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A novel extension of weighted hyper geometric functions and fractional derivative

Venkata Manikanta Batchu[1]*, Yudhveer Singh[2] and Vinod Gill[3]

Abstract

In this paper we present a number of weighted hyper geometric functions and the suitable generalization of Caputo fractional derivation. We look into the viable outcomes to find out arrangements of incomplete differential conditions or differential conditions regarding our outcomes. The parameters of the different Erd'elyi-Kober $(E-K)$ cut off basic fulfill the conditions $\beta_k(\gamma_k+1) > \frac{\mu}{p}-1, \delta_k > 0, k=1,\ldots\ldots,m$, at that point $I^{(\gamma_k),(\delta_k)}_{(\beta_k),m}$, $f(z)$ exists wherever on top of $(0,\infty)$ plus it is a partial direct administrator. In addition, a number of straight and bilinear relatives are acquired by methods like Mellin Function, H-Function, G-Function, and Appell Functions for the referenced inference administrator. At that point a portion of the considered hyper geometric functions are resolved as far as the summed up Mittag-Leffler work $E^{(\gamma_j)(l_j)}_{(\rho_j)\lambda}[z_1,z_2,\ldots,z_k]$ and the summed up polynomials $S^m_n[x]$. The limit conduct of some different class of weighted hyper geometric functions is portrayed as far as Frost man's α -limit. It manages results of driving E-K partial integrals both of structures (R-L type) and their right-hand sided analogs. At long last, a function is known utilizing our partial administrator in the issue of fractional analytics of varieties some out of the ordinary cases of the main result here are also well thought-out.

Keywords

Weighted Hyper geometric Functions, Weighted Caputo Derivative, Mellin Function, H-Function, G-Function, Apell Functions.

AMS Subject Classification

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[1] *Department of Mathematics, Amity University Rajasthan, Jaipur-303002, India.*
[2] *Amity Institute of information Technology, Amity University, Rajasthan, Jaipur-303002, India.*
[3] *Department of Mathematics, Govt. College Nalwa (Hisar), Haryana-125001, India.*
***Corresponding author**: [1]venkatamanikanta.b@gmail.com; [2]yudhvir.chahal@gmail.com; [3]vinod.gill08@gmail.com
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Contents

1. Introduction

Numerous authorities have examined unique functions because of the significant employment of these capacities in scientific, physical and designing issues. Various extensions of some novel capacities were pondered in various works.

For exhibiting some new weighted hyper geometric capacities, we use the weighted development of Euler's beta-work [7]:

$$B^{(\alpha,\beta)}_{\omega}(x,y)=\int_0^1 t^{x-1}(1-t)^{y-1}\omega^{(\alpha,\beta)}(zt,u,v)dt, \qquad (1.1)$$

where $\mathrm{Re}\,x>0, \mathrm{Re}\,y>0$ plus α,β,z,u,v be genuine or else compound parameters plus $\omega(\alpha,\beta)$ (zt,u,v) be an element of the group of pupil's Ω, for example to such an extent that the basic is completely merged. In addition, by composing $B_\omega(x,y)$ we imply that this capacity and the capacity ω don't rely upon α and β. One can see that, if

$$w(t,p,o)=e^{\frac{-p}{t(1-t)}}$$

with $\mathrm{Re}\,p>0$, then $B_w(x,y)(\min\{\mathrm{Re}\,x,\mathrm{Re}\,y\}>0)$. Turns in to the expansion of Euler's beta-work wary through [8], Furthermore, through a clear count, it follow inside the common container so as to

$$B_w(x,y+1)+B_w(x+1,y)=B_w(x,y), w\in\Omega \tag{1.2}$$

In the most recent years, the enthusiasm to utilization of the partial subsidiary administrators a significant development. Various distributions have been committed to the arrangements of various issues by applying these administrators. Review that the outstanding Caputo partial subordinate is characterized as

$$D_z^{\mu}f(z)=\frac{1}{\Gamma(m-\mu)}\int_0^z(z-t)^{m-\mu-1}\frac{d^m}{dt^m}f(t)dt \tag{1.3}$$

Where at this time plus $m-1<\mathrm{Re}\,\mu<m, m\in N$ inside the accompanying, permit C, R, N, and $Z0^-$ are the arrangements of multi faceted statistics, genuine numbers, positive whole numbers, plus negative numbers, separately. We present the broad Caputo fragmentary subsidiary appropriate by means of the capacity through the accompanying speculation of so as to characterize in [13]

$$D_z^{\mu,\tau}f(z)=\frac{1}{\Gamma(m-\mu)}\int_0^z(z-t)^{m-\mu-1}\tau(t,u,v)\frac{d^m}{dt^m}f(t)dt, \tag{1.4}$$

where $m-1<\mathrm{Re}\,\mu<m, m\in N$ plus v, u is genuine otherwise multi faceted parameter plus τ is thought toward be alive an element of the group of pupils Λ, for example with the end goal that the essential is totally focalized. It is noticed that turns into the all-encompassing Caputo fragmentary subordinate [12] when

$$\tau(t,p,z)=e^{\frac{-pz^2}{t(z-t)}},\ (\mathrm{Re}\,p>0),$$

while it turns into the traditional Caputo fragmentary subsidiary when $\tau\equiv 1$.

