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An analysis of retrial queue with setup time in single server general service model for two different group of customers

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Abstract

We consider a single server general service model with setup time retrial queue, for two different group of customers. Also in this paper the probability generating function of number of customers in two types of groups is obtained by using supplementary variable technique.

Keywords: Retrial Queue, Priority groups, Non Priority groups.

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1 Introduction

Retrial queues have been widely used to model many problems arising in telephone switching systems, telecommunication networks, computer networks and computer systems.

Retrial queueing model are characterized by the feature that arriving calls which find a server busy, do not line up or leave the system immediately forever, but go to some virtual place called as orbit and try their luck again after some random time.

During the last two decades considerable attention has been paid to the analysis of queueing system with repeated calls also it is called as retrial queues.

For example, M/G/1 retrial queue with two types of customers in which the service distributions for both types of customers are same is investigated by choi and park[4].

A single line system with secondary orders was studied by G.I.Falin [1]. A survey on retrial queue analysed by T.Yang and J.G.C.Templeton [3]. B.D Choi, K.K.Park and Y.W.Lee [5] considers

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the retrial queues with Bernoulli feedback. Also, A survey of retrial queue was investigated by G.I.Falin [6]. G.I.Falin, J.R.Artalejo, and M.Martin [8] discussed about the single server retrial queue with priority customers.

And M/G/1 retrial queueing system with two types of calls and finite capacity paper investigates by B.D.Choi and K.B.Choi and Y.W.Lee [9]. B.D.Choi and V.G.Kulkarni [7], have been analysed about the Feedback retrial queueing system.

Yong War Lee [10], considers about the M/G/1 feedback retrial queue with two types of customers.

In this paper, a single server retrial queue with setup time includes general service for two different group of customers is considered. And the probability generating function is obtained by using supplementary variable technique.

2 Mathematical Model

In this paper we consider a single server retrial queueing system with setup time in which two different types of customers arrive according to independent poisson streams with rates of λ_1 and λ_2 respectively.

Customers can be identified as priority customers with rate of λ_1 and non-priority customers with rate of λ_2 from the Poisson flow.

In priority group the arriving customer finds the server idle, he immediately gets service. If he finds the server busy, he will joins in the priority group and then served in FCFS queue discipline (or) random order.

And in non-priority group customer finds the server idle, he obtains service immediately. If the server busy, he joins the retrial group. This process is continued until the customer is eventually served.

In this system the retrial time is exponentially distributed with mean $\frac{1}{v}$ and is independent of all previous retrial times and all other stochastic process.

The following diagram shows the retrial queue with setup time in single server. General service for two different types of customer.

The service times of both types of customers are independent. The service time B_k has a general distribution with p.d.f $b_k(x)$ and mean $b_k, k = 1$ is related to the priority customers and $k = 2$ is related to the non-priority customers.

$1 - \delta_1$ is the probability of an priority customer who has received service departs the system and δ_1 is the priority group for more. A non-priority customer who has received service leaves the system with probability $1 - \delta_2$ or rejoins the retrial group with probability δ_2.

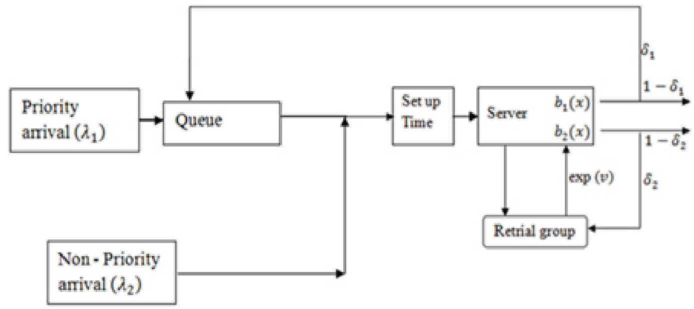

Figure 1 Retrial queue with setup time in single server, General Service for two different types of customers

3 Notations

The following notations are used in this paper.

λ_1 - Arrival rate of priority customer.

λ_2 - Arrival rate of non-priority customer.

$X(t)$ - The residual service time of the customer in the service at time $''t''$.

$S(t)$ - Setup time $''t''$.

$$\xi(t) - \begin{cases} 0 - & \text{when server is idle at time } ''t'' \\ 1 - & \text{when server services the priority customers at time } ''t'' \\ 2 - & \text{when server services the non-priority customers at time } ''t'' \end{cases}$$

N_1 - The number of customers in the Priority group at time $''t''$.

