Modified Bernoulli vacation batch arrival and retrial clients in a single server queuing model with server utilization

V. Rajam[1]* and S. Uma[2]

Abstract
This paper investigates a batch arrival feedback retrial queue with two types of service under modified Bernoulli vacation where each type consists of an optional re-service where the busy server is subjected to starting failures In Poisson form, the consumer comes to the system in batches but can also baulk at certain specific times. Customers may re-service the same type without joining the orbit after the completion of each type of service or may leave the system. The server either goes on vacation at the completion stage of each service or can wait for the next client to serve. The model is analysed during the supplementary variable technique and the probability generating function of system size, the server utilization and the probability that the system is empty are found. Stochastic decomposition law is shown to hold good for this model also when there is no bulking permitted along with other performance measures to predict the behaviour of the system are derived. Further, we carry out some special cases for the proposed model.

Keywords
Modified Bernoulli Vacation, Retrial Queue, Single server Queue, Server Utilization.

AMS Subject Classification
60K25.

[1] *Department of Mathematics, Rajah Serfoji Government College, Affiliated to Bharathidasan University, Thanjavur-613005, Tamil Nadu, India.*
[2] *Department of Mathematics, Dharmapuram Gannambigai Government Women's College, Mayilduthurai-609001, Tamil Nadu, India.*
***Corresponding author**: [1]rajamramv@gmail.com; [2]umamaths@gmail.com

Contents

1. Introduction

Queuing system is a powerful tool in many real life problems. It has been used to optimize the model to predict the queue length and waiting time.[1] It plays a vital role in communication systems, production line, supermarkets, restaurants, hospitals, wireless sensor networks etc. The fundamental idea of a queue is to provide a structured and coordinated method to sort people based on the precedence of their respective time of arrival. Queue happens when a node's processing time is higher than the arrival rate, allowing the people to have a queue waiting time. A queue therefore provides a systematic approach to assigning node preference; this reduces chaos in the operation of the service [2, 3]. The first client to come in is serviced first, and the client to come in last is added to the end of the queue. A FIFO (First in First out) technique follows real world queues [4]. A significant characteristic of healthcare procedures (or facilities in general) is that resource demand is mostly unplanned. Patients can be directed to various facilities during consultation. It is not deterministic to route a patient through health centers. Instead, there is probabilistic routing during the diagnosis stage. In addition, in many cases, patients need several appointments before leaving the medical centre. In other words, to construct the queuing model, the re-entry of patients needs to be taken care of. The server will become unavailable in vacation queuing systems for a random period of time when there are no patients in the waiting queue at the time of service completion [5]. This is seen in maintenance activities such

as telecommunication networks, customized manufacturing systems, production systems, etc. The queuing models are helpful in determining the levels of capacity (and capacity allocation) necessary to respond to demands in a timely manner (minimizing the delay). The queuing model of server vacations (server absences) has been well studied and successfully implemented in many sectors, such as manufacturing / service and computer / communication network systems. The arrival processes, operation practices, and vacation policies can be classified according to these vacation queuing models.[6]

2. Literature Survey

The classical vacation method with the discipline of the Bernoulli schedule was introduced and studied by Keilson and Servi [7]. Pavai Madheswari et al [8] investigated the M / G/1 retrial queuing method with two phases of operation on which the phase two is optional and the server operating under the Bernoulli vacation schedule that the server will wait for the next customer to arrive with probability 1-a even if there are no customers in the method, and choose to go on vacation with probability a. This particular form of Bernoulli vacation is referred to in queuing literature as the modified Bernoulli vacation schedule. A single server batch arrival retrial queue model has been analyzed by Nila and Sumitha [9] with the server subject to starting failures, customer baulks and renege at unique times, the server offers two phases of operation and customer search from the orbit. When a client's service is unsatisfied, the service can be retried again before the service is successfully completed.

