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Total eccentricity index of some composite graphs

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Abstract

The total eccentricity index of a graph G is the sum of eccentricities of all the vertices of G. In this paper, we first derive some sharp upper and lower bounds of total eccentricity index of different subdivision graphs and then determine some explicit expression of the total eccentricity index of the double graph, extended double cover graph and some generalized thorn graphs.

Keywords: Eccentricity, graph invariant, total eccentricity index, composite graphs, graph operations.

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1 Introduction

Let $G = (V(G), E(G))$ be a connected graph with number of vertices $|V(G)| = n$ and number of edges $|E(G)| = m$. For any two vertices $u, v \in V(G)$, the distance between u and v, denoted by $d_G(u, v)$, is defined as the number of edges in the shortest path connecting u and v. The eccentricity of a vertex v, denoted by $\varepsilon_G(v)$, is the largest distance of v and any other vertex u of G. The degree of a vertex v is the number of vertices adjacent with the vertex v. A vertex v is called well-connected if $\deg_G(v) = n-1$, *i.e.*, it is adjacent to any other vertex of G. A number of topological indices based on vertex eccentricity are already subject to various studies. The total eccentricity index of G is defined as $\zeta(G) = \sum_{v \in V(G)} \varepsilon_G(v)$. Similar to this index, Dankelmann et.al. [3] and Tang et al. [16] studied average eccentricity of graphs. Fathalikhani et al. in [12], studied total eccentricity of some graph operations. In [7], the present authors present total eccentricity of the generalized hierarchical product graphs. As usual, let $K_n, S_n, C_n, K_{m,n}$ denote the complete graph with n vertices, the star graph on $(n+1)$ vertices, the cycle on n vertices and the complete bipartite graph with $(m+n)$ vertices respectively. In this paper, we first find some sharp upper and lower bounds of total eccentricity index of different subdivision graphs and then determine some explicit expression of the total eccentricity index of the double graph, extended double cover graph and some generalized thorn graphs.

2 Total eccentricity index of subdivision graphs

In this section, we derive sharp upper and lower bounds of total eccentricity index of four types of graphs resulting from edge subdivisions, such as $S(G)$, $R(G)$, $Q(G)$ and $T(G)$. For different study of these subdivision graphs see [11, 17, 18]. For a given graph G, the line graph $L(G)$ is the graph whose vertices are the edges of G and two vertices of $L(G)$ are adjacent if and only if the corresponding edges are adjacent in G. The subdivision graph $S(G)$ of a graph G is the graph obtained by inserting an additional vertex in each edge of G so that $|V(S(G))| = |V(G)| + |E(G)|$and $|E(S(G))| = 2|E(G)|$.

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Lemma 2.1. *[18] Let G be a connected graph. Then*
(i) For each $v \in V(G)$, $\varepsilon_{S(G)}(v) = 2\varepsilon_G(v)$,
(ii) For each $e \in E(G)$, $2\varepsilon_{L(G)}(e) \leq \varepsilon_{S(G)}(e) \leq 2\varepsilon_{L(G)}(e) + 1$.

Theorem 2.1. *Let G be a connected graph. Then*
(i) $\zeta(S(G)) \leq 2\zeta(L(G)) + 2\zeta(G) + |E(G)|$,
(ii) $\zeta(S(G)) \geq 2\zeta(L(G)) + 2\zeta(G)$.

Proof. From definition of $S(G)$, we have

$$\begin{aligned}\zeta(S(G)) &= \sum_{v \in V(S(G))} \varepsilon_{S(G)}(v) \\ &= \sum_{v \in V(G)} \varepsilon_{S(G)}(v) + \sum_{e \in E(G)} \varepsilon_{S(G)}(e) \\ &= \sum_{v \in V(G)} 2\varepsilon_G(v) + \sum_{e \in E(G)} \varepsilon_{S(G)}(e) \\ &= 2\zeta(G) + \sum_{e \in E(G)} \varepsilon_{S(G)}(e).\end{aligned}$$

Now using Lemma 2.1, the above sum $\sum_{e \in E(G)} \varepsilon_{S(G)}(e)$ is bounded above by $2 \sum_{v \in V(G)} \varepsilon_{L(G)}(v) + |E(G)|$ and bounded below by $2 \sum_{v \in V(G)} \varepsilon_{L(G)}(v)$. This completes the proof. □

As explained in [18], the equality attains in Theorem 2.1 when the eccentricities of all the vertices attained at the vertices of degree one (including trees). Therefore the following corollary follows.