N_2 - The number of customers in the non-priority group at time $''t''$.

4 Probability generating function of Queue sizes

The stochastic process $X(t) = (\xi(t), N_1(t), N_2(t), X(t), S(t); t \geq 0)$ is the Markovian process, and denoted by (ξ, N_1, N_2, X, S) which is the limiting random variable of $(\xi(t), N_1(t), N_2(t), X(t), S(t))$.

The probabilities are defined by

$$q_j = p\left\{\xi = 0, N_2 = j\right\}, j = 0, 1, 2, \cdots$$
$$p_{kij}(x)dx = p\left\{\xi = k, N_1 = i, N_2 = j, S = \eta, X_{\in}(x,\ x + dx)\right\}$$
$$k = 1, 2, i, j = 0, 1, 2, \cdots \text{ and } x \geq 0$$

and their Laplace-stieltjes transforms are

$$p^*_{kij}(\theta) = \int_0^\infty e^{-\theta x} p_{kij}(x)dx, k = 1, 2, \cdots i, j = 0, 1, 2, \cdots$$

$$p^*_{kij}(0) = \int_0^\infty p_{kij}(x)dx = p\{\xi = k, N_1 = i, N_2 = j, S = \eta\} \qquad \longrightarrow (A)$$

$$R^*_\eta(\theta) = \int_0^\infty e^{-\theta x} R_\eta(x)dx \qquad \longrightarrow (B)$$

Equation (A) is the steady state probability that there are $''t''$ customers in the priority group, $''j''$ customers in the retrial group and the server services the $''k''$ type customer.

The system of difference equations are given by.

$$(\lambda_1 + \lambda_2 + jv)\, qj = (1 - \delta_1)\, p_{10j}(0)$$
$$+ (1 - \delta_2)\, p_{20j}(0) + \delta_2 p_{20j-1}(0) \qquad \longrightarrow (1a)$$
$$-p'_{10j}(x) = -(\lambda_1 + \lambda_2)\, p_{10j}(x) + \lambda_1 b_1(x) q_j + \lambda_2 p_{10j-1}(x)$$
$$+\delta_1 b_1(x) p_{10j}(0) + (1 - \delta_1)\, b_1(x) p_{11j}(0)$$
$$+\delta_2 b_1(x) p_{21j-1}(0) + (1 - \delta_1)\, b_1(x) p_{21j}(0) \qquad \longrightarrow (1b)$$
$$-p'_{1ij}(x) = -(\lambda_1 + \lambda_2)\, p_{1ij}(x) + \lambda_1 p_{1i-1j}(x) + \lambda_2 p_{1ij-1}(x)$$
$$+\delta_1 b_1(x) p_{1ij}(0) + (1 - \delta_1)\, b_1(x) p_{1i+1j}(0)$$
$$+\delta_2 b_1(x) p_{2i+1j-1}(0) + (1 - \delta_2)\, b_1(x) p_{2i+1j}(0) \qquad \longrightarrow (1c)$$
$$-p'_{20j}(x) = -(\lambda_1 + \lambda_2)\, p_{20j}(x) + \lambda_2 b_2(x) q_j$$
$$+ (j + 1)\, v b_2(x) q_{j+1} + \lambda_2 p_{20j-1}(x) \qquad \longrightarrow (1d)$$
$$-p'_{2ij}(x) = -(\lambda_1 + \lambda_2)\, p_{2ij}(x) + \lambda_1 p_{2i-1j}(x) + \lambda_2 p_{2ij-1}(x)$$
$$+\lambda_2 p_{2ij-1}(x) \qquad \longrightarrow (1e)$$
$$-R'_\eta(x) = -(\lambda_1 + \lambda_2)\, R_\eta(x) + \delta_1 [b_1(x) p_{10j}(x) + b_1(x) p_{1ij}(0)]$$
$$+\delta_2 [b_1(x) p_{20j-1}(0) + b_1(x) p_{2i+ij-1}(0)] \qquad \longrightarrow (1f)$$

where $i = 1, 2, \cdots \quad j = 0, 1, 2, \cdots \quad p_{kij} = 0$ for $i, j < 0, k = 1, 2$ and any $x \geq 0$. By taking Laplace transform Stieltjes transform of $(1b) - (1e)$, we obtain.