In the service facility of a queuing system, a remarkable and inevitable occurrence is its breakdown. The waiting period for consumers in the system rises until the failed service facility is recovered, resulting in customer impatience [10]. The M / G/1 retrial queue was discussed by Yang and Li [11] with the server subjected to initial failures in which they obtained empirical results for the distribution of queue length and proved the stochastic decomposition law in which the distribution of retrial time is presumed to be exponential.

Haight first researched the notion of baulking in 1957, Hence, assumes in this model that an arriving batch can enter or leave the system to enter the system for a service. The definition of batch arrival retrial queuing method and baulking has been used in most of the research work. Singh et al[12] addressed on vacation and baulking, it offers service at lower rates or stops the service altogether. Wu et al[13] and Gilbert[14] presented retrials and baulks with baulking in the M / G/1. Re-service is also an important factor in the theory of queuing system and have many applications in the real world.

Re-service is also an significant element in queuing system theory and has many real-world implementations. There may be circumstances where re-service is needed, such as while visiting a doctor, the patient may be referred for some inquiries, during which he might need to see the doctor again while a customer might find his service unsatisfied in other

cases providing public service. Authors such as Madan et al [15], Jeyakumar and Arumuganathan [16] have actually been researching queues with re-service. We presume that clients arrive in lots and the system offers standard and optional re-service, the client may choose re-service for the same service. We will find many implementations of this model in real life. These circumstances are primarily found in the telephone consultation of medical care systems and in the fields of information systems, banks, post offices, etc.

3. The Proposed Mathematical Model Description

Consider the Modified Bernoulli vacation of batch arrival and retrial queue with baulking customers, starting failures and two forms of services are discussed in this article. As follows, a detailed description of the model is given:

1. Customers arrives in batches according to an arrival of a job follows a compound Poisson process with λ. Let X_i denotes the number of customers belonging to the ith arrival batch where X_i, $i = 1, 2, 3, \ldots$ $P[X_i = n] = C_i$, $n = 1, 2, 3, \ldots$. The probability generating function $X(z)$ and having the first two moments are $E(X)$, $E(X(X-1))$.

2. If there is no waiting space, and then if an arriving customer realises that the server is busy fulfilling its request immediately, all these customers exit the service area and enter a pool of blocked customers called orbit. Later, the clients in the orbit try to get their operation. Successive retrial times of every customer have an arbitrary distribution of probabilities. $A(x)$ with the corresponding density function $a(x)$ and Laplace-Stieljies transform (LST) $A^\star(x)$. For retrial time, the conditional completion rates is $a(x)dx = \dfrac{dA(x)}{1 - A(x)}$.

3. There is a single server which provides two types of service and there is a option for reservice to each job. FCFS(First come first served) is the service discipline. As soon as regular service is completed may opt for optional reservice for the same service taken without joining the orbit with probability r or may leaves the system with probability $\bar{r} = 1 - r$. It is assumed that the service time follows general random variable S_1 and S_2 with distribution function $S_1(t)$ for normal service and $S_2(t)$ for optional re-service and LST $S_1^\star(x)$, $S_2^\star(x)$ and the first and second moments are $E(S_1)$, $E(S_1^2)$, $E(S_2)$, $E(S_2^2)$ respectively.The conditional completion rates for service is $\mu_1(x)dx = \dfrac{dS_1(x)}{1 - S_1(x)}$, $\mu_2(x)dx = \dfrac{dS_2(x)}{1 - S_2(x)}$

4. After the customer is served completed, he will decide either to join the retrial group for another service with probability p or to leave the system forever with probability $(q = 1 - p)$

5. The server may go for a vacation of random length V with probability $0 \leq a \leq 1$ an after the completion of service to each customer or may wait in the service facility to serve the next customer with probability $1 - a$. It is assumed that when there are no customers waiting in the orbit, the server waits in the system for a new customer with probability $1 - a$. At the end of vacation, if there are no customers waiting in

the orbit, the server waits for the one to arrive. The vacation time of the server V, vacation time has $V(t)$ and LST $V^\star(x)$ distribution functions. $E(V)$, $E(V^2)$ are the first and second moments. The conditional completion rates for vacation is

$$\upsilon(x)dx = \frac{dV(x)}{1-V(x)}$$

6. During repair, the server stops providing service to the arriving batch of customers. With the probability distribution function R(x), the successive repair time is independent and identically distributed and the first and second moments are $E(R)$, $E(R^2)$. The conditional completion rates $r(x)dx = \frac{dR(x)}{1-R(x)}$.

