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On $\tilde{\mu}$-open sets in generalized topological spaces

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Abstract

In this paper, we introduce the notion of $\tilde{\mu}$-open sets in generalized topological spaces. Further, we introduce the notions of interior, closure, boundary, exterior and study some of their properties. In addition, we introduce the concepts of $\tilde{\mu}$-T_i $(i = 0, \frac{1}{2}, 1, 2)$ spaces are characterized them using $\tilde{\mu}$-open and $\tilde{\mu}$-closed sets.

Keywords: $\tilde{\mu}$-open, $\tilde{\mu}$-closed, $\tilde{\mu}$-interior, $\tilde{\mu}$-closure, $\tilde{\mu}$-boundary, $\tilde{\mu}$-exterior and $\tilde{\mu}$-T_i $(i = 0, \frac{1}{2}, 1, 2)$.

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1 Introduction

Generalized topologies were introduced by A. Csaszar. Further, he defined the concepts of μ-open sets and their corresponding interior and closure operators in generalized topological spaces. Also, he obtained and studied the notions of μ-semi-open sets, μ-preopen sets, μ-α-open sets and μ-β-open sets in generalized topological spaces. In this paper in section 3, we introduced the concept of $\tilde{\mu}$-open sets, which is analogous to μ-semi-open sets and introduced the notion $\tilde{\mu}O(X)$ which is the set of all $\tilde{\mu}$-open sets in a generalized topological space (X, μ). Further, we introduced the concepts of $\tilde{\mu}$-interior, $\tilde{\mu}$-closure, $\tilde{\mu}$-boundary and $\tilde{\mu}$-exterior operators and studied some of their fundamental properties. In section 4, we introduced the notion of $\tilde{\mu}$-T_i spaces $(i = 0, \frac{1}{2}, 1, 2)$ and characterized $\tilde{\mu}$-T_i spaces using $\tilde{\mu}$-closed and $\tilde{\mu}$-open sets.

2 Preliminaries

We recall some basic definitions and notations. Let X be a nonempty set and $\exp(X)$ the power set of X. We called a class $\mu \subseteq \exp(X)$ a generalized topology (briefly, GT) if $\emptyset \in \mu$ and the arbitrary union of elements of μ belongs to μ [4]. We called the pair (X, μ) a generalized topological space (briefly, GTS). For a generalized topological space (X, μ), the elements of μ are called μ-open sets and the complements of μ-open sets are called μ-closed sets [4]. For $A \subseteq X$, we denote by $c_\mu(A)$ the intersection of all μ-closed sets containing A, i.e., the smallest μ-closed set containing A [7]; and by $i_\mu(A)$ the union of all μ-open sets contained in A, i.e., the largest μ-open set contained in A [7]. It is easy to observe that i_μ and c_μ are idempotent and monotonic, where $\gamma : exp(X) \rightarrow exp(X)$ is said to be idempotent iff $A \subseteq B \subseteq X$ implies $\gamma(\gamma(A)) = \gamma(A)$ and monotonic iff $\gamma(A) \subseteq \gamma(B)$ [2]. According to [9], let μ be a generalized topology on X, $A \subseteq X$ and $x \in X$, then (1) $x \in c_\mu(A)$ if and only if $M \cap A \neq \emptyset$ for each $M \in \mu$ containing x; (2) $c_\mu(X \setminus A) = X \setminus i_\mu(A)$ and (3) $c_\mu(c_\mu(A)) = c_\mu(A)$. A subset A of a generalized topological space (X, μ) is said to be μ-semi-open (resp. μ-preopen, μ-α-open, μ-β-open) if $A \subseteq c_\mu(i_\mu(A))$ (resp. $A \subseteq i_\mu(c_\mu(A))$, $A \subseteq i_\mu(c_\mu(i_\mu(A)))$, $A \subseteq c_\mu(i_\mu(c_\mu(A)))$). The complement of

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a μ-semi-open (resp. μ-preopen, μ-α-open, μ-β-open) set is said to be μ-semi-closed (resp. μ-preclosed, μ-α-closed, μ-β-closed) [4]. For $A \subseteq X$, we denote by $c_{s_\mu}(A)$ the intersection of all μ-semi-closed sets containing A, i.e., the smallest μ-semi-closed set containing A [7]; and by $i_{s_\mu}(A)$ the union of all μ-semi-open sets contained in A, i.e., the largest μ-semi-open set contained in A [7]. According to [9], let μ be a generalized topology on X, $A \subseteq X$ and $x \in X$, then (1) $x \in c_{s_\mu}(A)$ if and only if $M \cap A \neq \emptyset$ for each μ-semi-open set M containing x; (2) $c_{s_\mu}(X \setminus A) = X \setminus i_{s_\mu}(A)$ and (3) $c_{s_\mu}(c_{s_\mu}(A)) = c_{s_\mu}(A)$.

3 $\tilde{\mu}$-open sets

Definition 3.1. *Let (X, μ) be a generalized topological space. A subset A of X is said to be a $\tilde{\mu}$-open set, if there exists a μ-open set U of X such that $U \subseteq A \subseteq c_{s_\mu}(U)$. The set of all $\tilde{\mu}$-open sets is denoted by $\tilde{\mu}O(X)$.*

Example 3.1. *Let $X = \{a, b, c\}$ and $\mu = \{\emptyset, \{a\}\}$. Then $\tilde{\mu}$-open sets are $\{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$.*

Theorem 3.1. *Let (X, μ) be a generalized topological space, A be a subset of X. If A is $\tilde{\mu}$-open in (X, μ) if and only if $A \subseteq c_{s_\mu}(i_\mu(A))$.*

Proof. If A is a $\tilde{\mu}$-open of X, then there exists a μ-open set U such that $U \subseteq A \subseteq c_{s_\mu}(U)$. Since U is $\tilde{\mu}$-open, we have that $U = i_\mu(U) \subseteq i_\mu(A)$. Therefore $A \subseteq c_{s_\mu}(U) \subseteq c_{s_\mu}(i_\mu(A))$ and hence $A \subseteq c_{s_\mu}(i_\mu(A))$. Conversely, assume that $A \subseteq c_{s_\mu}(i_\mu(A))$. To prove that A is a $\tilde{\mu}$-open set in (X, μ). Take $U = i_\mu(A)$. Then $i_\mu(A) \subseteq A \subseteq c_{s_\mu}(i_\mu(A))$. Hence A is $\tilde{\mu}$-open in (X, μ). □

Theorem 3.2. *Let (X, μ) be a generalized topological space, A be a subset of X. If A is a μ-open set in (X, μ), then A is $\tilde{\mu}$-open in (X, μ).*

Proof. If A is a μ-open set in (X, μ), then $A = i_\mu(A)$. Since $A \subseteq c_{s_\mu}(A)$, we have that $A \subseteq c_{s_\mu}(i_\mu(A))$. Then by Theorem 3.1 A is $\tilde{\mu}$-open in (X, μ). □

Remark 3.1. *The following example shows that the converse of the above theorem need not be true.*

Let $X = \{a, b, c, d\}$ and $\mu = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Then $A = \{a, b, d\}$ is a $\tilde{\mu}$-open set in (X, μ) but not μ-open.