$$\{\theta - (\lambda_1 + \lambda_2)\}\, p^*_{10j}(\theta) + \lambda_2 p^*_{10j-1}(\theta)$$
$$= p_{10j}(0) - \lambda_1 b^*_1(\theta) q_j - \delta_1 b^*_1(\theta) p_{10j}(0)$$
$$- (1 - \delta_1)\, b^*_1(\theta) p_{11j}(0) - \delta_2 b^*_1(\theta) p_{21j-1}(0)$$
$$- (1 - \delta_2)\, b^*_1(\theta) p_{21j}(0) \qquad \longrightarrow (2b)$$
$$\{\theta - (\lambda_1 + \lambda_2)\}\, p^*_{1ij}(\theta) + \lambda_1 p^*_{1i-1j}(\theta) + \lambda_2 p^*_{1ij-1}(\theta)$$
$$= p_{1ij}(0) - \lambda_1 b^*_1(\theta) p_{1ij}(0) - (1 - \delta_1)\, b^*_1(\theta) p_{1i+1j}(0)$$
$$-\delta_2 b^*_1(\theta) p_{2i+1j-1}(0) - (1 - \delta_2)\, b^*_1(\theta) p_{2i+1j}(0) \qquad \longrightarrow (2c)$$
$$\{\theta - (\lambda_1 + \lambda_2)\}\, p^*_{20j}(\theta) + \lambda_2 p^*_{20j-1}(\theta)$$
$$= p_{20j}(0) - \lambda_2 b^*_2(\theta) q_j - (j + 1) v b^*_2(\theta) q_{j+1} \qquad \longrightarrow (2d)$$
$$\{\theta - (\lambda_1 + \lambda_2)\}\, p^*_{2ij}(\theta) + \lambda_1 p^*_{2i-1j}(\theta)$$
$$+\lambda_2 p^*_{2ij-1}(\theta) = p^*_{2ij}(0) \qquad \longrightarrow (2e)$$
$$\{\theta - (\lambda_1 + \lambda_2)\}\, R^*_\eta(\theta) = -\delta_1 [b^*_1(\theta) p_{1ij}(0) + b^*_1(\theta) p_{1ij}(0)]$$

$$-\delta_2\left[b_1^*(\theta)p_{2ij-1}(0)+b_1^*(\theta)p_{2i+1j-1}(0)\right] \longrightarrow (2f)$$

Introducing the following generating function for complex z with $|z|\le 1$

$$Q(z_2)=\sum_{j=0}^{\infty}q_j\ z_2^j,$$
$$P_{ki}^*(\theta,z_2)=\sum_{j=0}^{\infty}P_{kij}^*(\theta)\ z_2^j, k=1,2,$$
$$P_{ki}(0,z_2)=\sum_{j=0}^{\infty}P_{kij}(0)\ z_2^j, k=1,2,$$
$$R_\eta^*(\theta,z_2)=\sum_{j=0}^{\infty}R_\eta^*(\theta)\ z_2^j$$
$$R_\eta(0,z_2)=\sum_{j=0}^{\infty}R_\eta(0)\ z_2^j$$

Multiplying equations $(1a)$ and $(2b)-(2e)$ by z_2^j and summing over all j, we obtain the following basic system of equations.

$$(\lambda_1+\lambda_2)Q(z_2)+vz_2Q'(z_2)$$
$$=(1-\delta_1)P_{10}(0,z_2)+(1-\delta_2+\delta_2z_2)P_{20}(0,z_2) \longrightarrow (3a)$$
$$\{\theta-(\lambda_1+\lambda_2)+\lambda_2z_2\}\,P_{10}^*(\theta,z_2)$$
$$=(1-\delta_1b_1^*(\theta))P_{10}(0,z_2)-\lambda_1b_1^*(\theta)Q(z_2)$$
$$-(1-\delta_1)b_1^*(\theta)P_{11}(0,z_2)-(1-\delta_2+\delta_2z_2)b_1^*(\theta)P_{21}(0,z_2) \longrightarrow (3b)$$
$$\{\theta-(\lambda_1+\lambda_2)+\lambda_2z_2\}\,P_{1i}^*(\theta,z_2)+\lambda_1P_{1i-1}^*(\theta,z_2)$$
$$=(1-\delta_1b_1^*(\theta))P_{1i}(0,z_2)-(1-\delta_1)b_1^*(\theta)P_{1i+1}(0,z_2)$$
$$-(1-\delta_2+\delta_2z_2)b_1^*(\delta)P_{2i+1}(0,z_2) \longrightarrow (3c)$$
$$\{\theta-(\lambda_1+\lambda_2)+\lambda_2z_2\}\,P_{20}^*(\theta,z_2)$$
$$=P_{20}(0,z_2)-\lambda_2b_2^*(\theta)Q(z_2)-vb_2^*(\theta)Q'(z_2) \longrightarrow (3d)$$