2. Weighted Hypergeometric Functions

Inside a comparable structure as characterized inside [12], we present a numeral of weighted forms of Gauss hyper geometric capacity ${}_2F_1$, the Appell hyper geometric capacities F_1, F_2 [14] the Lauricella hyper Geometric work $F^3_{D;w'}$, the summed up Gauss hyper geometric work $F_\omega F_\omega$ and the summed up blended hyper geometric work ${}_1F_1{}^{\omega}$. Wherever inside this article we expect so as to $m\in N, \omega\in\Omega$ plus $B(x,y)(\min\{$ Rex, Rey $\}>0$ *min* is the traditional Beta capacity.

Definition 2.1. *The ω -weighted expanded Gauss hyper geometric capacity is*

$${}_2F_1(a,b;c;z;\omega)=\sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(b-m)_n}\frac{B_\omega(b-m+n,c-b+m)}{B(b-m,c-b+m)}\frac{z^n}{n!}, \tag{2.1}$$

where, $|z|<1$ *and* $m<\mathrm{Re}, b<\mathrm{Re}\,c$ *For a representation of Gauss hyper geometric capacity* [15,16].

Definition 2.2. *The ω -weighted broadened Appell hyper geometric capacity F_1 is*

$$\begin{aligned}&{}_2F_1(a,b,c;d;x;y;\omega)\\&=\sum_{n,k=0}^{\infty}\frac{(a)_{n+k}(b)_n(c)_k}{(a-m)_{n+k}}\frac{B_\omega(a-m+n+k,d-a+m)}{B(a-m,d-a+m)}\frac{x^n}{n!}\frac{y^k}{k!}\end{aligned} \tag{2.2}$$

Where $|x|<1|, |y|<1$ *and* $m<\mathrm{Re}, a<\mathrm{Re}\,d$.

Definition 2.3. *The ω-weighted expanded Appell hyper geometric capacity F_2 is*

$$\begin{aligned}&F_2(a,b,c;d,e;x,y;\omega)\\&=\sum_{n,k=0}^{\infty}\frac{(a)_{n+k}(b)_n(c)_k}{(b-m)_n(c-m)_k}\frac{B_\omega(b-m+n,d-b+m)}{B(b-m,d-b+m)}\\&\times\frac{B_\omega(c-m+k,e-c+m)}{B(c-m,e-c+m)}\frac{x^n}{n!}\frac{y^k}{k!}.\end{aligned} \tag{2.3}$$

Anywhere $|x|+|y|<1, m<\mathrm{Re}\,b<\mathrm{Re}\,d$, *and* $m<\mathrm{Re}\,c<\mathrm{Re}\,e$. *Furthermore, we simply believe one of the categories of Appell hyper geometric capacity* [12,17].

Definition 2.4. *The ω -weighted comprehensive Appell hyper geometric purpose $F^3_{D,\omega}$ is*

$$\begin{aligned}&F^3_{D,\omega}(a,b,c;d,e;x,y,z;\omega)\\&=\sum_{n,k,r=0}^{\infty}\frac{(a)_{n+k+r}(b)_n(c)_k(d)_r}{(a-m)_{n+k+r}(c-m)_k}\\&\times\frac{B_\omega(a-m+n+k+r,e-a+m)}{B(a-m,e-a+m)}\frac{x^n}{n!}\frac{y^k}{k!}\frac{z^r}{r!},\end{aligned} \tag{2.4}$$

where $\sqrt{|x|}+\sqrt{|y|}+\sqrt{|z|}<1$ *plus* $m<\mathrm{Re}\,a<\mathrm{Re}\,e$. *Reminder so as to the capacities characterized on top of turn into persons inside* [12] *when* $\omega(t,p,o)=e^{\frac{-p}{t(1-t)}}$ *and* $\mathrm{Re}p>0$. *Also, for* $\omega\equiv 1$, *these capacities decrease to the notable Gauss hyper geometric capacity* ${}_2F_1$. *Appell capacities* F_1, F_2 *and Lauricella work* $F^3_{D,\omega}$, *separately.*

Definition 2.5. *The ω -weighted comprehensive Gauss hyper geometric purpose F_ω is*

$$F_\omega(a,b,c;z)=\sum_{n=0}^{\infty}(a)_n\frac{B_\omega(b+n,c-b)}{B(b,c-b)}\frac{z^n}{n!} \tag{2.5}$$

Anywhere $|z|<1$ *and* $\mathrm{Re}, c>\mathrm{Re}, b>0$.

Definition 2.6. *The ω-weighted summed up blended hyper geometric capacity ${}_1F_1{}^{\omega}$be*

$${}_1F_1^{\omega}(b,c;z)=\sum_{n=0}^{\infty}\frac{B_\omega(b+n,c-b)}{B(b,c-b)}\frac{z^n}{n!} \tag{2.6}$$

Anywhere $|z|<1$ *and* $\mathrm{Re}\,c>\mathrm{Re}\,b>0$, *For certain instances of capacities* F_ω *and* ${}_1F_1{}^{\omega}$, [7].

Remark 2.7. *If*

$$\omega^{(\alpha,\beta)}(t,p,0) = {}_1F_1\left(\alpha;\beta;\frac{-p}{t(1-t)}\right).$$

where $\min\{\operatorname{Re} x, \operatorname{Re} y, \operatorname{Re}\alpha, \operatorname{Re}\beta\} > 0$ *and* $\operatorname{Re} p \geq 0$ *then* $B_\omega^{(\alpha,\beta)}(x,y)$ *is the function* $B_p^{(\alpha,\beta)}(x,y)$ *defined in* [5], *also, in* [18]. *Hence,* $F_\omega(a,b,c;z) = F_p^{(\alpha,\beta)}(a,b;c;z)$ *and* ${}_1F_1^\omega(b;c;z) = {}_1F_1^{(\alpha,\beta,p)}(b;c;z)$ *are equivalent to those in* [5,18]. *The next description, it determination exist helpful toward present a common outcome.*

Definition 2.8. *Permit*

$$f(z) := \sum_{n=0}^{\infty} a_n z^n \tag{2.7}$$

$$g(z) := \sum_{n=0}^{\infty} b_n z^n \tag{2.8}$$

be two power arrangements whose union or radii are R_f *and* R_g *separately. At that point Hadamard result of* $f(z)$ *plus* $g(z)$ *be the authority arrangement* [18,19]

$$(f^* g)(z) = \sum_{n=0}^{\infty} a_n b_n z^n. \tag{2.9}$$

Whose assembly range R fulfills the imbalance $R_f, R_g \leq R$.

