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Mixed domination in an M-strong fuzzy graph

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Abstract

In this paper, mixed dominating set, mixed domination number , mixed strong domination number and mixed weak domination number of an M-strong fuzzy graph $G = (\sigma, \mu)$ are defined. Also these numbers are determined for various standard fuzzy graphs. The relationship between these numbers and other well known numbers are derived.

Keywords: Fuzzy graph, M-strong fuzzy graph, mixed domination number, mixed strong domination number, mixed weak domination number.

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1 Introduction

Zadeh [13] introduced the concept of Fuzzy sets in the year 1965. In 1975, Fuzzy graph was introduced by Rosenfeld [7]. Rosenfeld has obtained the fuzzy analogues of several basic graph-theoretic concepts like bridges, paths, cycles, trees, connectedness and established some of their properties. Fuzzy trees were characterized by Sunitha and Vijayakumar [11]. They have obtained a characterization for blocks in fuzzy graphs using the concept of strongest paths [12]. Bhutani and Rosenfeld have introduced the concepts of strong arcs , fuzzy end nodes and geodesics in fuzzy graphs [2]. Mordeson and Peng [6] introduced strong fuzzy graph using effective edges. Bhutani and Battou [1] consider the strong fuzzy graph of Mordeson and Peng as M-strong fuzzy graph.

The concept of domination in fuzzy graphs was defined by Somasundaram and Somasundaram [9]. The vertex neighbourhood number and edge neighbourhood number of an M-strong fuzzy graphs are introduced by S. Ismail Mohideen and A. Mohamed Ismayil [3, 4].

Mixed domination in crisp graph was introduced by E. Sampathkumar and S.S. Kamath [8]. In this paper, Mixed dominating set and mixed domination number in an M-strong fuzzy graph are defined. Mixed strong domination number and mixed weak domination number in an M-strong fuzzy graph are also defined. Theorems related to these mixed dominating sets and mixed domination numbers are stated and proved. The relation between these numbers and other well known parameters are derived.

2 Preliminaries

Definition 2.1. *Let V be a finite non empty set and E be the collection of two element subsets of V. A fuzzy graph $G = (\sigma, \mu)$ is a set with two functions $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$ such that $\mu(u, v) \le \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.*

Definition 2.2. *Let $G = (\sigma, \mu)$ be a fuzzy graph defined on V and $S \subseteq V$. Then the* scalar cardinality *of S is defined by $\sum_{u \in S} \sigma(u)$. The* order *(denoted by p) and* size *(denoted by q) of a fuzzy graph $G = (\sigma, \mu)$ are the scalar cardinality of σ and μ respectively.*

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Definition 2.3. *A fuzzy graph $G_1 = (\sigma_1, \mu_1)$ is called the fuzzy sub graph induced by V_1 if $\sigma_1(u) \leq \sigma(u)$ for all $u \in V_1$ and $\mu_1(u,v) \leq \sigma_1(u) \wedge \sigma_1(v) \wedge \mu(u,v)$ for all $u, v \in V_1$ and is denoted by $\langle V_1 \rangle$. A fuzzy graph $G_1 = (\sigma_1, \mu_1)$ is called the full fuzzy sub graph induced by V_1 if $\sigma_1(u) = \sigma(u)$ for all $u \in V_1$ and $\mu_1(u,v) = \mu(u,v)$ for all $u, v \in V_1$ and is denoted by $\langle\langle V_1 \rangle\rangle$.*

Definition 2.4. *An edge $e = (u,v)$ of a fuzzy graph is called an effective edge if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$. If $e = (u,v)$ is an effective edge, then u and v are adjacent vertices and e is incident with u and v. A fuzzy graph $G = (\sigma, \mu)$ is said to be M-strong fuzzy graph [1] if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for all $(u,v) \in E$. That is, In an M-strong fuzzy graph every edge is an effective edge.*

Definition 2.5. *A fuzzy graph $G = (\sigma, \mu)$ is said to be* complete fuzzy graph *if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$. That is, In a complete fuzzy graph every pair of verices should have an effective edge.*

Definition 2.6. *Let $u, v \in V$ and $e = (u,v) \in E$ then $N(u) = \{v \in V : \mu(u,v) = \sigma(u) \wedge \sigma(v)\}$ is called* open neighbourhood *of u and $N[u] = N(u) \cup \{u\}$ is called* closed neighbourhood *of u. $N[e] = N(u) \cup N(v)$ is called* closed neighbourhood *of e. If $N(u) = \phi$ then u is said to be* isolated vertex.