$$\{\theta-(\lambda_1+\lambda_2)+\lambda_2z_2\}\,P_{2i}^*(\theta,z_2)+\lambda_1P_{2i-1}^*(\theta,z_2)=P_{2i}(0,z_2) \longrightarrow (3e)$$

$$\{\theta-(\lambda_1+\lambda_2)+\lambda_2z_2\}\,R_\eta^*(\theta,z_2)$$
$$=-\delta_1[b_1^*(\theta,z_2)P_{10}(0)+b_1^*(\theta,z_2)P_{1i}(0)]$$
$$-\delta_2[b_1^*(\theta,z_2)P_{21}(0)-b_1^*(\theta,z_2)P_{2i+1}^*(0)] \longrightarrow (3f)$$

Define the generating functions of $P_k^*(\theta,z_1,z_2)$ and $P_k(0,z_1,z_2), R_\eta^*(\theta,z_1,z_2)$ and $R_\eta(0,z_1,z_2)$ for $k=1,2$ as follows

$$P_k^*(\theta,z_1,z_2)=\sum_{i=0}^{\infty}P_{ki}^*\ (\theta,z_2)\ z_1^i$$
$$P_k(\theta,z_1,z_2)=\sum_{i=0}^{\infty}P_{ki}\ (0,z_2)\ z_1^i$$

$$R^*_\eta(\theta, z_1, z_2) = \sum_{i=0}^{\infty} R^*_{\eta i}(\theta, z_2)\ z_1^i$$
$$R_\eta(0, z_1, z_2) = \sum_{i=0}^{\infty} R_{\eta i}(0, z_2)\ z_1^i$$

Here $P^*_k(0, z_1, z_2) = E(z_1^{N_1}, z_2^{N_2})\ ; \xi = k, S = \eta)$ which is the joint generating function of (N_1, N_2) when the server services the $k-$type customer.

Multiplying equations $(3b)$ $-$ $(3e)$ by z_1^i and summing over all i, and then comparing with $(3f)$.

$$\begin{aligned}
&\{\theta - \lambda_1(1 - z_1) - \lambda_2(1 - z_2)\}\ P^*_1(\theta, z_1, z_2)\\
&= \left\{1 - \delta_1\ b^*_1(\theta) - \frac{(1 - \delta_1)b^*_1(\theta)}{z_1}\right\} P_1(0, z_1, z_2)\\
&- \frac{(1 - \delta_2 + \delta_2 z_2)b^*_1(\theta)}{z_1}\ P_2(0, z_1, z_2) + \frac{(1 - \delta_1)b^*_1(\theta)}{z_1}\ P_{10}(0, z_2)\\
&+ \frac{(1 - \delta_2 + \delta_2\ z_2)b^*_1(\theta)}{z_1}\ P_{20}(0, z_2) - \lambda_1 b^*_1(\theta)\ Q(z_2) + \frac{(1 - \eta)R^*_\eta(\theta)}{z_1} \qquad \longrightarrow (4a)\\
&\{\theta - \lambda_1(1 - z_1) - \lambda_2(1 - z_2)\}\ P^*_2(\theta, z_1, z_2)\\
&= P_2(0, z_1, z_2) - \lambda_2 b^*_2(\theta)\ Q(z_2) - v b^*_2(\theta) Q'(z_2) + R^*_\eta(\theta) \qquad \longrightarrow (4b)
\end{aligned}$$

Let $\theta - \lambda_1(1 - z_1) + \lambda_2(1 - z_2)$ into $(4a)$ and $(4b)$ eliminating $P^*_1(\theta, z_1, z_2)$ and $P^*_2(\theta, z_1, z_2)$ from $(4a)$ and $(4b)$ respectively and obtain.