7. The server may fail with a probability prior to the commencement of the service $\bar{\alpha} = 1 - \alpha$. If the server is successfully started, the customer gets service immediately, which is the probability of a successful start of service. Otherwise, the time for server maintenance begins and a later retrial is rendered by the service field. The state of the system at time t can be described by the Markov process $\{N(t) : t \geq 0\} = \{C(t), X(t), \xi(t) : t \geq 0\}$ where $C(t)$ denotes the state of the server defined as $C(t) = 0$, if the server is idle

$\Rightarrow 1$, if the server is busy with regular service

$\Rightarrow 2$, if the server is busy with optional re-service

$\Rightarrow 3$, if the server is under repair

$\Rightarrow 4$, if the server is on vacation

$X(t)$ represents the number of customers in orbit at time t

$\Rightarrow C(t) = 0$, represents the elapsed retrial time at time t

$\Rightarrow C(t) = 1$, represents the elapsed service time in regular service at time t

$\Rightarrow C(t) = 2$, represents the elapsed service time in optional re-service at time t

$\Rightarrow C(t) = 3$, represents the elapsed repair time at time t

$\Rightarrow C(t) = 4$, represents the elapsed vacation time at time t

Ergodicity Condition:

At departure completion epochs, we test the embedded Markov chain, Let be the series of epochs where either a service cycle is ended or a vacation period ends or a repair period ends. Epoch series where either a service cycle is finished or a vacation time finishes or a maintenance period ends. The sequence of random vectors $Z_n = \{C(t_n+), X(t_n+)\}$ form a Markov chain, which is embedded Markov chain for our retrial queuing system **Define the following probabilities**

$P_0(t)$ is the probability that the server is idle at time t, and there is no customer in the retrial orbit $P_n(x,t)dx$ is the joint probability that at time t there are exactly n customers in the orbit, the server is idle and the elapsed retrial time of a customer in the orbit is between x and $x+dx$ $\pi_{1,n}(x,t)dx$ is the joint probability that at time t there are exactly n customers in the orbit, the server is providing service with the elapsed regular service time is between x and $x+dx$ $\pi_{2,n}(x,t)dx$ is the joint probability that at time t there are exactly n customers in the orbit, the server is providing service with the elapsed re-service time is between x and $x+dx$ $R_n(x,t)dx$ is the joint probability that at time t there are exactly n customers in the orbit with the elapsed service time of the test customer un-

dergoing service is x and the elapsed repair time is between x and $x+dx$ $V_n(x,t)dx$ is the probability that at time t there are exactly n customers in the orbit with the elapsed vacation time is x.

4. Steady State Distribution

$P_0(t) = P\{C(t) = 0, X(t) = 0\}$

$P_n(x,t)dx = P\{C(t) = 0, X(t) = n, x \leq \xi(t) < x+dx\}, n \geq 0, t \geq 0$

$\pi_{1,n}(x,t)dx = P\{C(t) = 1, X(t) = n, x \leq \xi(t) < x+dx\}, t \geq 0, x \geq 0, n \geq 0$

$\pi_{2,n}(x,t)dx = P\{C(t) = 2, X(t) = n, x \leq \xi(t) < x+dx\}, t \geq 0, x \geq 0, n \geq 0$

$R_n(x,t)dx = P\{C(t) = 3, X(t) = n, x \leq \xi(t) < x+dx\}, t \geq 0, (x,y) \geq 0, n \geq 1$

$V_n(x,t)dx = P\{C(t) = 4, X(t) = n, x \leq \xi(t) < x+dx\}, n \geq 0$

We assume that the stability condition is fulfilled in the sequence and so that the limiting probabilities

