_1-T_1)+e^{-R(t_1-T_1)}-1)e^{-RT_1}] \tag{3.16}$$

or

$$TC(n,k) = Ae^{-RH} + W[A + \frac{\alpha c_{hr}}{p^{\beta}R^2}(e^{-Rt_r} - 1 + Rt_r) + c_{ho}\frac{Q_1}{R}(1-e^{-R\lambda}) + \frac{Q_1(c_{ho}+\theta c_d)}{\theta+R}(e^{-R\lambda} - e^{\theta\lambda-(\theta+R)t_r}) + (\frac{c_{ho}}{\theta}+c_d)\frac{\alpha}{p^{\beta}}(\frac{e^{\theta\frac{kH}{n}-(\theta+R)t_r} - e^{\frac{-RkH}{n}}}{\theta+R} + \frac{e^{\frac{-RkH}{n}} - e^{-Rt_r}}{R}) + c_p(Q_1 + \frac{\alpha}{p^{\beta}}t_r) + \frac{c_p\alpha}{p^{\beta}}(1-k)\frac{H}{n}e^{\frac{-RH}{n}} + \frac{c_s\alpha}{R^2p^{\beta}}(R(k-1)\frac{H}{n} + e^{-R(k-1)\frac{H}{n}} - 1)e^{\frac{-RH}{n}}] \quad (3.17)$$

(because $t_1 = \frac{kH}{n}$ and $T_1 = \frac{H}{n}$)
To find the optimized values of different parameters, we differentiate above calculated total cost of the inventory considering the effect of inflation and time value of money, with respect to k, and we get

$$\frac{d(TC)}{dk} = W[\frac{\alpha c_{hr}}{p^{\beta}R}(1-e^{-Rt_r}) + Q_1(c_{ho}+\theta c_d)e^{\theta\lambda-(\theta+R)t_r} + (\frac{c_{ho}}{\theta}+c_d)\frac{\alpha}{p^{\beta}}(\frac{-Re^{\frac{\theta KH}{n}-(\theta+R)t_r}}{\theta+R} + e^{-Rt_r} - e^{\frac{-RkH}{n}}) + c_p\frac{\alpha}{p^{\beta}} - \frac{c_p\alpha}{p^{\beta}}e^{\frac{-RH}{n}} + \frac{c_s\alpha}{R^2p^{\beta}}(1-e^{-R(k-1)\frac{H}{n}})e^{\frac{-RH}{n}}] \quad (3.18)$$

and

$$\frac{d^2TC}{dk^2} = \frac{HW}{n}[c_{hr}\frac{\alpha}{p^{\beta}}e^{-Rt_r} - Q_1(\theta+R)(c_{ho}+\theta c_d)e^{\theta\lambda-(\theta+R)t_r} + (\frac{c_{ho}}{\theta}+c_d)\frac{\alpha}{p^{\beta}}(\frac{R^2e^{\frac{\theta kH}{n}-(\theta+R)t_r} - R^2e^{\frac{-RkH}{n}}}{\theta+R} - Re^{-Rt_r} + Re^{\frac{-RkH}{n}}) + \frac{c_s\alpha}{Rp^{\beta}}e^{\frac{-RkH}{n}}] \quad (3.19)$$

4. Numerical Example

The numerical results for the input parameters as below are displayed in Table1
$A = 150$ currency units, $c_{hr} = 2$ currency units, $c_{ho} = 1.2$ currency units, $c_d = 1.5$ currency units, $c_p = 5$ currency units, $c_2 = 3$ currency units, Q_1=50 units, $\lambda = \frac{8}{12}$years, $\theta = 0.8$, R=0.2, H=20 years, p=15 currency units, $\beta = 1.2$, $\alpha = 2578$

Table 1

Table 1							
n	k	t_r	t_1	T	I	TC	$\frac{d^2(TC)}{dk^2}$
1	0.51108	9.36934	10.22164	20	1964.77	11037.54	1403.488
2	0.49585	4.10618	4.95848	10	964.77	5712.198	1977.493
3	0.48062	2.35180	3.20410	6.667	631.4367	4404.64	1774.506
4	0.46538	1.47461	2.32691	5	464.7701	3847.322	1468.097
5	0.45015	0.94829	1.80059	4	364.77	3537.158	1195.957
6	0.43491	0.59741	1.44971	3.333	298.1033	3333.369	977.291
7	0.41968	0.34678	1.19909	2.857	250.4843	3183.17	805.6973
8	0.40445	0.15882	1.01112	2.5	214.77	3062.963	671.1007
9	0.389213	0.01262	0.86492	2.222	186.9922	2960.865	564.7058