$$\begin{aligned}
&\{z_1 - (1 - \delta_1 + \delta_1 z_1)\beta_1(z_1, z_2)\}\ P_1(0, z_1, z_2) = \beta_1(z_1, z_2)\\
&\{(1 - \delta_2 + \delta_2 z_2)P_2(0, z_1, z_2) + \lambda_1 z_1 Q(z_2) - (1 - \delta_1)P_{10}(0, z_2)\\
&-(1 - \delta_2 + \delta_2 z_2)P_{20}(0, z_2)\} + (1 - \eta)R_\eta(\theta) \qquad \longrightarrow (5)
\end{aligned}$$

$$P_2(0, z_1, z_2) = \beta_2(z_1, z_2)\left\{\lambda_2 Q(z_2) + v Q'(z_2)\right\} + \eta R_\eta(\theta) \qquad \longrightarrow (6)$$

Here

$$\beta_k(z_1, z_2) = b^*_k(\lambda_1(1 - z_1) + \lambda_2(1 - z_2)), k = 1, 2, \cdots$$

Considering the function

$$h(z_1, z_2) = z_1 - (1 - \delta_1 + \delta_1 z_1)\beta_1(z_1, z_2) \qquad \longrightarrow (7)$$

By using Rouches theorem it follows that for each z_2 with $|z_2| < 1$, there is a unique solution $z_1 = \phi(z_2)$ of the equation $h(z_1, z_2) = 0$ in the unit circle, $(ie.,)$;

$$h(\phi(z_2), z_2) = \phi(z_2) - (1 - \delta_1 + \delta_1\phi(z_2))\beta_1(\phi(z_2), z_2) + (1 - \eta)R_\eta(\theta), z_2 = 0$$

and

$$\left.\frac{\partial h(z_1, z_2)}{\partial z_1}\right|_{z_1 = z_2 = 1} = 1 - (\delta_1 + \lambda_1 b_1) > 0,$$

Hence $z_1 = \phi(z_2)$, is analytic on $|z_2| < 1$ and is continuous at $z_2 = 1$ and $\phi(1) = 1$ by the implicit function theorem.

It is necessary to show that the first and second derivatives of $\phi(z_2)$ at $z_2 = 1$. The derivatives are

$$\phi^{'}(1) = \frac{\lambda_2 b_1 + \eta R_\eta}{1 - (\delta_1 + \lambda_1 b_1)}$$

$$\phi^{''}(1) = \frac{2\,\delta_1\,(1-\delta_1)\,\lambda_2^2\,b_1^2 + (1-\delta_1)^2\,\lambda_2^2\,E(b_1^2) + \eta^2\,R_\eta^2}{\{1 - (\delta_1 + \lambda_1 b_1)\}^3} \longrightarrow (8)$$

Sub $z_1 = \phi(z_2)$ in $(5) P_1(0, z_1, z_2)$ is eliminated, and we get

$$(1 - \delta_2 + \delta_2 z_2) P_2(0, \phi(z_2), z_2) + \lambda_1 \phi(z_2)\, Q(z_2)$$
$$= (1 - \delta_1) P_{10}(0, z_2) + (1 - \delta_2 + \delta_2 z_2) P_{20}(0, z_2) + (1 - \eta) R_\eta(\theta) \longrightarrow (9)$$

Sub $z_1 = \phi(z_2)$ in (6), we get

$$P_2(0, \phi(z_2), z_2) = \beta_2(\phi(z_2), z_2) \left\{\lambda_2 Q(z_2) + v Q^{'}(z_2)\right\} + \eta\, R_\eta(\theta) \longrightarrow (10)$$

From (9) and (10) we obtain

$$(1 - \delta_1) P_{10}(0, z_2) + (1 - \delta_2 + \delta_2 z_2) P_{20}(0, z_2)$$
$$= \{\lambda_1 \phi(z_2) + (1 - \delta_2 + \delta_2 z_2) \lambda_2 \beta_2(\phi(z_2), z_2)\}\, Q(z_2)$$
$$+ (1 - \delta_2 + \delta_2 z_2) v \beta_2(\phi(z_2), z_2) Q^{'}(z_2) + R_\eta(\theta) \longrightarrow (11)$$