Corollary 2.1. *Let G be a connected graph such that eccentricity of each vertex is attained just at a pendant vertex. Then* $\zeta(S(G)) = 2\zeta(L(G)) + 2\zeta(G) + |E(G)|$.

Example 2.1. *Let* S_n *and* $P_n (n \geq 1)$ *be star and path on n vertices respectively. Then we have,* $\zeta(S_n) = 2n - 1$ *and*

$$\zeta(P_n) = \begin{cases} \frac{3}{4}n^2 - \frac{1}{2}n, & \text{when } n \text{ is even} \\ \frac{3}{4}n^2 - \frac{1}{2}n - \frac{1}{4}, & \text{when } n \text{ is odd.} \end{cases}$$

(i) Since $L(S_n) = K_{1,n}$, *the total eccentricity index of subdivision graph of star graph is given by*

$$\zeta(S(S_n)) = 2\zeta(L(S_n)) + 2\zeta(S_n) + |E(S_n)| = 7n - 5.$$

(ii) Since $S(P_n) = P_{2n-1}, L(P_n) = P_{n-1}$, $|E(P_n)| = n - 1$ *so the total eccentricity index of subdivision graph of path graph is given by*

$$\zeta(S(P_n)) = 2\zeta(L(P_n)) + 2\zeta(P_n) + |E(P_n)| = 3n^2 - 4n + 1.$$

The triangle parallel graph of a graph G is denoted by $R(G)$ and is obtained by replacing each edge of G by a triangle, so that $|V(R(G))| = |V(G)| + |E(G)|$ and $|E(R(G))| = 3|E(G)|$.

Lemma 2.2. *[18] Let G be a connected graph. Then*
(i) For each $v \in V(G)$, $\varepsilon_G(v) \leq \varepsilon_{R(G)}(v) \leq \varepsilon_G(v) + 1$,
(ii) For each $e \in E(G)$, $\varepsilon_{R(G)}(e) = \varepsilon_{L(G)}(e) + 1$.

Theorem 2.2. *Let G be a connected graph. Then*
(i) $\zeta(R(G)) \leq \zeta(L(G)) + \zeta(G) + |V(G)| + |E(G)|$,
(ii) $\zeta(R(G)) \geq \zeta(L(G)) + \zeta(G) + |E(G)|$.

Proof. From definition of $R(G)$, we have

$$\zeta(R(G)) = \sum_{v \in V(R(G))} \varepsilon_{R(G)}(v) = \sum_{v \in V(G)} \varepsilon_{R(G)}(v) + \sum_{e \in E(G)} \varepsilon_{R(G)}(e).$$

Using Lemma 2.2, we have $\zeta(G) \leq \sum_{v \in V(G)} \varepsilon_{R(G)}(v) \leq \zeta(G) + |V(G)|$ and $\sum_{e \in E(G)} \varepsilon_{R(G)}(e) = \zeta(L(G)) + |E(G)|$. Combining these, the desired result follows. □

Like previous corollary, the equality attains in Theorem 2.2 when the eccentricities of all the vertices attained only at the pendent vertices. So the following corollary follows.

Corollary 2.2. *Let G be a connected graph such that eccentricity of each vertex is attained only at a pendant vertex. Then,*

$$\zeta(R(G)) = \zeta(L(G)) + \zeta(G) + |E(G)|.$$

Example 2.2. *Similarly using the last corollary the total eccentricity index of triangle parallel graph of star graph and path graphs are given respectively as*

(i) $\zeta(R(S_n)) = \zeta(L(S_n)) + \zeta(S_n) + |E(S_n)| = 4n - 3$.

(ii) $\zeta(R(P_n)) = \zeta(L(P_n)) + \zeta(P_n) + |E(P_n)| = \frac{3}{2}n(n-1)$.