Definition 2.7. *The* neighbourhood degree *of a vertex u is defined to be the sum of the weights of the vertices adjacent to u and is denoted by $d_N(u)$, the* minimum neighbourhood degree *is $\delta_N(u) = min\{d_N(u) : u \in V\}$ and the* maximum neighbourhood degree *is $\Delta_N(G) = max\{d_N(u) : u \in V\}$.*

Definition 2.8. *A fuzzy graph $G = (\sigma, \mu)$ is said to be* bipartite *if the vertex set V can be partitioned into two sets V_1 defined on σ_1 and V_2 defined on σ_2 such that $\mu(v_1, v_2) = 0$ if $(v_1, v_2) \in V_1 \times V_1$ or $(v_1, v_2) \in V_2 X V_2$.*

Definition 2.9. *A bipatite fuzzy graph $G = (\sigma, \mu)$ is said to be* complete bipartite *if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for all $u \in V_1$ defined on σ_1 and $v \in V_2$ defined on σ_2 and is denoted by K_{σ_1, σ_2}.*

Definition 2.10. *A* path *in a fuzzy graph G is a sequence of distinct vertices $u_0, u_1, u_2, \ldots, u_n$ such that $\mu(u_{i-1}, u_i) = \sigma(u_{i-1}) \wedge \sigma(u_i), 1 \leq i \leq n$, $n > 0$ is called the length of the path. The path in a fuzzy graph is called a* fuzzy cycle *if $u_0 = u_n, n \geq 3$.*

Definition 2.11. *A fuzzy graph is said to be* cyclic *if it contains at least one cycle, otherwise it is called* acyclic.

Definition 2.12. *A fuzzy graph is said to be connected if there exists at least one path between every pair of vertices.*

Definition 2.13. *A connected acyclic fuzzy graph is said to be a* tree.

Definition 2.14. *A vertex in a fuzzy graph having only one neighbour is called a* pendent vertex. *Otherwise it is called non-pendent vertex.*

Definition 2.15. *An edge in a fuzzy graph incident with a pendent vertex is called a* pendent edge. *Otherwise it is called non-pendent edge.*

Definition 2.16. *A vertex in a fuzzy graph adjacent to the pendent vertices is called a* support *of the pendent edges.*

Definition 2.17. *[10] A* vertex covering *of fuzzy graph G is a subset K of V such that every effective edge of G has at least one end in K. The minimum scalar cardinality of vertices in K is called a vertex covering number of G and is denoted by α_0. α_0-set is a vertex cover with minimum scalar cardinality. Similarly edge cover number(α_1), vertex independence number(β_0) and edge independence number (β_1) can be defined.*

Theorem 2.1. *[10] For any fuzzy graph G, $\alpha_0 + \beta_0 = p$.*

Definition 2.18. *Let $G = (\sigma, \mu)$ be a fuzzy graph and let $u, v \in V$. If $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ then u dominates v (or v is dominated by u) in G. A subset D of V is called a* dominating set *in G if for every $v \notin D$ there exist $u \in D$ such that u dominates v. The minimum scalar cardinality taken over all dominating set is called* domination number *and is denoted by the symbol γ.*