$P_0(t) = \lim\limits_{n\to\infty} P_0(t)$, for $n \geq 1$

$P_n(x) = \lim\limits_{n\to\infty} P_n(x,t)$, for $x \geq 0, n \geq 1$

$\pi_{1,n}(x) = \lim\limits_{n\to\infty} \pi_{1,n}(x,t)$, for $t \geq 0, x \geq 0, n \geq 0$

$\pi_{2,n}(x) = \lim\limits_{n\to\infty} \pi_{2,n}(x,t)$, for $t \geq 0, x \geq 0, n \geq 0$

$R_n(x) = \lim\limits_{n\to\infty} R_n(x,t)$, for $t \geq 0$

$V_n(x) = \lim\limits_{n\to\infty} V_n(x,t)$, for $t \geq 0$

Steady state equations By the method of supplementary variable technique, we obtain the system of equations that govern the dynamics of the system behavior under steady state as

$$\lambda P_0 = (1-a)\bar{r}\int_0^\infty \pi_{1,0}(x)\mu_1(x)dx + (1-a)q\int_0^\infty \pi_{2,0}(x)\mu_2(x)dx + \int_0^\infty V_0(x)\upsilon(x)dx$$

$$\frac{dP_n(x)}{dx} + (\lambda + \theta(x))P_n(x) = 0, n \geq 1$$

$$\frac{d\pi_{1,0}(x)}{dx} + (\lambda(1-b) + \mu_1(x))\pi_{1,0}(x) = 0, n = 0$$

$$\frac{d\pi_{1,n}(x)}{dx} + [\lambda(1-b) + \mu_1(x)]\pi_{1,n}(x) = \lambda(1-b)\sum_{k=1}^{n} C_k\pi_{1,n-k}(x), n \geq 1$$

$$\frac{d\pi_{2,0}(x)}{dx} + (\lambda(1-b) + \mu_2(x))\pi_{2,0}(x) = 0, n = 0$$

$$\frac{d\pi_{2,n}(x)}{dx} + (\lambda(1-b) + \mu_2(x))\pi_{2,n}(x) = \lambda(1-b)\sum_{k=1}^{n} C_k\pi_{2,n-k}(x), n \geq 1$$

$$\frac{dR_1(x)}{dx} + (\lambda(1-b) + r(x))R_1(x) = 0, n = 0$$

$$\frac{dR_n(x)}{dx} + (\lambda(1-b) + r(x))R_n(x) = \lambda(1-b)\sum_{k=1}^{n} C_k R_{n-k}(x), n \geq 2$$

$$\frac{dV_0(x)}{dx} + (\lambda(1-b) + \upsilon(x))V_0(x) = 0, n = 0$$

$$\frac{dV_n(x)}{dx} + (\lambda(1-b) + \upsilon(x))V_n(x) = \lambda(1-b)\sum_{k=1}^{n} C_k V_{n-k}(x),$$

$n \geq 1$

The steady state boundary conditions are

$$P_n(x) = (1-a)\bar{r} \int_0^\infty \pi_{1,n}(x)\mu_1(x)dx + (1-a)q \int_0^\infty \pi_{2,n}(x)\mu_2(x)dx$$

$$+ (1-a)p \int_0^\infty \pi_{2,n-1}(x)\mu_2(x)dx + \int_0^\infty V_n(x)\upsilon(x)dx + \int_0^\infty R_n(x)r(x)dx,$$

$n \geq 1$

$$\pi_{1,n}(0) = \alpha \int_0^\infty P_1(x)\theta(x)dx + \alpha\lambda C_1 P_0$$

$$\pi_{1,n}(0) = \alpha \int_0^\infty P_{n+1}(x)\theta(x)dx + \alpha\lambda C_{n+1}P_0, \, n \geq 1$$

$$\pi_{2,n}(0) = r \int_0^\infty \pi_{1,n}(x)\mu_1(x)dx, \, n \geq 0$$

$$R_1(0) = \bar{\alpha} \int_0^\infty P_1(x)\theta(x)dx + \bar{\alpha}\lambda P_0, \, n = 1$$