Remark 2.9. *The above definitions can be considered with* ${B_\omega}^{(\alpha,\beta)}$ *rather than* B_ω.

3. Weighted Caputo Derivative

We set up a number of valuable proclamations; anywhere we exist relevant the weighted Caputo fragmentary subordinate. The subsequent outcomes be a few speculations of those in [12,14,18] and some others. Subsequently, the apparatuses to demonstrate them are comparable.

Lemma 3.1. *If* $m-1 < \operatorname{Re}\,\mu < m, \omega \in \Lambda, \omega \in \Omega, s, \omega \in C$ *and* $\operatorname{Re}\mu < \operatorname{Re}\lambda$ *then*

$$D_z^{\mu,\omega}\left[z^\lambda\right] = \frac{\Gamma(\lambda+1)B_\omega(\lambda-m+1,m-\mu)}{\Gamma(\lambda-\mu+1)B(\lambda-m+1,m-\mu)} z^{\lambda-\mu}. \tag{3.1}$$

Proof. Indeed,

$$\begin{aligned}
D_z^{\mu,\omega}&\left[z^\lambda\right] \\
&= \frac{1}{\Gamma(m-\mu)}\int_0^z (z-t)^{m-\mu-1}\omega(t,s,\omega)\frac{d^m}{dt^m}t^\lambda dt \\
&= \frac{\Gamma(\lambda+1)}{\Gamma(m-\mu)\Gamma(\lambda-m+1)} \\
&\quad \times \int_0^z (z-t)^{m-\mu-1}t^{\lambda-m}\omega(t,s,\omega)dt \\
&= \frac{\Gamma(\lambda+1)z^{\lambda-m}}{\Gamma(m-\mu)\Gamma(\lambda-m+1)} \\
&\quad \times \int_0^z (1-u)^{m-\mu-1}u^{\lambda-m}\omega(zu,s,\omega)du \\
&= \frac{\Gamma(\lambda+1)B_\omega(\lambda-m+1,m-\mu)}{\Gamma(\lambda-\mu+1)B(\lambda-m+1,m-\mu)} z^{\lambda-\mu}
\end{aligned} \tag{3.2}$$

□

Theorem 3.2. *If* $f(z) := \sum_{n=0}^{\infty} a_n z^n$ *is a scientific capacity in the circle* $|z| < p, \omega \in \Lambda$ *and* $\omega \in \Omega$ *then*

$$D_z^{\mu,\omega}\{f(z)\} = \sum_{n=0}^{\infty} D_z^{\mu,\omega}\{z^n\}. \tag{3.3}$$

Anywhere, $m-1 < \operatorname{Re}\mu < m$.

Proof. Since the authority arrangement combines consistently, vital of $D_z^{\mu,\omega}\{f(z)\}$ meets totally thus the ideal outcome pursues by a clear figuring. □

Theorem 3.3. *Let* $m-1 < \operatorname{Re}(\lambda-\mu) < m < \operatorname{Re}\lambda, K \in C, \omega \in \Lambda$ *and* $\omega \in \Omega$. *Then*

$$D_z^{\lambda-\mu,\omega}\left\{z^{\lambda-1}(1-z)^{-k}\right\} = z^{k-1}\frac{\Gamma(\lambda)}{\Gamma(\mu)}{}_2F_1(k,\lambda;\mu;z;\omega),\ |z|<1. \tag{3.4}$$

Proof. Utilizing authority arrangement of $(1-z)^{-x}$, Theorem, Lemma plus condition, we acquire

$$\begin{aligned}
D_z^{\lambda-\mu,\omega}\left\{z^{\lambda-1}(1-z)^{-k}\right\} &= D_z^{\lambda-\mu,\omega}\left\{z^{\lambda-1}\sum_{n=0}^{\infty}(k)_n\frac{z^n}{n!}\right\} \\
&= \sum_{n=0}^{\infty}\frac{(k)_n}{n!}D_z^{\lambda-\mu,\omega}\left\{z^{\lambda+n-1}\right\} \\
&= z^{\mu-1}\sum_{n=0}^{\infty}\frac{(k)_n\Gamma(\lambda+n)B_\omega(\lambda-m+n,m-\lambda+\mu)}{\Gamma(\mu+n)B(\lambda-m+n,m-\lambda+\mu)}\frac{z^n}{n!}
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
&= z^{\mu-1}\sum_{n=0}^{\infty}\frac{(k)_n(\lambda)_n}{(\lambda-m)_n}\frac{B_\omega(\lambda-m+n,m-\lambda+\mu)}{B(\lambda-m,\mu-\lambda+m)}\frac{z^n}{n!} \\
&= z^{k-1}\frac{\Gamma(\lambda)}{\Gamma(\mu)}{}_2F_1(k,\lambda;\mu;z;\omega)
\end{aligned}$$

□

Theorem 3.4. *If* $m-1<\operatorname{Re}(\lambda-\mu)<m<\operatorname{Re}\lambda, \omega\in\Lambda$ *and* $\omega\in\Omega$. *Then*

$$D_z^{\lambda-\mu,\omega}\left\{z^{\lambda-1}(1-rz)^{-k}(1-sz)^{-\theta}\right\} = z^{k-1}\frac{\Gamma(\lambda)}{\Gamma(\mu)}F_1(k,\lambda,\theta;\mu;rz;sz;\omega), \tag{3.6}$$

for $r,s,k,\theta\in C, |rz|<1$, *and* $|sz|<1$.