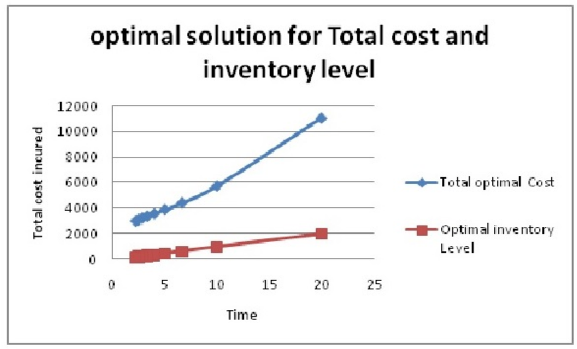


Figure 1

Table 2

For λ=0, means deterioration in OW starts at t=0						
n	k	t_r	t_1	T	I	TC
1	0.51579	9.81579	10.31579	20	2000	11771.33
2	0.50526	4.55263	5.05263	10	1000	6358.695
3	0.49474	2.79824	3.29824	6.667	666.667	5040.491
4	0.48421	1.92105	2.42105	5	500	4514.522
5	0.47368	1.39474	1.89474	4	400	4261.505
6	0.46316	1.04386	1.54386	3.333	333.333	4132.07
7	0.45263	0.79323	1.29323	2.857	285.7143	4068.297
8	0.44210	0.60526	1.10526	2.5	250	4043.304
9	0.43158	0.45906	0.95906	2.222	222.222	4043.007
10	0.42105	0.34210	0.84210	2	200	4059.348
11	0.41053	0.24641	0.74641	1.818	181.8182	4087.406

Table 3

For θ=0, means there is no deterioration in OW						
n	k	t_r	t_1	T	I	TC
1	0.404	7.58	8.08	20	2000	11771.33
2	0.408	3.58	4.08	10	1000	6358.695
3	0.412	2.2467	2.7467	6.667	666.667	5040.491
4	0.416	1.58	2.08	5	500	4514.522
5	0.420	1.18	1.68	4	400	4261.505
6	0.424	0.9133	1.4133	3.333	333.333	4132.07
7	0.428	0.7228	1.2228	2.857	285.7143	4068.297
8	0.432	0.58	1.08	2.5	250	4043.304
9	0.436	0.4689	0.9689	2.222	222.222	4043.007
10	0.440	0.38	0.88	2	200	4059.348

Figure 2

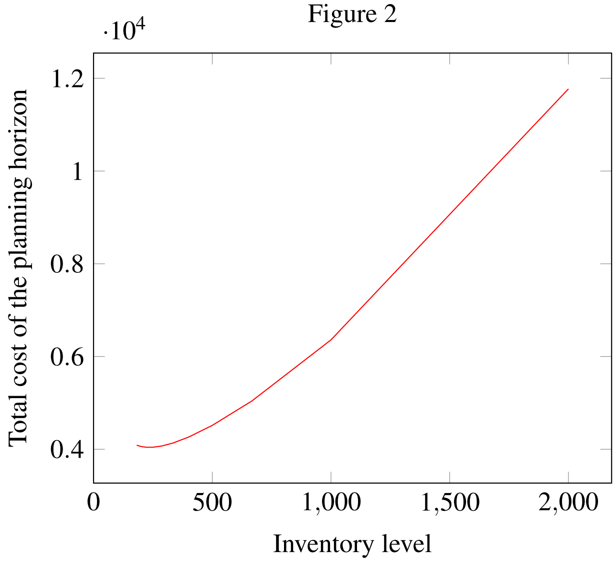
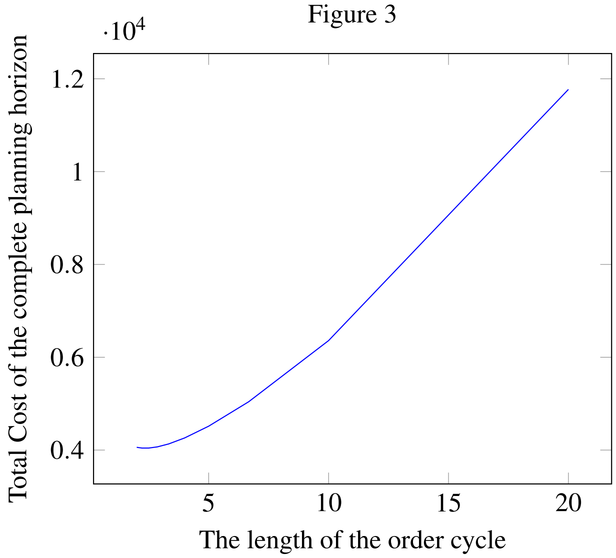


Figure 3



5. Observations

1. Table 1 provides the optimal values of inventory to be stocked when initially for some period $0 \leq t \leq \lambda$ there is no deterioration in Owned ware house and deterioration starts at $t = \lambda$.
2. Table 1 show that there is positive relation between optimal cycle time and optimal value of inventory level and total cost incurred. Figure 1 depicts graphical representation of situation.
3. Table 2 provides optimal inventory level when deterioration in owned ware house starts from the beginning of the cycle. Figure 2 shows relationship between optimal inventory level and corresponding optimal value of total cost incurred.
4. Table 3 depicts the situation when there is no deterioration in owned ware house and rented ware house at any stage.
5. The study finds optimal decision policy for stock level to be maintained in rented warehouse commensurate with various other parameters such as space available in owned warehouse, demand rate, selling price, cost at rented warehouse, etc.