Theorem 3.3. *Let (X, μ) be a generalized topological space, A be a subset of X. If A is a $\tilde{\mu}$-open set in (X, μ), then A is μ-semi-open in (X, μ).*

Proof. If A is a $\tilde{\mu}$-open set in (X, μ), then by Theorem 3.1 $A \subseteq c_{s_\mu}(i_\mu(A))$. Since every μ-closed set is μ-semi-closed and $c_{s_\mu}(i_\mu(A))$ is a least μ-semi-closed set containing $i_\mu(A)$, this implies that $c_{s_\mu}(i_\mu(A)) \subseteq c_\mu(i_\mu(A))$. Therefore $A \subseteq c_\mu(i_\mu(A))$ and hence A is a μ-semi-open set in (X, μ). □

Remark 3.2. *The following example shows that the converse of the above theorem need not be true.*

Let $X = \{a, b, c, d\}$ and $\mu = \{\emptyset, \{a\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then $A = \{a, d\}$ is a μ-semi-open set in (X, μ) but not $\tilde{\mu}$-open.

Theorem 3.4. *Let $\{A_\alpha : \alpha \in J\}$ be the collection of $\tilde{\mu}$-open sets in a generalized topological space (X, μ). Then $\bigcup_{\alpha \in J} A_\alpha$ is also a $\tilde{\mu}$-open set in (X, μ).*

Proof. Since A_α is $\tilde{\mu}$-open, then there exists a μ-open set U_α of X such that $U_\alpha \subseteq A \subseteq c_{s_\mu}(U_\alpha)$. This implies that $\bigcup_{\alpha \in J} U_\alpha \subseteq \bigcup_{\alpha \in J} A_\alpha \subseteq \bigcup_{\alpha \in J} c_{s_\mu}(U_\alpha) \subseteq c_{s_\mu}(\bigcup_{\alpha \in J} U_\alpha)$ since union of all μ-open sets is μ-open. Therefore $\bigcup_{\alpha \in J} A_\alpha$ is a $\tilde{\mu}$-open set in (X, μ). □

Remark 3.3. *If A and B are two $\tilde{\mu}$-open sets in (X, μ), then $A \cap B$ need not be $\tilde{\mu}$-open in (X, μ).*

(i) Let $X = \{1, 2, 3, ..., n\}$ and $\mu = \{\emptyset, X\} \cup \{M \subseteq X \mid M = X - \{i\}$ for some $i \in X\}$. Take $A = X - \{1\}$ and $B = X - \{2\}$. Then A and B are $\tilde{\mu}$-open sets in (X, μ) but $A \cap B = X - \{1, 2\}$ is not $\tilde{\mu}$-open in (X, μ).

(ii) Let $X = R$, with the usual topology. If A = [-1, 0] and B = [0, 1], then A and B are $\tilde{\mu}$-open sets in (X, μ) but $A \cap B = \{0\}$ is not $\tilde{\mu}$-open in (X, μ).

Theorem 3.5. *Let A be a $\tilde{\mu}$-open set in (X, μ) and B be any set such that $A \subseteq B \subseteq c_{s_\mu}(i_\mu(A))$. Then B is also a $\tilde{\mu}$-open set in (X, μ).*

Proof. If A is a $\tilde{\mu}$-open set in (X, μ), then by Theorem 3.1 $A \subseteq c_{s_\mu}(i_\mu(A))$. Since $A \subseteq B$, this implies that $c_{s_\mu}(i_\mu(A)) \subseteq c_{s_\mu}(i_\mu(B))$. By hypothesis $B \subseteq c_{s_\mu}(i_\mu(A)) \subseteq c_{s_\mu}(i_\mu(B))$ and hence $B \subseteq c_{s_\mu}(i_\mu(B))$. This shows that B is a $\tilde{\mu}$-open set in (X, μ). $\square$

Definition 3.2. *Let (X, μ) be a generalized topological space. A subset A of X is called $\tilde{\mu}$-closed if its complement $X \setminus A$ is $\tilde{\mu}$-open.*

Theorem 3.6. *Let (X, μ) be a generalized topological space, A be a subset of X. Then A is $\tilde{\mu}$-closed in (X, μ) if and only if $i_{s_\mu}(c_\mu(A)) \subseteq A$.*

Proof. If A is a $\tilde{\mu}$-closed set in (X, μ), then $X \setminus A$ is $\tilde{\mu}$-open. Therefore $X \setminus A \subseteq c_{s_\mu}(i_\mu(X \setminus A))$ (by Theorem 3.1) = $c_{s_\mu}(X \setminus c_\mu(A))$ = $X \setminus i_{s_\mu}(c_\mu(A))$. This implies that $i_{s_\mu}(c_\mu(A)) \subseteq A$. Conversely, suppose that $i_{s_\mu}(c_\mu(A)) \subseteq A$. Then $X \setminus A \subseteq X \setminus i_{s_\mu}(c_\mu(A)) = c_{s_\mu}(X \setminus c_\mu(A))$ = $c_{s_\mu}(i_\mu(X \setminus A))$. Therefore $X \setminus A$ is $\tilde{\mu}$-open set in (X, μ) and this shows that A is $\tilde{\mu}$-closed set in (X, μ). $\square$

Theorem 3.7. *Let (X, μ) be a generalized topological space, A be a subset of X. If $i_{s_\mu}(F) \subseteq A \subseteq F$, then A is $\tilde{\mu}$-closed in (X, μ) for any μ-closed set F of (X, μ).*

Proof. Let $i_{s_\mu}(F) \subseteq A \subseteq F$ where F is μ-closed subset of X. Then $X \setminus F \subseteq X \setminus A \subseteq X \setminus i_{s_\mu}(F) = c_{s_\mu}(X \setminus F)$. Let $U = X \setminus F$. Then U is μ-open and $U \subseteq X \setminus A \subseteq c_{s_\mu}(U)$. This implies that $X \setminus A$ is a $\tilde{\mu}$-open set in (X, μ) and hence A is a $\tilde{\mu}$-closed set in (X, μ). $\square$

Remark 3.4. *The converse of the above theorem need not be true.*

In Example 3.1 for the $\tilde{\mu}$-closed set $\{b\}$, does not exist any μ-closed set in (X, μ).