Definition 2.19. *A set $S \subseteq V$ in an M-strong fuzzy graph $G(\sigma, \mu)$ is a* vertex neighbourhood set *of G if $G = \cup_{u \in S} \langle\langle N[u] \rangle\rangle$, where $\langle\langle N[u] \rangle\rangle$ is the full fuzzy sub graph of G induced by $N[u]$ and is denoted by n-set. The minimum scalar cardinality taken over all n-set of G is called* vertex neighbourhood number *and is denoted by n_0.*

Theorem 2.2. *[3] For any M-strong fuzzy graph G without isolated vertices. Then*

1. $\gamma(G) \leq n_0(G) \leq \alpha_0(G)$
2. $n_0(G) \leq \alpha_1(G)$

Corollary 2.1. *[3] If G is a M-strong fuzzy graph without isolated vertices and having no triangles, then* $n_0(G) = \alpha_0(G)$.

Definition 2.20. *Let* $e = (u, v)$ *be an edge in an M-strong fuzzy graph* $G(\sigma, \mu)$. *A set* $M \subseteq E$ *in G is an* edge neighbourhood set *of G if* $G = \cup_{u \in S} \langle\langle N[e] \rangle\rangle$ *, where* $\langle\langle N[e] \rangle\rangle$ *is a full induced fuzzy sub graph of G and is denoted by en-set. The minimum scalar cardinality taken over all en-set of G is called* edge neighbourhood number *and is denoted by* n_0.

Theorem 2.3. *[4] For any M-strong fuzzy graph G*

1. $\gamma - m \leq n_1 \leq n_0$ *, where m is the number of edges in minimum en-set.*
2. $n_1 \leq \gamma_1 \leq min(\alpha_0, \alpha_1, \beta_1)$.
3. $n_1 \leq \beta_0$.
4. $n_1 \leq p/2$*, where p is the order of G.*

3 Mixed Domination in an M-strong fuzzy graph

Definition 3.21. *Let* $G = (\sigma, \mu)$ *be an* $M-$*strong fuzzy graph defined on* V. *A vertex* $v \in V$ *dominates an edge* $e \in E$ *if* $e \in \langle\langle N[v] \rangle\rangle$ *Where* $\langle\langle N[v] \rangle\rangle$ *is a full induced fuzzy sub graph of G. An edge* $e = (u, v) \in E$ *dominates* $v \in V$ *if* $v \in N[e]$*, where* $N[e] = N(u) \cup N(v)$.

Note 1. *If v dominates e, then e dominates v but the converse is not true.*

Example 3.1. *In the fuzzy graph given in Figure 1, Here* e_1 *dominates* v_3 *but* v_3 *does not dominate* e_1 .

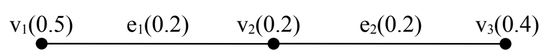

Figure 1:

Now, using this concept the vertex-edge dominating set , edge-vertex dominating set and mixed dominating sets in an M-strong fuzzy graphs are defined.

Definition 3.22. *A set* $S \subset V$ *in an M-strong fuzzy graph G is a* vertex-edge dominating set $(ved - set)$ *if every edge of G is dominated by a vertex in S. The minimum scalar cardinality taken over all ved-set is called* ve-domination number *and is denoted by the symbol* γ_{ve}. *A ved-set with minimum scalar cardinality is called* γ_{ve}*-set. The* Γ_{ve} *is called the maximum scalar cardinality of a minimal ved-set of G.*

Remark 3.1. *Every* $n_0 - set$ *in an M-strong fuzzy graph without isolated vertices is an* $\gamma_{ve} - set$ *and converse also true. That is* $\gamma_{ve} = n_0$.

Definition 3.23. *A set* $M \subseteq E$ *in an M-strong fuzzy graph G is an* edge-vertex dominating set $(evd - set)$ *if every vertex of G is dominated by an edge in M. The scalar cardinality taken over all evd-set is called* ev-domination number *and is denoted by the symbol* γ_{ev}. *An evd-set with minimum scalar cardinality is called* γ_{ev}*-set. The* Γ_{ev} *is called the maximum scalar cardinality of a minimal ved-set of G.*

Remark 3.2. *Every* $n_1 - set$ *in an M-strong fuzzy graph without isolated vertices is an* $evd - set$ *but converse not true. That is* $\gamma_{ev} \leq n_1$.