6. Conclusion

This model derives optimal policies for situation when stock can be maintained in two ware houses owned warehouse and rented ware house. The owned warehouse has fixed capacity and storage conditions are not very good. To accommodate excess stock due to demand rented ware house with good storage conditions is being utilized. Keeping in view the cost incurred in rented ware house, demand rate and conditions of owned warehouse, optimal decision policy is derived which minimizes the total cost incurred. Varied situations related to deteriorating conditions in owned ware house and corresponding optimal decisions are explained through numerical example. The nature of varied situations in different combinations affects the management standpoint on appropriate choice of stock maintenance.

References

[1] A. Banerjee, A joint economic-lot-size model for purchaser and vendor, *Decision sciences,* 17 (1986) 292-311.

[2] L. Benkherouf, A deterministic order level inventory model for deteriorating items with two storage facilities, *International Journal of Production Economics,* 48 (1997) 167-175.

[3] M. Bonney, Trends in inventory management, *International Journal of Production Economics,* 35 (1994) 107-114.

[4] S. Bose, A. Goswami, K. Chaudhuri, An eoq model for deteriorating items with linear time-dependent demand rate and shortages under inflation and time discounting, *Journal of the Operational Research Society,* 46 (1995) 771-782.

[5] R. R. Chowdhury, S. Ghosh, K. Chaudhuri, An order-level inventory model for a deteriorating item with time-quadratic demand and time dependent partial backlogging

with shortages in all cycles, *American Journal of Mathematical and Management Sciences,* 33 (2014) 75-97.

[6] S.-L. Chung, H.-M. Wee, P.-C. Yang, Optimal policy for a closed-loop supply chain inventory system with remanufacturing, *Mathematical and computer modelling,* 48 (2008) 867-881.

[7] M. Goh, Eoq models with general demand and holding cost functions, *European Journal of Operational Research,* 73 (1994) 50-54.

[8] R. Gupta, P. Vrat, Inventory model for stock-dependent consumption rate, *Opsearch,* 23 (1986) 19-24.

[9] K.-L. Hou, An inventory model for deteriorating items with stock- dependent consumption rate and shortages under inflation and time discounting, *European Journal of Operational Research,* 168 (2006) 463-474.

[10] C. C. Lee, C. Ying, Optimal inventory policy for deteriorating items with two-warehouse and time-dependent demands, *Production Planning and Control,* 11 (2000) 689-696.

[11] C. C. Lee, S.-L. Hsu, A two-warehouse production model for deteriorating inventory items with time-dependent demands, *European Journal of Operational Research,* 194 (2009) 700-710.

[12] R. I. Levin, Production operations management: contemporary policy for managing operating systems, *McGraw-Hill Companies,* 1972.

[13] B. a. Mandal, S. Phaujdar, An inventory model for deteriorating items and stock-dependent consumption rate, *Journal of the operational Research Society,* 40 (1989) 483-488.

[14] S. Pal, G. Mahapatra, G. Samanta, A production inventory model for deteriorating item with ramp type demand allowing inflation and shortages under fuzziness, *Economic modelling,* 46 (2015) 334-345.

[15] M. Palanivel, R. Uthayakumar, Two-warehouse inventory model for non-instantaneous deteriorating items with partial backlogging and inflation over a finite time horizon, *Opsearch,* 53 (2016) 278-302.

[16] R. Pandey, S. Singh, B. Vaish, S. Tayal, An eoq model with quantity incentive strategy for deteriorating items and partial backlogging, *Uncertain Supply Chain Management,* 5 (2017) 135-142.

[17] V. Pando, L. A. San-Jose, J. Garcia-Laguna, J. Sicilia, Optimal lot-size policy for deteriorating items with stock-dependent demand considering profit maximization, *Computers and Industrial Engineering,* 117 (2018) 81-93.

[18] M. Rastogi, S. Singh, P. Kushwah, S. Tayal, Two warehouse inventory policy with price dependent demand and deterioration under partial backlogging, *Decision Science Letters,* 6 (2017) 11-22.

[19] S. S. Sana, K. Chaudhuri, An inventory model for stock with advertising sensitive demand, *IMA Journal of management mathematics,* 19 (2008) 51-62.

[20] K. Sarma, M. Sastry, Optimum inventory for systems with two levels of storage, *Industrial Engineering Journal,* 8 (1988) 12-19.

[21] K. Sarma, A deterministic inventory model with two levels of storage and an optimal release rule, *Opsearch,* 20 (1983) 175-180.

[22] N. H. Shah, C. R. Vaghela, Economic order quantity for deteriorating items under inflation with time and advertisement dependent demand, *Opsearch,* 54 (2017) 168-180.

[23] J.-T. Teng, C.-T. Chang, Economic production quantity models for de- teriorating items with price-and stock-dependent demand, *Computers and Operations Research,* 32 (2005) 297-308.

[24] H.-L. Yang, Two-warehouse inventory models for deteriorating items with shortages under inflation, *European Journal of Operational Research,* 157 (2004) 344-356.

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