Theorem 3.8. *Let (X, μ) be a generalized topological space, A be a subset of X. Then (i) $i_{s_\mu}(c_\mu(A))$ is $\tilde{\mu}$-closed; (ii) $c_{s_\mu}(i_\mu(A))$ is $\tilde{\mu}$-open.*

Proof. (i) Obviously $i_{s_\mu}(c_\mu(i_{s_\mu}(c_\mu(A)))) \subseteq i_{s_\mu}(c_\mu(c_\mu(A))) = i_{s_\mu}(c_\mu(A))$. Hence $i_{s_\mu}(c_\mu(A))$ is $\tilde{\mu}$-closed.
(ii) Follows from (i) and Theorem 3.1. $\square$

Theorem 3.9. *Let $\{A_\alpha : \alpha \in J\}$ be the collection of $\tilde{\mu}$-closed sets in a generalized topological space (X, μ). Then $\bigcap_{\alpha \in J} A_\alpha$ is also a $\tilde{\mu}$-closed set in (X, μ).*

Proof. Let A_α be $\tilde{\mu}$-closed in (X, μ). Then $X \setminus A_\alpha$ is $\tilde{\mu}$-open. By Theorem 3.4 $\bigcup_{\alpha \in J}(X \setminus A_\alpha)$ is also $\tilde{\mu}$-open. This implies that $\bigcup_{\alpha \in J}(X \setminus A_\alpha) = X \setminus \bigcap_{\alpha \in J} A_\alpha$ is $\tilde{\mu}$-open and hence $\bigcap_{\alpha \in J} A_\alpha$ is $\tilde{\mu}$-closed in (X, μ). $\square$

Definition 3.3. *Let (X, μ) be a generalized topological space, A be a subset of X. Then $\tilde{\mu}$-interior of A is defined as union of all $\tilde{\mu}$-open sets contained in A. Thus $i_{\tilde{\mu}}(A) = \cup\{U : U \in \tilde{\mu}O(X)$ and $U \subseteq A\}$.*

Definition 3.4. *Let (X, μ) be a generalized topological space, A be a subset of X. Then $\tilde{\mu}$-closure of A is defined as intersection of all $\tilde{\mu}$-closed sets containing A. Thus $c_{\tilde{\mu}}(A) = \cap\{F : X \setminus F \in \tilde{\mu}O(X)$ and $A \subseteq F\}$.*

Theorem 3.10. *Let (X, μ) be a generalized topological space, A be a subset of X. Then (i) $i_{\tilde{\mu}}(A)$ is a $\tilde{\mu}$-open set contained in A;*
(ii) $c_{\tilde{\mu}}(A)$ is a $\tilde{\mu}$-closed set containing A;
(iii) A is $\tilde{\mu}$-closed if and only if $c_{\tilde{\mu}}(A) = A$;
(iv) A is $\tilde{\mu}$-open if and only if $i_{\tilde{\mu}}(A) = A$;
(v) $i_{\tilde{\mu}}(i_{\tilde{\mu}}(A)) = i_{\tilde{\mu}}(A)$;
(vi) $c_{\tilde{\mu}}(c_{\tilde{\mu}}(A)) = c_{\tilde{\mu}}(A)$;
(vii) $i_{\tilde{\mu}}(A) = X \setminus c_{\tilde{\mu}}(X \setminus A)$;
(viii) $c_{\tilde{\mu}}(A) = X \setminus i_{\tilde{\mu}}(X \setminus A)$.

Proof. (i) Follows from Definition 3.3 and Theorem 3.4.
(ii) Follows from Definition 3.4 and Theorem 3.9.
(iii) and (iv) Follows from Definition 3.5, (ii) and Definition 3.4, (i) respectively.
(v) and (vi) Follows from (i), (iv) and (ii), (iii) respectively.
(vii) and (viii) Follows from Definitions 3.2, 3.4 and 3.5. □

Theorem 3.11. *Let (X, μ) be a generalized topological space. If A and B are two subsets of X, then the following are hold:*
(i) If $A \subseteq B$, then $i_{\tilde{\mu}}(A) \subseteq i_{\tilde{\mu}}(B)$;
(ii) If $A \subseteq B$, then $c_{\tilde{\mu}}(A) \subseteq c_{\tilde{\mu}}(B)$;
(iii) $i_{\tilde{\mu}}(A \cup B) = i_{\tilde{\mu}}(A) \cup i_{\tilde{\mu}}(B)$;
(iv) $c_{\tilde{\mu}}(A \cap B) = c_{\tilde{\mu}}(A) \cap c_{\tilde{\mu}}(B)$;
(v) $i_{\tilde{\mu}}(A \cap B) \subseteq i_{\tilde{\mu}}(A) \cap i_{\tilde{\mu}}(B)$.
(vi) $c_{\tilde{\mu}}(A \cup B) \supseteq c_{\tilde{\mu}}(A) \cup c_{\tilde{\mu}}(B)$.

Proof. (i) Follows from Definition 3.3 and 3.4 respectively.
(ii) Follows from (i), Theorem 3.4 and (ii), Theorem 3.9 respectively.
(iii) Follows from (i) and (ii) respectively. □

Theorem 3.12. *Let (X, μ) be a generalized topological space, A be a subset of X. (i) If $A \subseteq i_{s_\mu}(c_\mu(A))$, then $c_{\tilde{\mu}}(A) \subseteq i_{s_\mu}(c_\mu(A))$;*
(ii) If $c_{s_\mu}(i_\mu(A)) \subseteq A$, then $i_{\tilde{\mu}}(A) \supseteq c_{s_\mu}(i_\mu(A))$.

Proof. (i) Since $c_{\tilde{\mu}}(A)$ is the least $\tilde{\mu}$-closed set containing A and Theorem 3.8(i) shows that $i_{s_\mu}(c_\mu(A))$ is $\tilde{\mu}$-closed. Therefore $c_{\tilde{\mu}}(A) \subseteq i_{s_\mu}(c_\mu(A))$.