Equating $(3a)$ and (11) we get the differential equation

$$Q^{'}(z_2) = \frac{R_\eta(\theta)}{v\,\{(1 - \delta_2 + \delta_2 z_2) \beta_2(\phi(z_2), z_2) - z_2\}}$$
$$\times [\lambda_1(1 - \phi(z_2)) + \lambda_2 \{1 - (1 - \delta_2 + \delta_2 z_2) \beta_2(\phi(z_2), z_2)\}]\, Q(z_2) \longrightarrow (12)$$

whose solution is

$$Q(z_2) = C.exp\left[-\frac{1}{v}\int_{z_2}^{1} \frac{R_\eta(\theta)}{(1 - \delta_2 + \delta_2 x)\beta_2(\phi(x), x) - x}\right.$$
$$\times \{\lambda_1(1 - \phi(x)) + \lambda_2 \{1 - (1 - \delta_2 + \delta_2 x) \beta_2(\phi(x), x)\}\}\, dx] \longrightarrow (13)$$

Sub (12) into (6) yields

$$P_2(0, z_1, z_2) = \frac{\{\lambda_1(1 - \phi(z_2)) + \lambda_2(1 - z_2)\}\, \beta_2(z_1, z_2) \eta R_\eta(\theta)}{(1 - \delta_2 + \delta_2 z_2) \beta_2(\phi(z_2), z_2) - z_2} Q(z_2) \longrightarrow (14)$$

Sub (11) and (12), (14) into (5) yields

$$P_1(0, z_1, z_2) = \beta_1(z_1, z_2) \left[\frac{(1 - \delta_2 + \delta_2 z_2)}{(1 - \delta_2 + \delta_2 z_2) \beta_2(\phi(z_2), z_2) - z_2}\right.$$
$$\times \frac{\{\lambda_1(1 - \phi(z_1)) + \lambda_2(1 - z_2)\}\, \{\beta_2(\phi(z_2), z_2) - \beta_2(z_1, z_2)\}}{(1 - \delta_1 + \delta_1 z_1) \beta_1(z_1, z_2) - z_1}$$
$$\left. + \frac{\lambda_1(\phi(z_2) - z_1)}{(1 - \delta_1 + \delta_1 z_1) \beta_1(z_1, z_2) - z_1}\right] Q(z_2) + (1 - \eta) R_\eta(\theta) \longrightarrow (15)$$

Letting $\theta = 0$ in $(4a)$ and $(4b)$ gives

$$
\begin{aligned}
&-\left\{\lambda_1(1-z_1)+\lambda_2(1-z_2)\right\}P_1^*(0,z_1,z_2)\\
&=\left(1-\delta_1-\frac{1-\delta_1}{z_1}\right)P_1(0,z_1,z_2)-\frac{1-\delta_2+\delta_2 z_2}{z_1}P_2(0,z_1,z_2)+\frac{1-\delta_1}{z_1}P_{10}(0,z_2)\\
&+\frac{1-\delta_2+\delta_2 z_2}{z_1}P_{20}(0,z_2)-\lambda_1 Q(z_2)+\frac{(1-\eta)R_\eta(0)}{z_1} \qquad \longrightarrow (16)
\end{aligned}
$$

$$
\begin{aligned}
&-\left\{\lambda_1(1-z_1)+\lambda_2(1-z_2)\right\}P_2^*(0,z_1,z_2)\\
&=P_2(0,z_1,z_2)-\lambda_2\, Q(z_2)-v\, Q'(z_2)+\eta R_\eta(0) \qquad \longrightarrow (17)
\end{aligned}
$$

We obtain from (16) using (11), (14) and (15)

$$
\begin{aligned}
&P_1^*(0,z_1,z_2)=\frac{\beta_1(z_1,z_2)-1}{\lambda_1(z_1-1)+\lambda_2(z_2-1)}\left[\frac{\lambda_1(\phi(z_2)-z_1)+(1-\eta)R_\eta(0)}{(1-\delta_1+\delta_1 z_1)\beta_1(z_1,z_2)-z_1}\right.\\
&+\frac{(1-\delta_2+\delta_2 z_2)\left\{\lambda_1(1-\phi(z_2))+\lambda_2(1-z_2)\right\}}{(1-\delta_2+\delta_2 z_2)\beta_2(\phi(z_2),z_2)-z_2}\\
&\left.\times\frac{\beta_2(\phi(z_2),z_2)-\beta_2(z_1,z_2)}{(1-\delta_1+\delta_1 z_1)\beta_1(z_1,z_2)-z_1}\right]Q(z_2) \qquad \longrightarrow (18)
\end{aligned}
$$