Another subdivision graph, the line superposition graph $Q(G)$ of a graph G is obtained by inserting a new vertex to each edge of G and then by joining each new vertex to the end vertices of the edge corresponding to it, so that $|V(Q(G))| = |V(G)| + |E(G)|$ and $|E(Q(G))| = 3|E(G)| + |E(L(G))|$.

Lemma 2.3. *[18] Let G be a connected graph. Then*

(i) For each $v \in V(G)$, $\varepsilon_{Q(G)}(v) = \varepsilon_G(v) + 1$,

(ii) For each $e \in E(G)$, $\varepsilon_{L(G)}(e) \le \varepsilon_{Q(G)}(e) \le \varepsilon_{L(G)}(e) + 1$.

Theorem 2.3. *Let G be a connected graph. Then*

(i) $\zeta(Q(G)) \le \zeta(L(G)) + \zeta(G) + |V(G)| + |E(G)|$,

(ii) $\zeta(Q(G)) \ge \zeta(L(G)) + \zeta(G) + |V(G)|$.

Proof. Using definition of $Q(G)$, we have

$$\zeta(Q(G)) = \sum_{v \in V(Q(G))} \varepsilon_{Q(G)}(v) = \sum_{v \in V(G)} \varepsilon_{Q(G)}(v) + \sum_{e \in E(G)} \varepsilon_{Q(G)}(e)$$

From Lemma 2.3, we have $\sum\limits_{v \in V(G)} \varepsilon_{Q(G)}(v) = \zeta(G) + |V(G)|$ and $\zeta(L(G)) \le \sum\limits_{e \in E(G)} \varepsilon_{Q(G)}(e) \le \zeta(L(G)) + |E(G)|$. From where, the desired result follows. □

Similarly, we get the following corollary.

Corollary 2.3. *Let G be a connected graph such that eccentricity of each vertex is attained only at a pendant vertex. Then*

$$\zeta(Q(G)) = \zeta(L(G)) + \zeta(G) + |V(G)| + |E(G)|.$$

Example 2.3. *The total eccentricity of line superposition graph of star graph and path graph are given by*

(i) $\zeta(Q(S_n)) = \zeta(L(S_n)) + \zeta(S_n) + |V(S_n)| + |E(S_n)| = 5n - 3$.

(ii) $\zeta(R(P_n)) = \zeta(L(P_n)) + \zeta(P_n) + |E(P_n)| = \frac{3}{2}n(n-1)$.

For the total graph $T(G)$ of a graph G, any two vertices being adjacent if and only if the corresponding elements of G are either adjacent or incident, so that $|V(T(G))| = |V(G)| + |E(G)|$ and $|E(T(G))| = 2|V(G)| + |E(L(G))|$.

Lemma 2.4. *[18] Let G be a connected graph. Then*

(i) For each $v \in V(G)$, $\varepsilon_G(v) \le \varepsilon_{T(G)}(v) \le \varepsilon_G(v) + 1$,

(ii) For each, $\varepsilon_{L(G)}(e) \le \varepsilon_{T(G)}(e) \le \varepsilon_{L(G)}(e) + 1$.

Theorem 2.4. *Let G be a connected graph. Then*

(i) $\zeta(T(G)) \le \zeta(L(G)) + \zeta(G) + |V(G)| + |E(G)|$,

(ii) $\zeta(T(G)) \ge \zeta(L(G)) + \zeta(G)$.

Proof. From the construction of $R(G)$, we have

$$\zeta(T(G)) = \sum_{v \in V(T(G))} \varepsilon_{T(G)}(v) = \sum_{v \in V(G)} \varepsilon_{T(G)}(v) + \sum_{e \in E(G)} \varepsilon_{T(G)}(e).$$

Now similar to the previous theorem, using Lemma 2.4, we have $\zeta(G) \le \sum\limits_{v \in V(G)} \varepsilon_{T(G)}(v) \le \zeta(G) + |V(G)|$ and $\zeta(L(G)) \le \sum\limits_{e \in E(G)} \varepsilon_{T(G)}(e) \le \zeta(L(G)) + |E(G)|$. Combining, the desired result follows. □