(ii) Since $i_{\tilde{\mu}}(A)$ is the greatest $\tilde{\mu}$-open set containing A and Theorem 3.8(ii) shows that $c_{s_\mu}(i_\mu(A))$ is $\tilde{\mu}$-open. Therefore $i_{\tilde{\mu}}(A) \supseteq c_{s_\mu}(i_\mu(A))$. □

Definition 3.5. *Let (X, μ) be a generalized topological space. A subset A of X is called $\tilde{\mu}$-regular if it is both $\tilde{\mu}$-open and $\tilde{\mu}$-closed. The class of all $\tilde{\mu}$-regular set of X is denoted by $\tilde{\mu}R(X)$.*

Remark 3.5. *If A is a $\tilde{\mu}$-regular set in (X, μ), then $X \setminus A$ is $\tilde{\mu}$-regular in (X, μ).*

Proof. Follows from Definition 3.5. □

Definition 3.6. *Let (X, μ) be a generalized topological space and A be a subset of X. Then $\tilde{\mu}$-boundary of A is denoted by $bd_{\tilde{\mu}}(A)$ and is defined as $bd_{\tilde{\mu}}(A) = c_{\tilde{\mu}}(A) \cap c_{\tilde{\mu}}(X \setminus A)$.*

Theorem 3.13. *For a subset A of X, $bd_{\tilde{\mu}}(A) = \emptyset$ if and only if A is $\tilde{\mu}$-regular in (X, μ).*

Proof. Let $bd_{\tilde{\mu}}(A) = \emptyset$. Then $c_{\tilde{\mu}}(A) \cap c_{\tilde{\mu}}(X \setminus A) = \emptyset$. This implies that $c_{\tilde{\mu}}(A) \subseteq X \setminus c_{\tilde{\mu}}(X \setminus A) = i_{\tilde{\mu}}(A)$ (by Theorem 3.10(vii)). Therefore $c_{\tilde{\mu}}(A) = A = i_{\tilde{\mu}}(A)$ and hence A is both $\tilde{\mu}$-closed and $\tilde{\mu}$-open in (X, μ). Conversely, assume that A is $\tilde{\mu}$-regular. Then A is both $\tilde{\mu}$-closed and $\tilde{\mu}$-open. This implies that $c_{\tilde{\mu}}(A) = A = i_{\tilde{\mu}}(A) = X \setminus c_{\tilde{\mu}}(X \setminus A)$ (by Theorem 3.10(vii)). Since $X \setminus c_{\tilde{\mu}}(X \setminus A) \cap c_{\tilde{\mu}}(X \setminus A) = \emptyset$, we have that $c_{\tilde{\mu}}(A) \cap c_{\tilde{\mu}}(X \setminus A) = \emptyset$. This shows that $bd_{\tilde{\mu}}(A) = \emptyset$. □

Theorem 3.14. *In any generalized topological space (X, μ), the following are equivalent:*
(i) $X \setminus bd_{\tilde{\mu}}(A) = i_{\tilde{\mu}}(A) \cup i_{\tilde{\mu}}(X \setminus A)$;
(ii) $c_{\tilde{\mu}}(A) = i_{\tilde{\mu}}(A) \cup bd_{\tilde{\mu}}(A)$;
(iii) $bd_{\tilde{\mu}}(A) = c_{\tilde{\mu}}(A) \cap c_{\tilde{\mu}}(X \setminus A) = c_{\tilde{\mu}}(A) \setminus i_{\tilde{\mu}}(A)$.

Proof. (i) ⇒ (ii). From (i) $X \setminus bd_{\tilde{\mu}}(A) = i_{\tilde{\mu}}(A) \cup i_{\tilde{\mu}}(X \setminus A)$ implies that $bd_{\tilde{\mu}}(A) = [X \setminus i_{\tilde{\mu}}(A)] \cap [X \setminus i_{\tilde{\mu}}(X \setminus A)]$. Therefore $i_{\tilde{\mu}}(A) \cup bd_{\tilde{\mu}}(A) = [i_{\tilde{\mu}}(A) \cup (X \setminus i_{\tilde{\mu}}(A))] \cap [i_{\tilde{\mu}}(A) \cup c_{\tilde{\mu}}(A)] = X \cap c_{\tilde{\mu}}(A) = c_{\tilde{\mu}}(A)$. Hence $c_{\tilde{\mu}}(A) = i_{\tilde{\mu}}(A) \cup bd_{\tilde{\mu}}(A)$.

(ii) ⇒ (iii). From (ii) $c_{\tilde{\mu}}(A) \setminus i_{\tilde{\mu}}(A) = [i_{\tilde{\mu}}(A) \cup bd_{\tilde{\mu}}(A)] \setminus i_{\tilde{\mu}}(A) = bd_{\tilde{\mu}}(A)$ ....(*1). Also from (ii) $X \cap c_{\tilde{\mu}}(A) = i_{\tilde{\mu}}(A) \cup bd_{\tilde{\mu}}(A)$ implies that $[i_{\tilde{\mu}}(A) \cup (X \setminus i_{\tilde{\mu}}(A))] \cap [i_{\tilde{\mu}}(A) \cup c_{\tilde{\mu}}(A)] = i_{\tilde{\mu}}(A) \cup bd_{\tilde{\mu}}(A)$ implies that $i_{\tilde{\mu}}(A) \cup [c_{\tilde{\mu}}(X \setminus A) \cap c_{\tilde{\mu}}(A)] = i_{\tilde{\mu}}(A) \cup bd_{\tilde{\mu}}(A)$. Therefore $bd_{\tilde{\mu}}(A) = c_{\tilde{\mu}}(A) \cap c_{\tilde{\mu}}(X \setminus A)$ ....(*2). From (*1) and (*2), we have that $bd_{\tilde{\mu}}(A) = c_{\tilde{\mu}}(A) \cap c_{\tilde{\mu}}(X \setminus A) = c_{\tilde{\mu}}(A) \setminus i_{\tilde{\mu}}(A)$.