From (12), (14) and (17) we obtain

$$
\begin{aligned}
&P_2^*(0,z_1,z_2)=\frac{\left\{\lambda_1(1-\phi(z_2))+\lambda_2(1-z_2)\right\}\left\{1-\beta_2(z_1,z_2)\right\}+\eta\, R_\eta(0)}{(1-\delta_2+\delta_2 z_2)\beta_2(\phi(z_2),z_2)-z_2}\\
&\times\frac{Q(z_2)}{\lambda_1(1-z_1)+\lambda_2(1-z_2)} \qquad \longrightarrow (19)
\end{aligned}
$$

Letting $z_2\rightarrow 1$, and then $z_1\rightarrow 1$ in (18) and (19).

Using $\phi(1)=1$ in (8) we obtain by using the $L'Hospital\ rule$ that

$$
\begin{aligned}
&P_1^*(0,1,1)=\lim_{z_1\rightarrow 1}C\cdot\frac{\beta(z_1,1)-1}{\lambda_1(z_1-1)}\left[\frac{\lambda_1(1-z_1)+(1-\eta)R_\eta(0)}{(1-\delta_1+\delta_1 z_1)\beta_1(z_1,1)-z_1}\right.\\
&\left.+\frac{1-\beta_2(z_1,1)}{(1-\delta_1+\delta_1 z_1)\beta_1(z_1,1)-z_1}\cdot\frac{-\lambda_1\phi'(1)-\lambda_2}{\delta_2+(\lambda_1\phi'(1)+\lambda_2)b_2-1}\right.\\
&=C\cdot\frac{\lambda_1 b_1(1-\delta_2)\left\{1-(\delta_1+\lambda_1 b_1)\right\}+(1-\eta)\,R_\eta(0)}{\left\{1-(\delta_1+\lambda_1 b_1)\right\}\left[(1-\delta_2)\left\{1-(\delta_1+\lambda_1 b_1)\right\}-\lambda_2 b_2(1-\delta_1)\right]} \qquad \longrightarrow (20)
\end{aligned}
$$

$$
\begin{aligned}
&P_2^*(0,1,1)=\lim_{z_1\rightarrow 1}C\cdot\frac{1-\beta_2(z_1,1)}{\lambda_1(1-z_1)}\frac{-\lambda_1\phi'(1)-\lambda_2+\eta\,R_\eta(0)}{\delta_2+(\lambda_1\phi'(1)+\lambda_2)b_2-1}\\
&=C\cdot\frac{\lambda_2 b_2(1-\delta_1)+\eta\,R_\eta(0)}{(1-\delta_2)\left\{1-(\delta_1+\lambda_1 b_1)\right\}-\lambda_2 b_2(1-\delta_1)} \qquad \longrightarrow (21)
\end{aligned}
$$

From the total probability $Q(1)+P_1^*(0,1,1)+P_2^*(0,1,1)=1$ we obtain

$$
C=\frac{(1-\delta_2)\left\{1-(\delta_1+\lambda_1 b_1)\right\}-\lambda_2 b_2(1-\delta_1)+\eta\,R_\eta(0)}{(1-\delta_1)\left\{1-(\delta_1+\lambda_1 b_1)\right\}+\lambda_1 b_1(1-\delta_2)}
$$

That is, the probability that the server is idle, and

$$
P_1^*(0,1,1)+P_2^*(0,1,1)
$$

$$= \frac{(1-\delta_1)\,\{1-(\delta_1+\lambda_1 b_1)+\lambda_2 b_2\}-(1-\delta_2)(1-\delta_1-2\lambda_1 b_1)+\eta\, R_\eta(0)}{(1-\delta_1)\,\{1-(\delta_1+\lambda_1 b_1)\}+\lambda_1 b_1(1-\delta_2)}$$

as the probability that the server is busy.

5 Conclusion

In this paper the behaviour of the single server with setup time general service model retrial queue for two different type of customers are analysed. The probability generating function of queue size at arbitrary time epoch and different completion epoch are obtained for both priority group and non-priority group of customers.

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