Corollary 2.4. *Let T be a tree. Then*

$$\zeta(T(T)) = \zeta(L(T)) + \zeta(T) + |E(T)|.$$

Proof. Since for any tree T, $v \in V(G)$, $\varepsilon_{T(T)}(v) = \varepsilon_T(v)$ and, $\varepsilon_{T(G)}(e) = \varepsilon_T(e) + 1$. □

Example 2.4. *From the above corollary the total eccentricity of the total graph of star graph and path graph are given by*
(i) $\zeta(T(S_n)) = \zeta(L(S_n)) + \zeta(S_n) + |V(S_n)| + |E(S_n)| = 5n - 3$.
(ii) $\zeta(T(P_n)) = \zeta(L(P_n)) + \zeta(P_n) + |E(P_n)| = \frac{3}{2}n(n-1)$.

3 Total eccentricity index of double graph and extended double cover

In this section, we derive total eccentricity index of double graph and extended double cover graph. The double graph of G denoted by G^*, constructed by making two copies of G and for each vertex $u_i \in V(G)$ there are two vertices x_i and $y_i$in $V(G^*)$, so that for any edge $u_iu_j \in E(G)$ there will be two edges x_iy_j and x_jy_i including the edges x_iy_i and x_jy_j in G^*. Different applications of double graph of a graph ware investigated in [2, 9, 10, 14].

Theorem 3.5. *The total eccentricity index of the double graph G^* is given by $\zeta(G^*) = 2\zeta(G) + 2\|n-1\|_G$ where, $\|n-1\|_G$ the number of vertices with eccentricity one i.e. of degree $(n-1)$.*

Proof. From the definition of double graph it is clear $\varepsilon_{G^*}(x_i) = \varepsilon_{G^*}(y_i) = \varepsilon_G(v_i)$, *when* $\varepsilon_G(v_i) \geq 2$ and $\varepsilon_{G^*}(x_i) = \varepsilon_{G^*}(y_i) = \varepsilon_G(v_i) + 1 = 2$, *when* $\varepsilon_G(v_i) = 1$. Thus the connective eccentric index of double graph G^* is

$$\begin{aligned}
\zeta(G^*) &= \sum_{i=1}^{n} \varepsilon_{G^*}(x_i) + \sum_{i=1}^{n} \varepsilon_{G^*}(y_i) = 2\left[\sum_{\varepsilon_G(v_i)\geq 2} \{\varepsilon_G(v_i) + 1\} + \sum_{\varepsilon_G(v_i)\geq 1} 2\right] \\
&= 2\left[\sum_{\varepsilon_G(v_i)\geq 2} \varepsilon_G(v_i) + \sum_{\varepsilon_G(v_i)=1} \{\varepsilon_G(v_i) + 1\}\right] \\
&= 2\left[\sum_{\varepsilon_G(v_i)\geq 2} \varepsilon_G(v_i) + \sum_{\varepsilon_G(v_i)=1} \varepsilon_G(v_i)\right] + 2\sum_{\varepsilon_G(v_i)=1} 1 \\
&= 2\zeta(G) + 2\|n-1\|_G.
\end{aligned}$$

where, $\|n-1\|_G$ the number of vertices with eccentricity one i.e. of degree $(n-1)$. □

From the above result the following corollary is obvious.

Corollary 3.5. *If G does not contain any well connected vertices then $\zeta(G^*) = 2\zeta(G)$.*

Example 3.5. *Let G_{2n} be the double graph of P_n. Then the total eccentricity index of G_{2n} is given by*

$$\zeta(G_{2n}) = \begin{cases} \frac{3}{2}n^2 - n & , if\, n\, is\, even \\ \frac{3}{2}n^2 - n - \frac{1}{2}, & if\, n\, is\, odd. \end{cases}$$

The extended double cover was introduced by Alon [1] in 1986. Let G be a simple connected graph with vertex set $V = \{v_1, v_2, ..., v_n\}$. The extended double cover of G, denoted by G^{**}, is the bipartite graph with bipartion (P, Q) where $P = \{x_1, x_2, ..., x_n\}$ and $Q = \{y_1, y_2, ..., y_n\}$ in which x_i and y_i are adjacent if and only if $i = j$.