Definition 3.24. *A set* $D \subseteq V \cup E$ *in an M-strong fuzzy graph G is a* mixed dominating set (md-set) *if*

1. *every vertex* $v \notin D$ *is dominated by at least an edge* $e \in D$ *and*

2. *every edge $e \notin D$ is dominated by at least one vertex in $v \in D$.*

The minimum scalar cardinality taken over all md-set is called mixed domination number *and is denoted by the symbol γ_m. Γ_m is called the maximum scalar cardinality of a minimal md-set of G.*

Note 2. *A ved-set with minimum scalar cardinality is called γ_{ve}-set, similarly γ_{ev}-set and γ_m-set.*

Observation 1. *Let K_σ be a complete fuzzy graph with more than two vertices defined on V, $\gamma_{ve} = \gamma_{ev} = min_{u \in V}\sigma(u)$ and $\gamma_m = 2min_{u \in V}\sigma(u)$.*

Observation 2. *Let K_σ be a complete fuzzy graph with two vertices defined on V, $\gamma_{ve} = \gamma_{ev} = \gamma_m = min_{u \in V}\sigma(u)$.*

Observation 3. *Let K_{σ_1,σ_2} be a complete bipartite fuzzy graph, σ_1 defined on V_1 and σ_2 defined on V_2 respectively and $V = V_1 \cup V_2$. Then $\gamma_{ve} = min\{|\sigma_1|, |\sigma_2|\}$, $\gamma_{ev} = min_{u \in V}\sigma(u)$ and $\gamma_m = \gamma_{ve} + \gamma_{ev}$.*

Theorem 3.4. *For any M-strong fuzzy graph without isolated vertices G*
$\gamma_{ev} \leq \gamma_{ve} \leq \gamma_m \leq \gamma_{ev} + \gamma_{ve}$.
proof: *The first inequality follows from the fact that an evd-set is obtained by choosing one edge incident at each vertex v in $\gamma_{ve} - set$. The second inequality follows from the fact that by replacing each of the edges in γ_m-set by one of its end vertices with minimum membership grade, we get a ved-set. The last inequality follows from the fact that the union of ved-set and an evd-set is an md-set.*

Note 3. *Let $\alpha_0, \alpha_1, \beta_0$ and β_1 are the vertex cover, edge cover, vertex independent and edge independent numbers of a fuzzy graph G.*

Theorem 3.5. *For any M-strong fuzzy graph G without isolated vertices , The following results are true:*

1. *$\gamma_{ve} \leq \alpha_0$, where α_0 is a vertex cover number of G.*
2. *$\gamma_{ev} \leq \beta_0$ where β_0 is a vertex independent number of G.*
3. *$\gamma_m \leq p$, where p is the order of G.*

proof: *(1)From the remark 3.1, $\gamma_{ve} = n_0$ and from the theorem 2.2(1), $n_0 \leq \alpha_0$. Hence $\gamma_{ve} \leq \alpha_0$.*
(2)From the remark 3.2, $\gamma_{ev} = n_1$ and from the theorem 2.3(3), $n_1 \leq \beta_0$. Hence $\gamma_{ev} \leq \beta_0$.
(3) By theorem 3.4, $\gamma_m \leq \gamma_{ev} + \gamma_{ve} \leq \alpha_0 + \beta_0$ and theorem 2.1 $\alpha_0 + \beta_0 = p$. Hence $\gamma_m \leq p$.