Proof. Utilizing authority arrangement of $(1-rz)^{-k},(1-sz)^{-\theta}$. Theorem, Lemma and, we get

$$\begin{aligned}
&D_z^{\lambda-\mu,\omega}\left\{z^{\lambda-1}(1-rz)^{-k}(1-sz)^{-\theta}\right\}\\
&=D_z^{\lambda-\mu,\omega}\left\{\sum_{n,k=0}^{\infty}\frac{(k)_n(\theta)_k r^n s^k z^{\lambda+n+k-1}}{n!k!}\right\}\\
&=\sum_{n,k=0}^{\infty}\frac{(k)_n(\theta)_k r^n s^k}{n!k!}D_z^{\lambda-\mu,\omega}\left\{z^{\lambda+n+k-1}\right\}\\
&=z^{\mu-1}\sum_{n,k=0}^{\infty}\frac{(k)_n(\theta)_k r^n s^k}{n!k!}\\
&\quad\times\frac{\Gamma(\lambda+n+k)B_\omega(\lambda-m+n+k,m-\lambda+\mu)}{\Gamma(\mu+n+k)B(\lambda-m+n+k,m-\lambda+\mu)}z^{n+k}
\end{aligned} \tag{3.7}$$

$$\begin{aligned}
&=z^{\mu-1}\frac{\Gamma(\lambda)}{\Gamma(\mu)}\sum_{n,k=0}^{\infty}\frac{(\lambda)_{n+k}(k)_n(\theta)_k}{(\lambda-m)_{n+k}}\\
&\quad\times\frac{B_\omega(\lambda-m+n+k,m-\lambda+\mu)}{B(\lambda-m,m-\lambda+\mu)}\frac{\Gamma(r)^n}{n!}\frac{(sz)^k}{k!}\\
&=z^{\mu-1}\frac{\Gamma(\lambda)}{\Gamma(\mu)}F_1(\lambda,k,\theta;\mu;rz;sz;\omega)
\end{aligned}$$

□

Theorem 3.5. *If* $m-1<\operatorname{Re}(\lambda-\mu)<m<\operatorname{Re}\lambda, \omega\in\Lambda\omega\in\Omega$ *and* $m<\operatorname{Re}\beta<\operatorname{Re}\gamma$. *Then,*

$$\begin{aligned}
&D_z^{\lambda-\mu,\omega}\left\{z^{\lambda-1}(1-z)^{-\alpha}{}_2F_1\left(\alpha,\beta,\gamma\frac{x}{1-z};\omega\right)\right\}\\
&\quad=z^{k-1}\frac{\Gamma(\lambda)}{\Gamma(\mu)}F_2(\alpha,\beta,\lambda,\gamma;\mu;x,z;\omega), |x|+|z|<1.
\end{aligned} \tag{3.8}$$

Proof. Through authority arrangement of $(1-az)^{-a}$ plus, we acquire

$$\begin{aligned}
&D_z^{\lambda-\mu,\omega}\left\{z^{\lambda-1}(1-z)^{-\alpha}{}_2F_1\left(\alpha,\beta,\gamma\frac{x}{1-z};\omega\right)\right\}\\
&=D_z^{\lambda-\mu,\omega}\left\{z^{\lambda-1}(1-z)^{-\alpha-n}\right\}\\
&\quad\times\sum_{n=0}^{\infty}\frac{(\alpha)_n(\beta)_n}{(\beta-m)_n}\frac{B_\omega(\beta-m+n,\gamma-\beta+m)}{B(\beta-m,\gamma-\beta+m)}\frac{x^n}{n!}\\
&=z^{\mu-1}\frac{\Gamma(\lambda)}{\Gamma(\mu)}\sum_{n,k=0}^{\infty}\frac{(\alpha)_{n+k}(\beta)_n(\lambda)_k}{(\beta-m)_n(\lambda-m)_k}\\
&\quad\times\frac{B_\omega(\beta-m+n,\gamma-\beta+m)}{B(\beta-m,\gamma-\beta+m)}\frac{B_\omega(\lambda-m+k,m-\lambda+\mu)}{B(\lambda-m,m-\lambda+\mu)}\frac{x^n}{n!}\frac{z^k}{k!}\\
&=z^{k-1}\frac{\Gamma(\lambda)}{\Gamma(\mu)}F_2(\alpha,\beta,\lambda,\gamma;\mu;x,z;\omega)
\end{aligned} \tag{3.9}$$