Theorem 3.6. *The total eccentricity index of the extended double cover G^{**} is given by $\zeta(G^{**}) = 2\zeta(G) + 2n$.*

Proof. If G is a graph with n vertices and m edges then from definition of extended double cover graph G^{**} consists of $2n$ vertices and $(n+2m)$ edges and $\varepsilon_{G^*}(x_i) = \varepsilon_{G^*}(y_i) = \varepsilon_G(v_i)+1$, for $i = 1,2,,n$. Thus the connective eccentric index of double graph G^{**} is

$$\begin{aligned}
\zeta(G^{**}) &= \sum_{i=1}^{n}\varepsilon_{G^{**}}(x_i) + \sum_{i=1}^{n}\varepsilon_{G^{**}}(y_i) \\
&= 2\sum_{i=1}^{n}\{\varepsilon_G(v_i)+1\} \\
&= 2\zeta(G)+2n
\end{aligned}$$

as desired. □

Example 3.6. *(i) Let H_{2n} be the extended double cover of P_n. Then the total eccentricity index of H_{2n} is given by*

$$\zeta(H_{2n}) = \begin{cases} \frac{3}{4}n^2+\frac{3}{2}n, & \text{when } n \text{ is even} \\ \frac{3}{4}n^2+\frac{3}{2}n-\frac{1}{4}, & \text{when } n \text{ is odd.} \end{cases}$$

(ii) Since the extended double cover of K_n is $K_{n,n}$, so from Theorem 3.6 it can be verified that

$$\zeta(K_n^{**}) = 2\zeta(K_n)+2n = 4n = \zeta(k_{n,n}).$$

4 Total eccentricity index of generalized thorn graphs

Now we determine total eccentricity index of two special types of graphs G_{K_p} and G_{P_m}(see [15]), named as generalized thorn graph as the thorny graph are special cases of these graphs. Let G_{K_p} be the graph obtained from G by attaching t complete graph of order p i.e. K_p at every vertex of G. Let the vertices of G are denoted by $v_1, v_2, ..., v_k$. Also the vertices attached to the vertex v_i are denoted by $v_{i1}^{(r)}, v_{i2}^{(r)}, ..., v_{im}^{(r)}$; $i = 1,2,...,k$; $r = 1,2,...,t$. Now we find the total eccentricity index of this graph G_{K_p}. Let the vertex v_i is identified with $v_{ip}^{(r)}$ $i = 1,2,...,k; r = 1,2,...,t$.

Theorem 4.7. *For any simple connected graph G the $\zeta(G_{K_p})$ and $\zeta(G)$ are related as*

$$\zeta(G_{K_p}) = (pt-t+1)\zeta(G) + k(2pt-2t+1)$$

(2) where G_{K_p} is the graph obtained from G by attaching t complete graphs at each vertex of G.

Proof. From the construction, the eccentricities of the vertices of G_{K_p} are given by

$$\varepsilon_{G_{K_p}}(v_i) = \varepsilon_G(v_i)+1, \text{for } i = 1,2,...,k;$$

$$\varepsilon_{G_{K_p}}(v_{ij}^{(r)}) = \varepsilon_G(v_i)+2, \text{for } i = 1,2,...,k; j = 1,2,...,m; r = 1,2,...,t.$$

Therefore the total eccentricity index of G_{K_p} is given by

$$\begin{aligned}
\zeta(G_{K_p}) &= \sum_{i=1}^{k}\varepsilon_{G_{K_p}}(v_i) + \sum_{i=1}^{k}\sum_{j=1}^{p-1}\sum_{r=1}^{t}\varepsilon_{G_{K_p}}(v_{ij}^{(r)}) \\
&= \sum_{i=1}^{k}\{\varepsilon_G(v_i)+1\} + \sum_{i=1}^{k}\sum_{j=1}^{p-1}\sum_{r=1}^{t}\{\varepsilon_G(v_i)+2\} \\
&= \sum_{i=1}^{k}\varepsilon_G(v_i)+k+t(p-1)\sum_{i=1}^{k}\{\varepsilon_G(v_i)+2\} \\
&= \{1+t(p-1)\}\,\zeta(G)+k+2kt(p-1)
\end{aligned}$$