Theorem 3.6. *If G is an M-strong fuzzy graph without isolated vertices and no triangles. Then*

1. $\gamma_{ve} = \alpha_0$
2. $\gamma_{ev} \leq \gamma \leq \gamma_{ve}$
3. $\gamma_m \leq \gamma_{ve} + \frac{p}{2}$
4. $\gamma_m \leq \gamma + \alpha_0$.

proof:*(1)From the remark 3.1, $\gamma_{ve} = n_0$ and from the corrolery 2.1, $n_0 = \alpha_0$. Hence $\alpha_0 = \gamma_{ve}$.*
(2) Every dominating set is an evd-set, because a vertex v in a dominating set dominates only adjacent vertices but en edge e in $\gamma_{ev} - set$ dominates adjacent vertices of both the end vertices of e. Hence $\gamma_{ev} \leq \gamma$. Every ved-set is a dominating set, since G has no triangles. Hence $\gamma \leq \gamma_{ve}$.
(3) An edge will definitely dominate at least two vertices in an evd-set, therefore $\gamma_{ev} \leq \frac{p}{2} \Rightarrow \gamma_{ev} \leq \frac{p}{2} + \gamma_{ve} + \gamma_{ev} - \gamma_m$ by theorem 3.4. Hence $\gamma_m \leq \gamma_{ve} + \frac{p}{2}$.
(4) From theorem 3.4, $\gamma_m \leq \gamma_{ev} + \gamma_{ve} \leq \gamma + \alpha_0$

Theorem 3.7. *For any M-strong fuzzy graph G without isolated vertices, $\gamma_m \leq min\{p, q\}$. Where p and q be the order and size of G respectively.*
proof: *Let G be an M-strong fuzzy graph without isolated vertices and let p and q be the order and size of G.*
The vertex set $V = \{v_1, v_2, v_3, \ldots, v_n\}$ is an $md - set$, since very edge of E is incident with at least two vertices in V. Hence $\gamma_m \leq p - - - -(i)$.
The edge set $E = \{e_1, e_2, e_3, \ldots, e_m\}$ is also an $md - set$, since every vertex of V is incident with at least one edge in E. Hence $\gamma_m \leq q - - - -(ii)$.
From (i) and (ii) we obtain $\gamma_m \leq min\{p, q\}$.

Theorem 3.8. *Let T_σ be a tree in an M-strong fuzzy graph G. If r and s are the scalar cardinality of the pendent vetices and supports of the pendent edges of T_σ respectively. Then $\gamma_m(T_\sigma) \leq p + s - r - \sigma_0$, where $\sigma_0 = min_{u \in V} \sigma(u)$.*
proof: *Let T_σ be a tree in an M-strong fuzzy graph. Given r and s are the scalar cardinalitry of the pendent vertices and supports of the pendent edges of T_σ respectively.*

Let M, N and R be the set of all non-pendent edges, supports of the pendent edges and non-pendent vertices in T_σ respectively. Then the Union of M and N form an md-set. Therefore $\gamma_m \leq |M| + |N|$ —–(1)
The set of non-pendent edges of T_σ also form a tree. Therefore $|M| \leq |R| - \sigma_0$, where $\sigma_0 = min_{u \in V} \sigma(u)$. From (1) $\gamma_m(T_\sigma) \leq |R| - \sigma_0 + |N| \leq p - r + s - \sigma_0$.

Theorem 3.9. *Let G be an M-strong fuzzy graph without isolated vertices and $|N(v)| = \Delta_N$, if $e_i = (v, v_i), 1 \leq i \leq n$, $r = \sum_{i=1}^{n} \mu(e_i)$ and $s = min\{\mu(e_i)\}$, $i = 1$ to n. Then $\gamma_m \leq p + q - \Delta_N - r + s$.*
proof: *Let v be a vertex of an M-strong fuzzy graph G and $\{v_1, v_2, \ldots, v_n\}$ open neighbourhood set of v. Let Δ_N be the maximum neighbourhood degree of G. That is $|N(v)| = \Delta_N$. If $e_i = (v, v_i), 1 \leq i \leq k$, $r = \sum_{i=1}^{k} \mu(e_i)$ and $s = min\{\mu(e_i)\}$, $i = 1$ to k . Then the set $\{V - \{v_1, v_2, \ldots, v_k\}\} \cup \{E - \{e_1, e_2, \ldots, e_k\}\} \cup \{e_i\}$ such that e_i is the minimum of $\mu(e_i), \forall i$, is an md-set. Therefore $\gamma_m \leq p + q - \Delta_N - r + s$.*