$$R_n(0) = \bar{\alpha} \int_0^\infty P_1(x)\theta(x)dx + \bar{\alpha}\lambda \sum_{k=1}^n C_k \int_0^\infty P_{n-k}(x)dx, \, n \geq 2$$

$$V_n(x) = a\bar{r} \int_0^\infty \pi_{1,n}(x)\mu_1(x)dx + aq \int_0^\infty \pi_{2,n}(x)\mu_2(x)dx$$

$$+ ap \int_0^\infty \pi_{2,n-1}(x)\mu_2(x)dx, \, n \geq 0$$

The normalizing condition is

$$P_0 + \sum_{n=1}^\infty \left[\int_0^\infty P_n(x)dx + \int_0^\infty R_n(x)dx \right]$$

$$+ \sum_{n=0}^\infty \left[\int_0^\infty \pi_{1,n}(x)dx + \int_0^\infty \pi_{2,n}dx + \int_0^\infty V_n(x)dx \right] = 1$$

To solve the above equations, then we define the generating functions for $|z| \leq 1$

$$P(x,z) = \sum_{n=1}^\infty P_n(x)z^n; \, P(0,z) = \sum_{n=1}^\infty P_n(0)z^n$$

$$\pi_1(x,z) = \sum_{n=0}^\infty \pi_{1,n}z^n; \, \pi_1(0,z) = \sum_{n=0}^\infty \pi_{1,n}(0)z^n$$

$$\pi_2(x,z) = \sum_{n=0}^\infty \pi_{2,n}z^n; \, \pi_2(0,z) = \sum_{n=0}^\infty \pi_{2,n}(0)z^n$$

$$R_n(x,z) = \sum_{n=1}^\infty R_n(x)z^n; \, R_n(0,z) = \sum_{n=0}^\infty R_n(0)z^n$$

$$V(x,z) = \sum_{n=0}^\infty V_n(x)z^n; \, V_n(0,z) = \sum_{n=0}^\infty V_n(0)z^n.$$

RESULT AND DISCUSSION

The study focused on a single server and delayed M /G/1 queuing method vacation with server state switching. It is assumed in this model that the patient arrival pattern (λ) is according to the Poisson process and the service is performed on a single server that is distributed exponentially with the parameter μ Patients can be directed to various facilities during consultation. It is not deterministic to route a patient through health centres. Instead, there is probabilistic routing during the diagnosis stage. In addition, in many cases, patients need several appointments before leaving the medical centre. In other words, the queuing model needs to take care of patients re-entry to build, additional work at the top on the new patients. It is related to re-entry in most cases.

The following assumptions are made in this model: FCFS is the queue discipline adhered to by each of the stations. Any variance in patient arrival (e.g. the early, late, unannounced or not showing up of patients) is assumed to be absorbed by the variance of the arrival process.
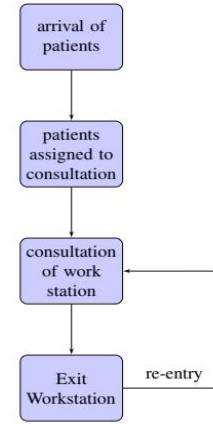

Figure -1 : Patients Arrival and Re-Entry Framework

1. If the server finds that only one server is allowed to go on vacation at a time.

2. If one of the servers detects patients on the system and the server is busy with $(n-1)$

3. Sever can only take a vacation in two consecutive hours of service.

4. Arrival Rate: Is the mean rate of arrivals per unit of time lambda (λ) 1 hour $\Rightarrow$ 6 patients (arrival rate 10 min per patients).