(iii) ⇒ (i). From (iii), we have that $X \setminus bd_{\tilde{\mu}}(A) = X \setminus [c_{\tilde{\mu}}(X \setminus A) \cap c_{\tilde{\mu}}(A)] = [X \setminus c_{\tilde{\mu}}(X \setminus A)] \cup [X \setminus c_{\tilde{\mu}}(A)] = i_{\tilde{\mu}}(A) \cup i_{\tilde{\mu}}(X \setminus A)$. Therefore $X \setminus bd_{\tilde{\mu}}(A) = i_{\tilde{\mu}}(A) \cup i_{\tilde{\mu}}(X \setminus A)$. □

Theorem 3.15. *For a subset A of generalized topological space (X, μ), we have the following conditions hold:*
(i) $bd_{\tilde{\mu}}(A) = bd_{\tilde{\mu}}(X \setminus A)$;
(ii) $bd_{\tilde{\mu}}(A) = c_{\tilde{\mu}}(A) \setminus i_{\tilde{\mu}}(A)$;
(iii) $bd_{\tilde{\mu}}(A) \cap i_{\tilde{\mu}}(A) = \emptyset$;
(iv) $c_{\tilde{\mu}}(A) = i_{\tilde{\mu}}(A) \cup bd_{\tilde{\mu}}(A)$;
(v) $bd_{\tilde{\mu}}(i_{\tilde{\mu}}(A)) \subseteq bd_{\tilde{\mu}}(A)$;
(vi) $bd_{\tilde{\mu}}(c_{\tilde{\mu}}(A)) \subseteq bd_{\tilde{\mu}}(A)$;
(vii) $X \setminus bd_{\tilde{\mu}}(A) = i_{\tilde{\mu}}(A) \cup i_{\tilde{\mu}}(X \setminus A)$;
(viii) $X = i_{\tilde{\mu}}(A) \cup i_{\tilde{\mu}}(X \setminus A) \cup bd_{\tilde{\mu}}(A)$.

Proof. (i) By Definition 3.6, we have that $bd_{\tilde{\mu}}(A) = c_{\tilde{\mu}}(X \setminus A) \cap c_{\tilde{\mu}}(A) = c_{\tilde{\mu}}(X \setminus A) \cap c_{\tilde{\mu}}(X \setminus (X \setminus A)) = bd_{\tilde{\mu}}(X \setminus A)$. Therefore $bd_{\tilde{\mu}}(A) = bd_{\tilde{\mu}}(X \setminus A)$.

(ii) By Definition 3.6, we have that $bd_{\tilde{\mu}}(A) = c_{\tilde{\mu}}(A) \cap c_{\tilde{\mu}}(X \setminus A) = c_{\tilde{\mu}}(A) \setminus (X \setminus c_{\tilde{\mu}}(X \setminus A)) = c_{\tilde{\mu}}(A) \setminus i_{\tilde{\mu}}(A)$ (by Theorem 3.10 (vii)). Therefore $bd_{\tilde{\mu}}(A) = c_{\tilde{\mu}}(A) \setminus i_{\tilde{\mu}}(A)$.

(iii) By Definition 3.6, we have that $bd_{\tilde{\mu}}(A) \cap i_{\tilde{\mu}}(A) = (c_{\tilde{\mu}}(A) \setminus i_{\tilde{\mu}}(A)) \cap i_{\tilde{\mu}}(A)$ (by (ii)) = $\emptyset$. Hence $bd_{\tilde{\mu}}(A) \cap i_{\tilde{\mu}}(A) = \emptyset$.

(iv) Follows from (ii) and Theorem 3.14.

(v) By Definition 3.6, we have that $bd_{\tilde{\mu}}(i_{\tilde{\mu}}(A)) = c_{\tilde{\mu}}(X \setminus i_{\tilde{\mu}}(A)) \cap c_{\tilde{\mu}}(i_{\tilde{\mu}}(A)) = c_{\tilde{\mu}}(c_{\tilde{\mu}}(X \setminus A)) \cap c_{\tilde{\mu}}(i_{\tilde{\mu}}(A)) = c_{\tilde{\mu}}(X \setminus A) \cap c_{\tilde{\mu}}(i_{\tilde{\mu}}(A))$ (by Theorem 3.10 (vi)) $\subseteq c_{\tilde{\mu}}(X \setminus A) \cap c_{\tilde{\mu}}(A) = bd_{\tilde{\mu}}(A)$. This shows that $bd_{\tilde{\mu}}(i_{\tilde{\mu}}(A)) \subseteq bd_{\tilde{\mu}}(A)$.

(vi) By Definition 3.6, we have that $bd_{\tilde{\mu}}(c_{\tilde{\mu}}(A)) = c_{\tilde{\mu}}(X \setminus c_{\tilde{\mu}}(A)) \cap c_{\tilde{\mu}}(c_{\tilde{\mu}}(A)) = c_{\tilde{\mu}}(i_{\tilde{\mu}}(X \setminus A)) \cap c_{\tilde{\mu}}(A)$ (by Theorem 3.10 (vi)) $\subseteq c_{\tilde{\mu}}(X \setminus A) \cap c_{\tilde{\mu}}(A) = bd_{\tilde{\mu}}(A)$. Therefore $bd_{\tilde{\mu}}(c_{\tilde{\mu}}(A)) \subseteq bd_{\tilde{\mu}}(A)$.

(vii) Follows from (iv) and Theorem 3.14.

(viii) Using (vii) $(X \setminus bd_{\tilde{\mu}}(A)) \cup bd_{\tilde{\mu}}(A) = [i_{\tilde{\mu}}(A) \cup i_{\tilde{\mu}}(X \setminus A)] \cup bd_{\tilde{\mu}}(A)$. This implies that $X = i_{\tilde{\mu}}(A) \cup i_{\tilde{\mu}}(X \setminus A) \cup bd_{\tilde{\mu}}(A)$. □

Theorem 3.16. *Let A be a subset of generalized topological space (X, μ). Then (i) A is $\tilde{\mu}$-open if and only if $A \cap bd_{\tilde{\mu}}(A) = \emptyset$;*
(ii) A is $\tilde{\mu}$-closed if and only if $bd_{\tilde{\mu}}(A) \subseteq A$.

Proof. Let A be a $\tilde{\mu}$-open set in (X, μ). Then $X \setminus A$ is $\tilde{\mu}$-closed and $c_{\tilde{\mu}}(X \setminus A) = X \setminus A$. Also $A \neq c_{\tilde{\mu}}(A)$. By Definition 3.6 $A \cap bd_{\tilde{\mu}}(A) = A \cap (c_{\tilde{\mu}}(A) \cap c_{\tilde{\mu}}(X \setminus A)) = A \cap c_{\tilde{\mu}}(A \cap (X \setminus A)) = A \cap \emptyset = \emptyset$. Thus $A \cap bd_{\tilde{\mu}}(A) = \emptyset$. Conversely, assume that $A \cap bd_{\tilde{\mu}}(A) = \emptyset$. Then $A \cap (c_{\tilde{\mu}}(A) \cap c_{\tilde{\mu}}(X \setminus A)) = \emptyset$. This implies that $A \cap c_{\tilde{\mu}}(X \setminus A) = \emptyset$ and hence $c_{\tilde{\mu}}(X \setminus A) \subseteq X \setminus A$. Therefore $c_{\tilde{\mu}}(X \setminus A) = X \setminus A$. This shows that $X \setminus A$ is $\tilde{\mu}$-closed in (X, μ) and hence A is $\tilde{\mu}$-open in (X, μ).