4 Mixed strong(Weak) Domination in an M-strong fuzzy graph

Definition 4.25. *Let $v \in V$ and $e = (u, v) \in E$ in an M-strong fuzzy graph G. Then*

1. *v and e* strongly dominates *each other if $e \in \langle\langle N[v] \rangle\rangle$ and*
2. *v and e* weakly dominates *each other if $v \in N[e]$.*

Definition 4.26. *A set $D \subseteq V$ in an M-strong fuzzy graph G is a* vertex-edge strong dominating set *of G, if every edge in G is strongly dominated by at least one vertex in D. It is denoted by $vesd - set$. The minimum scalar cardinality taken over all vesd-set is called* vertex-edge strong domination number *and it is denoted by the symbol γ_{ves}.*
Similarly edge-vertex strong domination number *(γ_{evs}),* vertex-edge weak domination number *(γ_{vew}) and* edge-vertex weak domination number *(γ_{evw}) can be defined.*

Observation 4. *For any M-strong fuzzy graph G without isolated vertices:*

1. *a vertex v dominates an edge e $\Leftrightarrow$ a vertex v strongly dominates an edge e. Therefore $\gamma_{ves} = \gamma_{ve}$.*
2. *an edge e dominates a vertex v $\Leftrightarrow$ an edge e weakly dominates a vertex v. Therefore $\gamma_{evw} = \gamma_{ev}$.*
3. *a vertex v dominates an edge e $\Rightarrow$ a vertex v weakly dominates an edge e. Therefore $\gamma_{vew} \leq \gamma_{ve}$.*
4. *an edge e strongly dominates a vertex v $\Rightarrow$ an edge e dominates a vertex v. Therefore $\gamma_{ev} \leq \gamma_{evs}$.*

Remark 4.3. *For some of the M-strong fuzzy graph G that we considered, there is no relation exist between γ_{vew} and γ_{evw}.*

Example 4.2. *Consider the M-strong fuzzy graphs given in Figures 2 and Figure 3.*

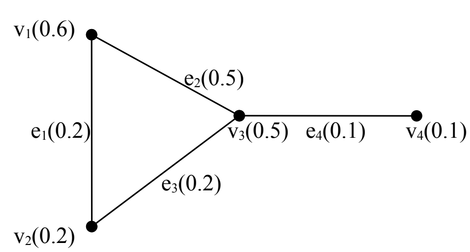

Figure 2:

From the Figure. $2\gamma_{vew} - set = \{v_2\} \Rightarrow \gamma_{vew} = 0.2$ *and*
$\gamma_{evw} = \{e_4\} \Rightarrow \gamma_{evw} = 0.1$

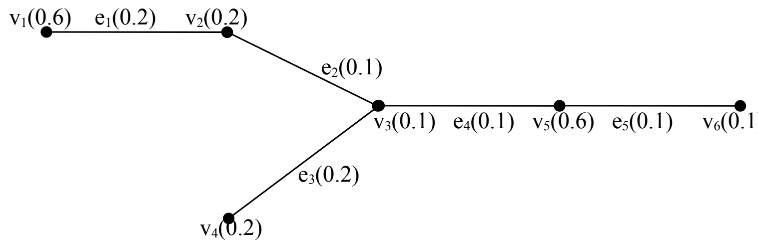

Figure 3:

Hence $\gamma_{evw} \leq \gamma_{vew}$.
From the Figure. 3 $\gamma_{vew} - set = \{v_3\} \Rightarrow \gamma_{vew} = 0.1$ *and*
$\gamma_{evw} = \{e_2, e_5\} \Rightarrow \gamma_{evw} = 0.2$
Hence $\gamma_{vew} \leq \gamma_{evw}$. *Therefore there is no relation exist between* γ_{vew} *and* γ_{evw}.