□

4. Basic Results of the GFC

The specific decision of the bit capacities guarantees a decay of our administrators into results of driving old style Erd'elyi-Kober administrators. In this way, entangled numerous integrals or vary indispensable articulations can be spoken to then again, by methods for single integrals including extraordinary capacities. The excellence and brevity of documentations plus property of these capacities permit advancement of a filled sequence of prepared principles, map property plus convolution association of summed up fragmentary integral just as a suitable express meaning of the comparing summed up subsidiaries. Then again, the successive appearance of organizations of old style partial administrators inside different issue of practical investigation give way in toward the extraordinary numeral of utilizations plus realized unique instances of our summed up fragmentary contrast integrals.
The primary useful spaces examined inside our articles resting on GFC be weighted spaces of persistent, Lebesgue integral or explanatory capacities: Let α,μ be subjective genuine, $k\geq 0$ and $1\leq p<\infty$ be numbers, the factors x,z be genuine or mind boggling, running resp. over the interim $[0,\infty)$ or in the area $\Omega\subset C$, star like as for the birthplace $z=0$ and let $H(\omega)$ represent the space of logical works in Ω. We utilize the significations:

$$\begin{aligned}
C_\alpha^{(k)}&=\left\{f(x)=x^p\tilde{f}(x):p>\alpha,\tilde{f}\in C^{(k)}[0,\infty)\right\}, C_\alpha^{(0)}=C_\alpha\\
L_{\mu,p}(0,\infty)&=\left\{f(x):\|f\|_{\mu,p}=\left[\int_0^\infty x^{\mu-1}|f(x)|dx\right]^{\frac{1}{p}}<\infty\right\}\\
H_\mu(\Omega)&=\left\{f(z)=z^\mu\tilde{f}(z);\tilde{f}(z)\in H(\Omega)\right\}, H_0(\Omega)=H(\Omega).
\end{aligned} \tag{4.1}$$

To consider the summed up fragmentary integrals, we have utilized basically the hypothesis of the G-and H-capacities, showing up as portion capacities. To this end, we elude the per user to the as of late showed up books on extraordinary capacities and fragmentary math, for instance [12] just as to Works of art. Note likewise, that the $G_{m,m^-}^{m,0}$ and $H_{m,m^-}^{m,0}$ capacities have three normal particular focuses $\sigma=0,1$ plus ∞, they evaporate intended for $|\sigma|>1$ plus be investigative capacities inside the component circle $|\sigma|<1$. Asymptotic conduct close $\sigma=0,1$ be as of now surely understood [28],[12] and guarantees the rightness of definitions inside the higher than places, under reasonable circumstances resting on parameter. A large portion of the essential results for the administrators of the summed up fragmentary math have been expressed in [17] independently for the instances of G-and H-capacities and for all the previously mentioned spaces. Here we uncover them in one form in particular and in the most trademark space and just notice the analogs for the others.

Theorem 4.1. *Each numerous $E-K$. Partial indispensable [16] saves the power capacities in $C_\alpha, \alpha \geq \max_k[-\beta(\gamma k+1)]$ up to a steady multiplier: $I^{(\gamma_k),(\delta_k)}_{(\beta_k),m}\{x^p\}=c_p x^p, p>\alpha$, Where*

$$c_p=\prod_{k=1}^{m}\frac{\Gamma\left(\gamma_k+\frac{p}{\beta_k}+1\right)}{\Gamma\left(\gamma_k+\delta_k+\frac{p}{\beta_k}+1\right)} \tag{4.2}$$

What's more, it is an invertible mapping

$$I^{(\gamma_k),(\delta_k)}_{(\beta_k),m}: c_\alpha \mapsto c_\alpha^{(\eta_k+\eta_2+\ldots+\eta_m)} \subset c_\alpha$$

If the file α of $C\alpha$ is fixed, at that point the conditions on the parameters are as per the following

$$\gamma_k \geq -\frac{\alpha}{\beta_k}-1, \delta_k>0,$$
$$\eta_k=\begin{cases}[\delta_k]+1, & \text{for noninteger } \delta_k \\ [\delta_k], & \text{for integer } \delta_k\end{cases} \quad k=1,\ldots\ldots,m$$

Comparable recommendation holds likewise in the space $H_\mu(\omega)$, expressed as pursues.

Theorem 4.2. *Let the conditions*

$$\gamma_k>-\frac{\mu}{\beta_k}-1, \delta_k>0, k=1,\ldots\ldots,m$$

Be fulfilled. At that point, the different $E.-K$. Administrator maps the class $H\mu(\omega)$ keen on itself, saving authority capacities awake toward steady multiple similar to in plus picture of the authority arrangement

$$f(z)=z^\mu\sum_{n=0}^{\infty}a_n z^n=z^\mu(a_0+a_1z+\ldots)\in H_\mu(\Delta_R), \tag{4.3}$$

where, $\Delta_R=\{|z|<R\}$, $R=\left\{\lim\operatorname{Sup}_{n\to\infty}\sqrt[n]{|a_n|}\right\}^{-1}$, is given by the arrangement

$$I^{(\gamma_k),(\delta_k)}_{(\beta_k,m}f(z)=z^\mu\sum_{n=0}^{\infty}\left\{a_n\prod_{k=1}^{m}\frac{\Gamma\left(\gamma_k+\frac{n+\mu}{\beta_k}+1\right)}{\Gamma\left(\gamma_k+\delta_k+\frac{n+\mu}{\beta_k}+1\right)}\right\}z^n. \tag{4.4}$$

Having a similar span of assembly $R>0$ and similar indications of the coefficients Administrator can be revamped in the structure

$$I^{(\gamma_k),(\delta_k)}_{(\beta_k),m}f(z)=\frac{1}{z}\int_0^z H^{m,0}_{m,m}\left[\frac{t}{z}\middle|\begin{array}{c}\left(\gamma_k+\delta_k+1-\frac{1}{\beta_k},\frac{1}{k_k}\right)_1^m \\ \left(\gamma_k+1-\frac{1}{\beta_k},\frac{1}{\beta_k}\right)_i^m\end{array}\right]f(t)dt. \tag{4.5}$$

Furthermore, along these lines, it tends to be placed as a convolution type necessary change, specifically:

$$I^{(\gamma_k),(\delta_k)}_{(\beta_k),m}f(z)=\int_0^\infty k\frac{z}{t}f(t)\frac{dt}{t}=(k\circ f)(z), \tag{4.6}$$

where $\circ$ means the Mellin convolution. In this way we get the accompanying.