(ii) Let A be a $\tilde{\mu}$-closed set in (X, μ). Then $A = c_{\tilde{\mu}}(A)$. Since $bd_{\tilde{\mu}}(A) = (c_{\tilde{\mu}}(A) \cap c_{\tilde{\mu}}(X \setminus A)) \subseteq c_{\tilde{\mu}}(A) = A$. Therefore $bd_{\tilde{\mu}}(A) \subseteq A$. Conversely, let $bd_{\tilde{\mu}}(A) \subseteq A$. Then $bd_{\tilde{\mu}}(A) \cap (X \setminus A) = \emptyset$. By Theorem 3.15 (i) $bd_{\tilde{\mu}}(X \setminus A) \cap (X \setminus A) = \emptyset$. By (i) $X \setminus A$ is $\tilde{\mu}$-open in (X, μ). Hence A is $\tilde{\mu}$-closed in (X, μ). □

Definition 3.7. *Let (X, μ) be a generalized topological space and A be a subset of X. Then $\tilde{\mu}$-exterior of A is denoted by $ext_{\tilde{\mu}}(A)$ and is defined as $ext_{\tilde{\mu}}(A) = i_{\tilde{\mu}}(X \setminus A)$.*

Theorem 3.17. *Let A and B be two subsets of generalized topological space (X, μ). Then (i) $ext_{\tilde{\mu}}(A \cup B) \subseteq ext_{\tilde{\mu}}(A) \cup ext_{\tilde{\mu}}(B)$;*
(ii) $bd_{\tilde{\mu}}(A \cup B) = c_{\tilde{\mu}}(A \cup B) \cap c_{\tilde{\mu}}(X - A) \cap c_{\tilde{\mu}}(X \setminus B)$;
(iii) $bd_{\tilde{\mu}}(A \cap B) = (bd_{\tilde{\mu}}(A) \cap c_{\tilde{\mu}}(B)) \cup (bd_{\tilde{\mu}}(B) \cap c_{\tilde{\mu}}(A))$.

Proof. By Definition 3.7, we have that $ext_{\tilde{\mu}}(A \cup B) = i_{\tilde{\mu}}(X \setminus (A \cup B)) = i_{\tilde{\mu}}((X \setminus A) \cap (X \setminus B)) \subseteq i_{\tilde{\mu}}(X \setminus A) \cap i_{\tilde{\mu}}(X \setminus B)$ (by Theorem 3.11 (v)) $= ext_{\tilde{\mu}}(A) \cup ext_{\tilde{\mu}}(B)$. Hence $ext_{\tilde{\mu}}(A \cup B) \subseteq ext_{\tilde{\mu}}(A) \cup ext_{\tilde{\mu}}(B)$.

(ii) By Definition 3.6, we have that $bd_{\tilde{\mu}}(A \cup B) = c_{\tilde{\mu}}(A \cup B) \cap c_{\tilde{\mu}}(X \setminus (A \cup B)) = c_{\tilde{\mu}}(A \cup B) \cap c_{\tilde{\mu}}((X \setminus A) \cap (X \setminus B)) = c_{\tilde{\mu}}(A \cup B) \cap c_{\tilde{\mu}}(X \setminus A) \cap c_{\tilde{\mu}}(X \setminus B)$ (by Theorem 3.11 (iv)). Hence $bd_{\tilde{\mu}}(A \cup B) = c_{\tilde{\mu}}(A \cup B) \cap c_{\tilde{\mu}}(X \setminus A) \cap c_{\tilde{\mu}}(X \setminus B)$.

(iii) By Definition 3.6, we have that $bd_{\tilde{\mu}}(A \cap B) = c_{\tilde{\mu}}(A \cap B) \cap c_{\tilde{\mu}}(X \setminus (A \cap B)) = (c_{\tilde{\mu}}(A) \cap c_{\tilde{\mu}}(B)) \cap (c_{\tilde{\mu}}(X \setminus A) \cup c_{\tilde{\mu}}(X \setminus B)) = ((c_{\tilde{\mu}}(A) \cap c_{\tilde{\mu}}(B)) \cap c_{\tilde{\mu}}(X \setminus A)) \cup ((c_{\tilde{\mu}}(A) \cap c_{\tilde{\mu}}(B)) \cap c_{\tilde{\mu}}(X \setminus B)) = ((c_{\tilde{\mu}}(A) \cap c_{\tilde{\mu}}(X \setminus A)) \cap c_{\tilde{\mu}}(B)) \cup (c_{\tilde{\mu}}(A) \cap (c_{\tilde{\mu}}(B) \cap c_{\tilde{\mu}}(X \setminus B))) = c_{\tilde{\mu}}(A \cup B) \cap c_{\tilde{\mu}}(X \setminus A) \cap c_{\tilde{\mu}}(X \setminus B)$. Hence $bd_{\tilde{\mu}}(A \cup B) = c_{\tilde{\mu}}(A \cup B) \cap c_{\tilde{\mu}}(X \setminus A) \cap c_{\tilde{\mu}}(X \setminus B)$. □

Theorem 3.18. *For any two subsets A and B of generalized topological space (X, μ), we have the following conditions hold:*
(i) $ext_{\tilde{\mu}}(X \setminus ext_{\tilde{\mu}}(A)) = ext_{\tilde{\mu}}(A)$;
(ii) $ext_{\tilde{\mu}}(A \cap B) = ext_{\tilde{\mu}}(A) \cup ext_{\tilde{\mu}}(B)$.

Proof. (i) By Definition 3.7, we have that $ext_{\tilde{\mu}}(X \setminus ext_{\tilde{\mu}}(A)) = i_{\tilde{\mu}}(X \setminus (X \setminus ext_{\tilde{\mu}}(A))) = i_{\tilde{\mu}}(ext_{\tilde{\mu}}(A)) = ext_{\tilde{\mu}}(A)$. Hence $ext_{\tilde{\mu}}(X \setminus ext_{\tilde{\mu}}(A)) = ext_{\tilde{\mu}}(A)$.