Remark 4.4. *Similarly, for some of the M-strong fuzzy graph G that we considered, there is no relation exist between* γ_{ves} *and* γ_{evs}.

Example 4.3. *Consider the M-strong fuzzy graphs given in Figure 4 and Figure 5.*

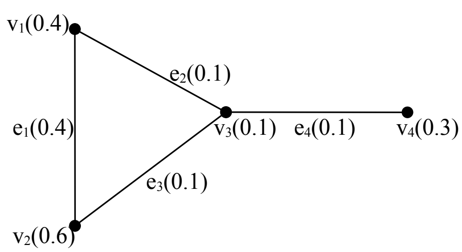

Figure 4:

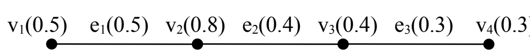

Figure 5:

From the Figure. 4 $\gamma_{ves} - set = \{v_3\} \Rightarrow \gamma_{ves} = 0.1$ *and*
$\gamma_{evs} = \{e_2, e_4\} \Rightarrow \gamma_{evs} = 0.2$
Hence $\gamma_{ves} \leq \gamma_{evs}$.

From the Figure. 5 $\gamma_{ves} - set = \{v_1, v_3\} \Rightarrow \gamma_{ves} = 0.9$ *and*
$\gamma_{evs} = \{e_1, e_3\} \Rightarrow \gamma_{evs} = 0.8$

Hence $\gamma_{evs} \leq \gamma_{ves}$. *Therefore there is no relation exist between* γ_{ves} *and* γ_{evs}.

Theorem 4.10. *For any M-strong fuzzy graph without isolated vertices G,*

1. $\gamma_{vew} \leq \gamma_{ves} = \gamma_{ve}$.
2. $\gamma_{ev} = \gamma_{evw} \leq \gamma_{evs}$.

Proof. (1) From the Observation 4(1) $\gamma_{ves} = \gamma_{ve}$ and the Observation 4(3)$\gamma_{vew} \leq \gamma ve$. Hence $\gamma_{vew} \leq \gamma_{ves} = \gamma_{ve}$.
(2) From the Observation 4(2) $\gamma_{evw} = \gamma_{ev}$ and the Observation 4(4)$\gamma_{ev} \leq \gamma evs$. Hence $\gamma_{ev} = \gamma_{evw} \leq \gamma_{evs}$. □

Definition 4.27. *A set* $D \subseteq V \cup E$ *in an M-strong fuzzy graph G is a* mixed strong(weak) dominating set *of G, if*

1. *every vertex* $v \in V$ *not in D is strongly(weakly) dominated by at least one edge in D and*

2. *every edge $e \in E$ not in D is strongly(weakly) dominated by at least one vertex in D.*

The mixed strong(weak) dominating set is denoted by msd-set(mwd-set). The minimum scalar cardinality taken over all msd-set(mwd-set) is called mixed strong(weak) domination number and it is denoted by the symbol $\gamma_{ms}(\gamma_{mw})$.

Theorem 4.11. *For any M-strong fuzzy graph without isolated vertices G,*

1. $\gamma_{vew} \leq \gamma_{mw} \leq \gamma_{vew} + \gamma_{evw}$.
2. $\gamma_{ves} \leq \gamma_{ms} \leq \gamma_{ves} + \gamma_{evs}$.