as desired. □

Any edge (u, v) of a graph G is called a thorn if and only if, either $\deg_G(u) = 1$ or $\deg_G(v) = 1$. The t-thorny graph G^t is obtained from G by attaching t thorns at each and every vertices of G. This type of graphs were introduced in [13] and for different eccentricity based topological indices of thorn graphs see [4–6, 8]. Since the thorn of a graph can be treated as K_2 , so it is easy to show that by substituting $p = 2$ in (2), we get the total eccentricity of the t-thorny graphs as follows, which already derived in [7].

Corollary 4.6. *[7] The total eccentricity index of the t-thorn graph G^t is computed as follows*

$$\zeta(G^t) = (t+1)\zeta(G) + n(2t+1).$$

Next, we construct another graph denoted by G_{P_m} by attaching t paths of order m (≥ 2) at each vertex v_i,$1 \leq i \leq p$ of G. The vertices of the r-th path attached at v_i are denoted by $v_{i1}^{(r)}, v_{i2}^{(r)}, ..., v_{im}^{(r)}$; $i = 1, 2, ..., p; r = 1, 2, ..., t$. Let the vertex $v_{i1}^{(r)}$ is identified with the i-th vertex v_i of G. Clearly the resulting graph G_{P_m} consists of $\{(m-1)t + p\}$ number of vertices.

Theorem 4.8. *For any simple connected graph G the $\zeta(G_{P_m})$ and $\zeta(G)$ are related as*

$$\zeta(G_{P_m}) = \{t(m-1)+1\}\,\zeta(G_{P_m}) + \frac{kt}{2}\left(3m^2 - 5m + 2\right) + k(m-1)$$

(3) where G_{P_m} is the graph obtained from G by attaching t paths each of length m at each vertex of G.

Proof. From the construction of G_{P_m}, eccentricities of the vertices are given by

$$\varepsilon_{G_{P_m}}(v_i) = \varepsilon_G(v_i) + (m-1), \text{for } i = 1, 2, ..., k;$$

$$\varepsilon_{G_{P_m}}(v_{ij}^{(r)}) = \varepsilon_G(v_i) + m + j - 2, \text{for } i = 1, 2, ..., k; j = 1, 2, ..., m; r = 1, 2, ..., t.$$

Therefore the total eccentricity index of G_{P_m} is given by

$$\begin{aligned}
\zeta(G_{P_m}) &= \sum_{i=1}^{k} \varepsilon_{G_{P_m}}(v_i) + \sum_{i=1}^{k}\sum_{j=2}^{m}\sum_{r=1}^{t} \varepsilon_{G_{P_m}}(v_{ij}^{(r)}) \\
&= \sum_{i=1}^{k} \{\varepsilon_G(v_i) + (m-1)\} + \sum_{i=1}^{k}\sum_{j=2}^{m}\sum_{r=1}^{t} \{\varepsilon_G(v_i) + m + j - 2\} \\
&= \sum_{i=1}^{k} \varepsilon_G(v_i) + k(m-1) + kt(m-1)(m-2) + t(m-1)\sum_{i=1}^{k} \varepsilon_G(v_i) + kt\sum_{j=2}^{m} j \\
&= \{t(m-1)+1\}\,\zeta(G_{P_m}) + k(m-1)\,\{t(m-2)+1\} + kt\left\{\frac{m(m+1)}{2} - 1\right\}
\end{aligned}$$

from where the desired result follows. □

Again, the thorn of the graph can be treated as P_2, so by substituting $m = 2$ in (3) we can obtain the Corollary 4.6.

5 Conclusion

In this paper, first we derive some sharp upper and lower bounds of total eccentricity index of different subdivision graphs and then apply those results to find total eccentricity index of some particular graphs. Then we determine some explicit expression of the total eccentricity index of the double graph, extended double cover graph and some generalized thorn graphs; from where we get the total eccentricity of the t-thorny graphs or t-fold bristled graph. For further study, total eccentricity index of some other graph operations can be computed.

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