(ii) By Definition 3.7, we have that $ext_{\tilde{\mu}}(A \cap B) = i_{\tilde{\mu}}(X \setminus (A \cap B)) = i_{\tilde{\mu}}((X \setminus A) \cup (X \setminus B)) = i_{\tilde{\mu}}(X \setminus A) \cup i_{\tilde{\mu}}(X \setminus B)$ (by Theorem 3.11 (iii) $= ext_{\tilde{\mu}}(A) \cup ext_{\tilde{\mu}}(B)$. Hence $ext_{\tilde{\mu}}(A \cap B) = ext_{\tilde{\mu}}(A) \cup ext_{\tilde{\mu}}(B)$. □

4 Separation axioms

Definition 4.8. *A generalized topological space (X, μ) is called a $\tilde{\mu}$-T_0 space if for each pair of distinct points $x, y \in X$, there exists a $\tilde{\mu}$-open set U such that either $x \in U$ and $y \notin U$ or $y \in U$ and $x \notin U$.*

Definition 4.9. *A generalized topological space (X, μ) is called a $\tilde{\mu}$-T_1 space if for each pair of distinct points $x, y \in X$, there exists a $\tilde{\mu}$-open sets U and V contain x and y respectively such that $y \notin U$ and $x \notin V$.*

Definition 4.10. *A generalized topological space (X, μ) is called a $\tilde{\mu}$-T_2 space if for each pair of distinct points $x, y \in X$, there exists a $\tilde{\mu}$-open sets U and V such that $x \in U$ and $y \in V$ and $U \cap V = \emptyset$.*

Definition 4.11. *Let (X, μ) be a generalized topological space and A be a subset of X. Then A is called a $\tilde{\mu}$-generalizedized closed (briefly $\tilde{\mu}$-g.closed) set if $c_{\tilde{\mu}}(A) \subseteq U$ whenever $A \subseteq U$ and U is a $\tilde{\mu}$-open set in (X, μ).*

Remark 4.6. *From Definition 4.4, every $\tilde{\mu}$-closed set is $\tilde{\mu}$-g.closed set. But, the converse need not be true.*

Definition 4.12. *A generalized topological space (X, μ) is called a $\tilde{\mu}$-$T_{\frac{1}{2}}$ space each $\tilde{\mu}$-g.closed set of (X, μ) is $\tilde{\mu}$-closed.*

Theorem 4.19. *Let (X, μ) be a generalized topological space. Then for a point $x \in X$, $x \in c_{\tilde{\mu}}(A)$ if and only if $V \cap A \neq \emptyset$ for any $V \in \tilde{\mu}O(X)$ such that $x \in V$.*

Proof. Let F_0 be the set of all $y \in X$ such that $V \cap A \neq \emptyset$ for any $V \in \tilde{\mu}O(X)$ and $y \in V$. Now, we prove that $c_{\tilde{\mu}}(A) = F_0$. Let us assume $x \in c_{\tilde{\mu}}(A)$ and $x \notin F_0$. Then there exists a $\tilde{\mu}$-open set U of x such that $U \cap A = \emptyset$. This implies that $A \subseteq X \setminus U$. Therefore $c_{\tilde{\mu}}(A) \subseteq X \setminus U$. Hence $x \notin c_{\tilde{\mu}}(A)$. This is a contradiction. Hence $c_{\tilde{\mu}}(A) \subseteq F_0$. Conversely, let F be a set such that $A \subseteq F$ and $X \setminus F \in \tilde{\mu}O(X)$. Let $x \notin F$. Then we have that $x \in X \setminus F$ and $(X \setminus F) \cap A = \emptyset$. This implies that $x \notin F_0$. Therefore $F_0 \subseteq F$. Hence $F_0 \subseteq c_{\tilde{\mu}}(A)$. □

Definition 4.13. *Let (X, μ) be a generalized topological space and A be a subset of X. Then A is $\tilde{\mu}$-g.closed if and only if $c_{\tilde{\mu}}(\{x\}) \cap A \neq \emptyset$ holds for every $x \in c_{\tilde{\mu}}(A)$.*

Proof. Let U be any $\tilde{\mu}$-open set in (X, μ) such that $A \subseteq U$. Let $x \in c_{\tilde{\mu}}(A)$. By assumption there exists a point $z \in c_{\tilde{\mu}}(\{x\})$ and $z \in A \subseteq U$. Therefore from Theorem 5.1, we have that $U \cap \{x\} \neq \emptyset$. This implies that $x \in U$. Hence A is a $\tilde{\mu}$-g.closed set in X. Conversely, suppose there exists a point $x \in c_{\tilde{\mu}}(A)$ such that $c_{\tilde{\mu}}(\{x\}) \cap A = \emptyset$. Since $c_{\tilde{\mu}}(\{x\})$ is a $\tilde{\mu}$-closed set implies that $X \setminus c_{\tilde{\mu}}(\{x\})$ is a $\tilde{\mu}$-open set. Since $A \subseteq X \setminus c_{\tilde{\mu}}(\{x\})$ and A is $\tilde{\mu}$-g.closed set, implies that $c_{\tilde{\mu}}(A) \subseteq X \setminus c_{\tilde{\mu}}(\{x\})$. Hence $x \notin c_{\tilde{\mu}}(A)$. This is a contradiction. □

Theorem 4.20. *Let (X, μ) be a generalized topological space and A be a subset of X. Then $c_{\tilde{\mu}}(\{x\}) \cap A \neq \emptyset$ for every $x \in c_{\tilde{\mu}}(A)$ if and only if $c_{\tilde{\mu}}(A) \subseteq ker_{\tilde{\mu}}(A)$ holds, where $ker_{\tilde{\mu}}(E) = \cap\{V : V \in \tilde{\mu}O(X)$ and $E \subseteq V\}$ for any subset E of X.*

Proof. Let $x \in c_{\tilde{\mu}}(A)$. By hypothesis, there exists a point z such that $z \in c_{\tilde{\mu}}(\{x\})$ and $z \in A$. Let $U \in \tilde{\mu}O(X)$ be a subset of X such that $A \subseteq U$. Since $z \in U$ and $z \in c_{\tilde{\mu}}(\{x\})$. By Theorem 4.2, we have that $U \cap \{x\} \neq \emptyset$, this implies that $x \in ker_{\tilde{\mu}}(A)$. Hence $c_{\tilde{\mu}}(A) \subseteq ker_{\tilde{\mu}}(A)$. Conversely, let U ba any $\tilde{\mu}$-open set such that $A \subseteq U$. Let x be a point such that $x \in c_{\tilde{\mu}}(A)$. By hypothesis, $x \in ker_{\tilde{\mu}}(A)$ holds. Namely, we have that $x \in U$, because $A \subseteq U$ and U is $\tilde{\mu}$-open set. Therefore $c_{\tilde{\mu}}(A) \subseteq U$. By Definition 4.4 A is $\tilde{\mu}$-g.closed. Then by Theorem 4.2 $c_{\tilde{\mu}}(\{x\}) \cap A \neq \emptyset$ holds for every $x \in c_{\tilde{\mu}}(A)$. □