Lemma 4.3. *Various $E.-K$. Partial vital [16] has the accompanying convolution type portrayal in $L_{\mu,p}$*

$$I^{(\gamma_k),(\delta_k)}_{(\beta_k),m}f(z)=H^{m,0}_{m,m}\left[\frac{1}{z}\middle|\begin{array}{c}\left(\gamma_k+\delta_k+1\frac{1}{\beta_k},\frac{1}{\beta_k}\right)_1^m \\ \left(\gamma_k+1-\frac{1}{\beta_k},\frac{1}{\beta_k}\right)_1^m\end{array}\right] of(z). \tag{4.7}$$

Furthermore, for $1\leq p\leq 2$ its Mellin change is given by the correspondence

$$\begin{aligned}&\mu\left\{I^{(\gamma_k),(\delta_k)}_{(\beta_k),m}f(z);s\right\}\\&=\left[\prod_{k=1}^{m}\frac{\Gamma\left(\gamma_k-\frac{s}{\beta_k}+1\right)}{\Gamma\left(\gamma_k+\delta_k-\frac{s}{\beta_k}+1\right)}\right]\mu\{f(z);s\}\end{aligned} \tag{4.8}$$

Utilizing portrayal and following the example of [33], it is anything but difficult to demonstrate the accompanying suggestion.

Theorem 4.4. *Let the parameters of the different $E.-K$. fragmentary basic fulfill the conditions*

$$\beta_k(\gamma_k+1)>\frac{\mu}{p}-1, \delta_k>0, k=1,\ldots\ldots,m$$

At that point $I^{(\gamma_k),(\delta_k)}_{(\beta_k),m}, f(z)$ exists wherever on top of $(0,\infty)$ plus it be a limited direct administrator as of the Banach gap $L\mu,p$ keen on itself. All the additional precisely,

$$\left\|I^{(\gamma_k),(\delta_k)}_{(\beta_k),m}f\right\|_{\mu,p}\leq h_{\mu,p}\|f\|_{\mu,p}. \tag{4.9}$$

For example, $\left\|I^{(\gamma_k),(\delta_k)}_{(\beta_k),m}f\right\|\leq h_{\mu,p}$, with

$$h_{\mu,p}=\prod_{k=1}^{m}\frac{\Gamma\left(\gamma_k-\frac{\mu}{p\beta_k}+1\right)}{\Gamma\left(\gamma_k+\delta_k-\frac{\mu}{p\beta_k}+1\right)}<\infty. \tag{4.10}$$

From the properties of the H-and G-works some quick conclusions of definitions pursue.

Theorem 4.5. *Assume conditions for $C\alpha$, for $H\mu$ or for $L_{\mu,p}$ hold. At that point, in the above spaces the accompanying essential operational standards of the different $E.-K.$ Partial integrals hold:*

$$I_{(\beta_k),m}^{(\gamma_k),(\delta_k)} f\{\lambda f(cz)+\eta g(cz)\} = \lambda \left\{I_{(\beta_k),m}^{(\gamma_k),(\delta_k)} f\right\}(cz)+\eta\left\{I_{(\beta_k),m}^{(\gamma_k),(\delta_k)} g\right\}(cz). \tag{4.11}$$

Bilinearity

$$I_{(\beta_1,\ldots,\beta_m),m}^{(\gamma_1,\ldots,\gamma_s,\gamma_{s+1},\ldots,\gamma_m),(0,\ldots,0,\delta_{s+1},\ldots,\delta_m)} f(z) = I_{(\beta_{s+1},\ldots,\beta_m),m}^{(\gamma_{s+1},\ldots,\gamma_m),(\delta_{s+1},\ldots,\delta_m)} f(z). \tag{4.12}$$

If $\delta_1,\delta_2,\ldots,\delta_\delta=0$, at that point variety decreases toward $m-s$

$$I_{(\beta_k),m}^{(\gamma_k),(\delta_k)} z^\lambda f(z) = z^\lambda I_{(\beta_k),m}^{\left(\gamma_k+\frac{\lambda}{\beta_k}\right),(\delta_k)} f(z),\ \lambda\in\Box \tag{4.13}$$

Commutability of administrators of structure. The left-hand side of

$$I_{\left((\beta_k)_1^m,(\varepsilon_j)_1^n\right),m+n}^{\left((\gamma_k)_1^m,(\tau_j)_1^n\right),\left((\delta_k)_1^m,(\alpha_j)_1^n\right)} f(z). \tag{4.14}$$

Creations of n-tuple and m-tuple integrals provide $m+n$ tuple of similar structure

$$I_{(\beta_k),m}^{(\gamma_k+\delta_k),(\sigma_k)} I_{(\beta_k),m}^{(\gamma_k),(\delta_k)} f(z) = I_{(\beta_k),m}^{(\gamma_k),(\sigma_k+\delta_k)} f(z) \tag{4.15}$$

$f(z)$, if $\delta_k>0,\sigma_k>0,k=1,2,\ldots,m$. Law of records, item rule or semi bunch property

$$\left\{I_{(\beta_k),m}^{(\gamma_k),(\delta_k)}\right\}^{-1} f(z) = I_{(\beta_k),m}^{(\gamma_k+\delta_k),(-\delta_k)} f(z) \tag{4.16}$$