5. Service Rate: The average amount of service per unit of time rendered. The Greek letter Mu (μ)

6. Service Discipline: Gives the importance of the collection of patients to be served.

patients	server service			re-entry time
	start	end	vacation	
P01	7:00:00	7:10:00	0:00:00	0:00:00
P02	7:11:00	7:21:00	0:01:00	7:31:00
P03	7:21:00	7:31:00	0:01:00	8:01:00
P02	7:31:00	7:41:00	0:01:00	0:00:00
P04	7:41:00	7:51:00	0:01:00	0:00:00
P05	7:51:00	8:01:00	0:01:00	0:00:00
P03	8:01:00	8:11:00	0:01:00	0:00:00
P06	8:11:00	8:21:00	0:01:00	0:00:00
P07	8:21:00	8:31:00	0:01:00	8:41:00
P08	8:31:00	8:41:00	0:01:00	0:00:00
P07	8:41:00	8:51:00	0:01:00	0:00:00

Table 1: Single Server Queue - Patients arrival and Server Service and Vacation rate

The Table 1 dataset further proves this. For the last patient, this queue system will see that the waiting period reaches 8 hours and 51 minutes and that the doctor is unable to take care of more than 11 patients within the specified period. The need

to apply the queuing principle in healthcare settings is very important because it affects someone's well-being and life. The time a patient spends waiting for a physician to attend is important for the patient and the reputation of the hospital before the public.

Considering that the buffers in front of the consultation workstation correspond to their respective waiting lists, it would be wrong to restrict them in size, In order to conduct a decomposition-based queuing analysis, aggregation of the arrival and service process is required in view of the re-entry environment of the queuing system. The following table 2 illustrates the re-entry, arrival, and service rate of the patient.

patients	server service			re-entry time
	start	end	vacation	
P01	7:00:00	7:10:00	0:00:00	0:00:00
P02	7:11:00	7:21:00	0:01:00	7:31:00
P03	7:21:00	7:31:00	0:01:00	8:01:00
P02	7:31:00	7:41:00	0:01:00	0:00:00
P04	7:41:00	7:51:00	0:01:00	0:00:00
P05	7:51:00	8:01:00	0:01:00	0:00:00
P03	8:01:00	8:11:00	0:01:00	0:00:00
P06	8:11:00	8:21:00	0:01:00	0:00:00
P07	8:21:00	8:31:00	0:01:00	8:41:00
P08	8:31:00	8:41:00	0:01:00	0:00:00
P07	8:41:00	8:51:00	0:01:00	0:00:00

Table 2: Single Server Queue - Server Service rate and Patient Re-Entry the system

If the numbers of persons that arrive are less than the expected number, most of the servers get wasted (idle). The queuing model must take care of patient re-entry to establish with care of Strength and reliability on the new patients at the level. In most instances, it is associated to re-entry. Table 2 shows the re-entry of P02, P03 and P08 patients in order to provide delayed care to other patients.

	Number of Patients Entering the system	System Total Service Time	Effective Service Time	Server Vacation Time
Without Re-Entry	11	02:00:00	01:41:00	00:19:00
Re-Entry	8	02:00:00	01:41:00	00:19:00

Table 3 : Comparison of Single Server System Re-entry

The findings refer to the numerical example shown in Table 3. The following Figure 2 graphically describes the phenomenon. The patient's re-entry is evident, so the total number of patient services decreased

Conclusion

This research paper analysed a modified Bernoulli vacation of batch arrival and retrial queue with balking customers,
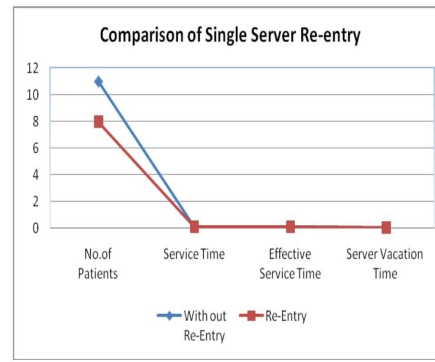

Figure 2: Comparing Single Server Delayed Vacation and Number of Patients Service

starting failures and two types of service. A vacation queuing models with server vacation depends on the batch sizes in this present study and the state of switching over is taken into consideration. Various performance measures like the probability that the server is idle, busy, repair in steady state and mean orbit size, mean system size are derived. The research on the present investigation can be further extended by including the concepts of working breakdown, Bernoulli vacation. Application of this paper results are useful to healthcare system, communication networks, manufacturing process and transportation, production lines and mail systems.

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