Theorem 4.21. *Let (X, μ) be a generalized topological space and A be the $\tilde{\mu}$-g.closed set in (X, μ). Then $c_{\tilde{\mu}}(A) \setminus A$ does not contain a non empty $\tilde{\mu}$-closed set.*

Proof. Suppose there exists a non empty $\tilde{\mu}$-closed set F such that $F \subseteq c_{\tilde{\mu}}(A) \setminus A$. Let $x \in F$. Then $x \in c_{\tilde{\mu}}(A)$, implies that $F \cap A = c_{\tilde{\mu}}(A) \cap A \supseteq c_{\tilde{\mu}}(\{x\}) \cap A \neq \emptyset$ and hence $F \cap A \neq \emptyset$. This is a contradiction. □

Theorem 4.22. *For each $x \in X$, $\{x\}$ is $\tilde{\mu}$-closed or $X \setminus \{x\}$ is $\tilde{\mu}$-g.closed.*

Proof. Suppose that $\{x\}$ is not $\tilde{\mu}$-closed. Then $X \setminus \{x\}$ is not $\tilde{\mu}$-open. This implies that X is the only $\tilde{\mu}$-open set containing $X \setminus \{x\}$ and hence $X \setminus \{x\}$ is $\tilde{\mu}$-g.closed. □

Theorem 4.23. *A generalized topological space (X, μ) is a $\tilde{\mu}$-$T_{\frac{1}{2}}$ space if and only if for each $x \in X$, $\{x\}$ is $\tilde{\mu}$-open or $\tilde{\mu}$-closed.*

Proof. Suppose that $\{x\}$ is not $\tilde{\mu}$-closed. Then it follows from the assumption and Theorem 4.5, $\{x\}$ is $\tilde{\mu}$-open. Conversely, let F be a $\tilde{\mu}$-g.closed set in (X, μ). Let $x \in c_{\tilde{\mu}}(F)$. Then by the assumption $\{x\}$ is either $\tilde{\mu}$-open or $\tilde{\mu}$-closed.

Case(i): Suppose that $\{x\}$ is $\tilde{\mu}$-open. Then by Theorem 4.1, $\{x\} \cap F \neq \emptyset$. This implies that $c_{\tilde{\mu}}(F) = F$. Therefore (X, μ) is a $\tilde{\mu}$-$T_{\frac{1}{2}}$ space.

Case(ii): Suppose that $\{x\}$ is $\tilde{\mu}$-closed. Let us assume $x \notin F$. Then $x \in c_{\tilde{\mu}}(F) \setminus F$. This is a contradiction. Hence $x \in F$. Therefore (X, μ) is a $\tilde{\mu}$-$T_{\frac{1}{2}}$ space. □

A space (X, μ) is $\tilde{\mu}$-T_1 if and only if for any $x \in X$, $\{x\}$ is $\tilde{\mu}$-closed.

Proof. Follows from Definitions 2.14 and 4.2. □

Remark 4.7. *(i) From the Theorems 4.5, 4.6 and 4.7, we have that every $\tilde{\mu}$-$T_{\frac{1}{2}}$ space is $\tilde{\mu}$-T_0, every $\tilde{\mu}$-T_1 space is $\tilde{\mu}$-$T_{\frac{1}{2}}$ and every $\tilde{\mu}$-T_2 space is $\tilde{\mu}$-T_1.*

(ii) Let $X = \{a, b, c, d\}$ and $\mu = \{\emptyset, X, \{a\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. For the distinct points $a, k \in X$, where $k \in \{b, c, d\}$, there exists a $\tilde{\mu}$-open set $\{a\}$ but which is not contain k; the pair $b, c \in X$, there exists a $\tilde{\mu}$-open set $\{b, d\}$ but which is not contain c; the points $b, d \in X$, there exists a $\tilde{\mu}$-open set $\{c, d\}$ but which is not contain b; more over for the points $c, d \in X$, there exists a $\tilde{\mu}$-open set $\{a, b, d\}$ but which is not contain c. This implies that (X, μ) is a $\tilde{\mu}$-T_0 space. Also $\{b, c\}, \{a, b, c\}$ are $\tilde{\mu}$-g.closed sets but not $\tilde{\mu}$-closed. Then by Definition 4.5 (X, μ) is not a $\tilde{\mu}$-$T_{\frac{1}{2}}$ space.

(iii) Let $X = \{a, b, c\}$, $\mu = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Then the $\tilde{\mu}$-g.closed sets $\emptyset, X, \{b\}, \{c\}, \{a, c\}, \{b, c\}$ are all $\tilde{\mu}$-closed. This implies that (X, μ) is a $\tilde{\mu}$-$T_{\frac{1}{2}}$ space. Also for the point $c \in X$, we have that two $\tilde{\mu}$-open sets $\{a, c\}$ and X but these sets containing the distinct point a. By Definition 4.2 (X, μ) is not a $\tilde{\mu}$-T_1 space. Then (X, μ) is a $\tilde{\mu}$-$T_{\frac{1}{2}}$ space but not $\tilde{\mu}$-T_1.

(iv) Let $X = \{a, b, c\}$, $\mu = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. For the distinct points $a, k \in X$, where $k \in \{b, c\}$, there exists $\tilde{\mu}$-open sets $\{a\}$ and $\{b, c\}$ containing a and k respectively such that $a \notin \{b, c\}$ and $k \notin \{a\}$. Also for the distinct points $b, c \in X$, there exists $\tilde{\mu}$-open sets $\{a, b\}$ and $\{a, c\}$ containing b and c respectively such that $b \notin \{a, c\}$ and $c \notin \{a, b\}$. This implies that (X, μ) is a $\tilde{\mu}$-T_1 space. More over for the distinct points $b, c \in X$, there does not exist disjoint $\tilde{\mu}$-open sets. By Definition 4.3 (X, μ) is not a $\tilde{\mu}$-T_2 space. Then (X, μ) is a $\tilde{\mu}$-T_1 space but not $\tilde{\mu}$-T_2.

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