Formal reversal equation.
The above reversal equation observes from the list law intended for $\sigma_k=-\delta_k<0,k=1,2,\ldots,m$ plus meaning intended for nil multi-request of incorporation, as:

$$I_{(\beta_k),m}^{(\gamma_k+\delta_k),(-\delta_k)} I_{(\beta_k),m}^{(\gamma_k),(\delta_k)} f(z) = I_{(\beta_k),m}^{(\gamma_k),(0,\ldots,0)} f(z) = f(z) \tag{4.17}$$

Be that as it may, images contain not so far be characterized intended for unconstructive multi requests of incorporation $-\delta_k<0$, $k=1,2,\ldots,m$. The issue has been toward suggest a fitting importance intended for them plus thus toward keep away from unique integral showing up. The circumstance is actually equivalent to in the traditional situation at what time. $E.-K.$ also $R.-L$, administrators of fragmentary request $\delta>0$ can be upset through engaging an extra separation of appropriate whole number request $\eta=[\delta]+1$. For this situation, we have utilized the accompanying differential recipe for the portion H-work, resp. for the H-or the G-work): Let η $k\geq 0,k=1,\ldots,m$ be discretionary numbers, at that point

$$H_{m,m}^{m,0}\left[\frac{t}{z}\left|\begin{matrix}\left(a_k,\frac{1}{\beta_k}\right)_1^m\\ \left(b_k,\frac{1}{\beta_k}\right)_1^m\end{matrix}\right.\right] = D_\eta H_{m,m}^{m,0}\left[\frac{t}{z}\left|\begin{matrix}\left(a_k+\eta_k,\frac{1}{\beta_k}\right)_1^m\\ \left(b_k,\frac{1}{\beta_k}\right)_1^m\end{matrix}\right.\right] \tag{4.18}$$

With differential administrator D_η organism polynomial of $z\left(\frac{d}{dz}\right)$ of quantity $\eta=\eta_1+\ldots+\eta_m$,

$$D_\eta = \prod_{r=1}^{m}\prod_{j=1}^{\eta_r}\left(\frac{1}{\beta_r}z\frac{z}{dz}+a_r-1+j\right) \tag{4.19}$$

This equation is expanding parameters $a_k,k=1,2,\ldots,m$ of the H-work inside higher line through self-assertive whole numbers, $\eta_k\geq 0,k=1,2,\ldots,m$ by utilizing reasonable administrator. D_η Picking properly the essential parameters, we have demonstrated that $D_{(\beta_k),m}^{(\gamma_k),(\delta_k)}$ the administrator of structure, is without a doubt a summed up fragmentary subsidiary with a straight right backwards administrator $I_{(\beta_k),m}^{(\gamma_k),(\delta_k)}$ to be specific:

$$D_{(\beta_k),m}^{(\gamma_k),(\delta_k)} I_{(\beta_k),m}^{(\gamma_k),(\delta_k)} f(z) = f(z), f\in L_{\mu,p}, C_\alpha \text{ or } H_\mu. \tag{4.20}$$

At the end of the day, we have for instance in $L_{\mu,p}$ the accompanying.

Theorem 4.6. *Let $f\in L_{\mu,p}$, let conditions (30) be fulfilled and $g(z)=I_{(\beta_k),m}^{(\gamma_k),(\delta_k)} f(z)$ At that point, the accompanying reversal recipe holds with the summed up fragmentary subsidiary characterized:*

$$f(z) = D_{(\beta_k),m}^{(\gamma_k),(\delta_k)} g(z). \tag{4.21}$$

Example 4.7.

$$f(z) = \left\{I_{(\beta_k),m}^{(\gamma_k),(\delta_k)}\right\}^{-1} g(z) = D_{(\beta_k),m}^{(\gamma_k),(\delta_k)} g(z) \tag{4.22}$$

for

$$g\in I_{(\beta_k),m}^{(\gamma_k),(\delta_k)}\left(L_{\mu,p}\right). \tag{4.23}$$

Next, we will express the fundamental result for the summed up partial integrals [14], [16] *proposing their elective name various (m-tuple) fragmentary integrals.*

Theorem 4.8. *Under the conditions, the traditional $E.-K.$ fragmentary integrals of structure $I_{(\beta_k)}^{(\gamma_k),(\delta_k)}$, $k=1,\ldots m$, drive in the space $L_{\mu,p}$ and their item*

$$\begin{aligned} I_{\beta_m}^{\gamma_m,\delta_m}\left\{I_{\beta_{m-1}}^{\gamma_{m-1},\delta_{m-1}}\ldots\left(I_{\beta_1}^{\gamma_1,\delta_1} f(z)\right)\right\} &= \left[\prod_{k=1}^{m} I_{\beta_k}^{\gamma_k,\delta_k}\right] f(z) \\ &= \int_0^1 (\dot{m}) \int_0^1 \left[\prod_{k=1}^{m}\frac{(1-\sigma_k)^{\delta_k-1}\sigma_k^{\gamma_k}}{\Gamma(\delta_k)}\right] \\ &\quad\times f\left(z\sigma_1^{\frac{1}{\beta_1}}\ldots\sigma_m^{\frac{1}{\beta_m}}\right) d\sigma_1\ldots d\sigma_m \end{aligned} \tag{4.24}$$