Proof. (1) Let $S = \{v_1, v_2, \ldots, v_m, e_{m+1}, e_{m+2}, \ldots, e_n\}$ be a $\gamma_{mw} - set$ in an M-strong fuzzy graph G. Replace each e_j of S by v_j such that $\sigma(v_j) = \mu(e_j)$, $m+1 \leq j \leq n$ and form the s'. Therefore $s' = \{v_1, v_2, \ldots, v_m, v_{m+1}, v_{m+2}, \ldots, v_n\}$. Hence $\gamma_{vew} \leq \gamma_{mw}$. Also the union of vewd-set and evwd-set forms an mwd-set. Therefore $\gamma_{mw} \leq \gamma_{vew} + \gamma_{evw}$.
(2) Similarly we can prove $\gamma_{ves} \leq \gamma_{ms} \leq \gamma_{ves} + \gamma_{evs}$. □

Theorem 4.12. *For any M-strong fuzzy graph without isolated vertices G , $\gamma_{mw} \leq \gamma_m \leq \gamma_{ms}$.*

Proof. Let $D = \{v_1, v_2, \ldots, v_m, e_1, e_2, \ldots, e_n\} - - - - (i)$ be any $\gamma_m - set$ in an M-strong fuzzy graph G. Let $v \in D = \gamma_m - set$. Then v dominates at least one edge $e \in E - D$. By Observation 4 (3), v weakly dominates at least one edge $e \in E - D. - - - (ii)$. Also, let $e \in D$. Then e dominates at least one vertex $v \in V - D$. By Observation 4 (2), e weakly dominates at least one vertex $v \in V - D. - - - - (iii)$ Hence by (i), (ii) and (iii) every $\gamma_m - set$ is a mixed weak dominating set. The scalar cardinality of mixed dominating set $\leq \gamma_m. - - - - (iv)$ Hence $\gamma_{mw} \leq \gamma_m$.

Let $S = \{v_1, v_2, \ldots, v_m, e_1, e_2, \ldots, e_n\} - - - - (v)$ be any $\gamma_{ms} - set$. Let $v \in S = \gamma_{ms} - set$. Then v strongly dominates at least one edge $e \in E - S$. By Observation 4 (1), v dominates at least one edge $e \in E - S. - - - - (vi)$ Also, let $e \in S$. Then e strongly dominates at least one vertex $v \in V - S$. By Observation 4 (4), e dominates at least one vertex $v \in V - S. - - - - (vii)$ Hence by (v), (vi) and (vii) every $\gamma_{ms} - set$ is a mixed dominating set . The scalar cardinality of mixed dominating set $\leq \gamma_{ms}. - - - - (viii)$ Hence $\gamma_{mw} \leq \gamma_m \leq \gamma_{ms}$. □

Example 4.4. *Consider an M-strong fuzzy graph given in Figure 6*
$\gamma_{ve} - set = \{v_1, v_3, v_5\} \Rightarrow \gamma_{ve} = 1.2.$

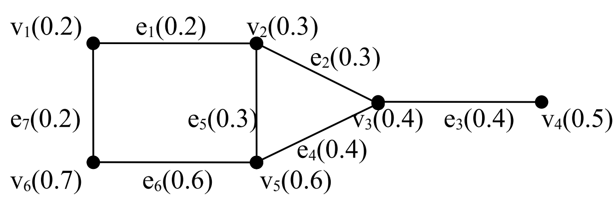

Figure 6:

$\gamma_{ev} - set = \{e_1, e_2\} \Rightarrow \gamma_{ev} = 0.5.$
$\gamma_m - set = \{v_3, v_5, e_1, e_2\} \Rightarrow \gamma_m = 1.5.$
$\gamma_{vew} - set = \{v_2\} \Rightarrow \gamma_{vew} = 0.3.$
$\gamma_{evw} - set = \{e_1, e_2\} \Rightarrow \gamma_{evw} = 0.5.$
$\gamma_{mw} - set = \{v_2, e_1, e_2\} \Rightarrow \gamma_{mw} = 0.8.$
$\gamma_{ves} - set = \{v_1, v_3, v_5\} \Rightarrow \gamma_{ves} = 1.2.$
$\gamma_{evs} - set = \{e_1, e_3, e_6, e_7\} \Rightarrow \gamma_{evs} = 1.4.$
$\gamma_{ms} - set = \{v_3, e_1, e_2, e_6, e_7\} \Rightarrow \gamma_{ms} = 1.8.$

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