Can be spoken to as a m-tuple E,K. administrator [16], *for example by methods for a solitary vital including the H-work:*

$$\left[\prod_{k=1}^{m} I_{\beta_k}^{\gamma_k,\delta_k}\right] f(z) = I_{(\beta_k),m}^{(\gamma_k),(\delta_k)} f(z)$$
$$= \int_0^1 H_{m,m}^{m,0}\left[\sigma \left| \begin{array}{c} \left(\gamma_k+\delta_k+1-\frac{1}{\beta_k},\frac{1}{\beta_k}\right)_1^m \\ \left(\gamma_k+1-\frac{1}{\beta_k},\frac{1}{\beta_k}\right)_1^m \end{array}\right.\right] f(z\sigma)d\sigma, \tag{4.25}$$

where, $f \in L_{\mu,p}$ (resp. C_α, H_μ). On the other hand, under similar conditions, each different $E-K$. Administrator of structure (16) can be spoken to as an item.
Give us a chance to take note of that a similar suggestion, under extra confinements, holds for the summed up partial subordinates [21], [22] *too: they can be viewed as results of E.- K. Fragmentary subordinates analogs of R-L subsidiaries [3] relating to E-K integrals [4] of the structure*

$$D_\beta^{\gamma,\delta} f(z) = D_{\beta,1}^{\gamma,\delta} f(z) = D_\eta I_\beta^{\gamma+\delta,\eta-\delta} f(z)$$
$$= \left[\prod_{j=1}^{\eta} \frac{1}{\beta} z\frac{d}{dz}+\gamma+j\right]_0^1 \frac{(1-\sigma)^{\eta-\delta-1}\sigma^{\gamma+\delta}}{\Gamma(\eta-\delta)} f\left(z\sigma^{\frac{1}{\beta}}\right)d\sigma.$$

In particular

$$D_{(\beta_k),m}^{(\gamma_k),(\delta_k)} = D_{\beta_1}^{\gamma_1,\delta_1} D_{\beta_2}^{\gamma_2,\delta_2} \ldots D_{\beta_m}^{\gamma_m,\delta_m}. \tag{4.26}$$

Remark 4.9. *Recently,* [21] *have presented and considered the supposed E-K partial subordinates of Caputo type, as expansions of the Caputo alteration of the R-L fragmentary subsidiaries. About the Caputo subsidiaries Blend of Theorems, prompts the subsequent stage in explaining the structure of assortment of known administrators: summed up or old style, fragmentary or whole number request reconciliations, separations or contrast incorporations. Specifically, in* [17] *we present a bound together hypothesis dependent on the regular thought summed up fragmentary vary integrals. At this point, administrators $I_{(\beta_k),m}^{(\gamma_k),(\delta_k)}$ by means of everyone $\delta_k \geq 0$, $k=1,2,\ldots,m$ contain be measured because integral as individuals by means of everyone $\delta_k \leq 0$, $k=1,2,\ldots,m$ contain be attempted because official significations intended for summed up fragmentary subordinates*

$$I_{(\beta_k),m}^{(\gamma_k+\delta_k'),(\delta_k')} = D_{(\beta_k),m}^{(\gamma_k),(\delta_k')} \tag{4.27}$$

Example 4.10. $I_{(\beta_k,m}^{(\gamma_k),(\delta_k)} = D_{(\beta_k),m}^{(\gamma_k+\delta_k),(-\delta_k)}$ *Presently, having a disintegration hypothesis as a primary concern, we may think about the two images $I_{(\beta_k),m}^{(\gamma_k)(\delta_k)}, D_{(\beta_k,m}^{(\gamma_k),(\delta_k)}$ as summed up partial vary integrals. If not the entirety of the segments of multi-request of vary joining $\delta = (\delta_1,\delta_2,\ldots,\delta_m)$ be of a similar symbol, we basically decipher them because "blended" results of E.K. partial integral plus subordinates. Intended for instance, Condition $\delta_1 < 0,\ldots,\delta_s < 0, \delta_{s+1} = \delta_{s+2} = \ldots = \delta_{s+j} = 0, \delta_{s+j+1} > 0,\ldots,\delta_m > 0$, at that point*

$$I_{(\beta_k),m}^{(\gamma_k),(\delta_k)} = D_{(\beta_1,\ldots,\beta_s),s}^{(\gamma_1+\delta_1,\ldots,\gamma_s+\delta_s),(-\delta_1,\ldots,-\delta_s)} I_{(\beta_{s+j+1},\ldots,\beta_m),m-s-j}^{(\gamma_{s+j+1},\ldots,\gamma_m),(\delta_{s+j+1},\ldots,\delta_m)}$$
$$= \prod_{i=1}^{s} D_{\beta_i}^{\gamma_i+\delta_i,-\delta_i} \prod_{k=s+j+1}^{m} I_{\beta_k}^{\gamma_k,\delta_k} \tag{4.28}$$

Remark 4.11. *An announcement more broad than Theorem, can be found for instance in* [9,10], *It manages results of driving E-K partial integrals both of structures* [4] *(R-L type) and their right-hand sided analogs. At that point the outcome is GFC administrator including portion elements of structure $H_{m+n,m+n}^{m,n}$ rather* [13]. *Hypothesis is the way in to the various uses of the GFC administrators. Some of them can be found in the monograph* [17] *and articles* [1,18,19,20]. *For different properties of these administrators, pictures of rudimentary and uncommon capacities and subtleties of the GFC outlined.*

5. Conclusion

We would like to discover some engineering applications identified with our new outcomes. Additionally, we investigate the potential outcomes to discover arrangements of incomplete differential conditions or differential conditions regarding our outcomes. In addition, we be attempting toward compose weighted hyper geometric function similar to Poisson integral thinking of several as uncommon loads to discover limit esteems, factorizations of this function.

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