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Reliability Measure of an Integrated H/W and S/W System with Redundancy and Preventive maintenance

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Abstract

The purpose of the study is to evaluate the reliability measures of an integrated h/w and s/w system with the concepts of redundancy and preventive maintenance. A stochastic model is developed considering two-identical units of the system- one unit is initially operative and other is in cold standby. In each unit h/w and s/w work together and may fail independently from normal mode. There is a single server who visits the system immediately to h/w repair and s/w up-gradation. The preventive maintenance of the system (unit) is conducted by the server after a maximum operation time. The failure time of h/w and s/w follows negative exponential distribution while the distributions of preventive maintenance, h/w repair and s/w up-gradation times are taken as arbitrary. The semi-Markov process and regenerative point technique is adopted to derive expressions for various measures of system effectiveness. The behaviour of some important reliability measures has been observed graphically giving particular values to various costs and parameters.

Keywords: Integrated h/w and s/w System, Redundancy, H/w Repair, S/w Up-gradation, Preventive Maintenance, Maximum Operation Time and Reliability Measures.

2010 MSC: 90B25, 60K10.

1 Introduction

Now a day's integrated h/w and s/w systems are of growing importance because of their use in almost all academic, business and industrial sectors. The continued operation and ageing of these systems gradually reduce their performance, reliability and safety. Therefore, a major challenge to the engineers and researchers is to develop such systems which can produce failure free services to the users with least cost. The method of redundancy has been used in many industrial systems not only to attain better reliability but also to reduce the frequency of failure up to a desired extent. Goel and Sharma[1] and Singh[2] discussed stochastically the two unit standby system under different repair policies of the server. But the technique of redundancy has not been used much more in case of integrated h/w and s/w systems. A few researchers including Malik and Anand[3] obtained reliability measures for a computer system by taking a redundant unit in cold standby. Further, it is proved that preventive maintenance can slow the deterioration process of operating system and restore them in a younger age or state. Thus, the method of preventive maintenance can be used to improve the performance of these systems. Recently, Malik and Kumar[4] investigated a reliability model for a computer system conducting preventive maintenance after a maximum operation time.

To strengthen the existing literature, here reliability measures for an integrated h/w and s/w system are obtained by introducing the concepts of redundancy and preventive maintenance. A stochastic model is developed considering two-identical units of the system- one unit is initially operative and other is in cold standby. In each unit h/w and s/w work together and may fail independently from normal mode. There

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is a single server who visits the system immediately to h/w repair and s/w up-gradation. The preventive maintenance of the system (unit) is conducted by the server after a maximum operation time. The failure time of h/w and s/w follows negative exponential distribution while the distributions of preventive maintenance, h/w repair and s/w up-gradation times are taken as arbitrary. The semi-Markov process and regenerative point technique is adopted to derive expressions for various measures of system effectiveness such as mean time to system failure, availability, busy period of the server due to preventive maintenance, busy period of the server due to h/w repair, busy period of the server due to software up-gradation, expected number of software up-gradations and expected number of visits of the server. The behaviour of some important reliability measures has been observed graphically giving particular values to various costs and parameters.

Notations

N_0	:	The unit is operative and in normal mode
C_s	:	The unit is cold standby
a/b	:	Probability that the system has hardware/software failure
λ^1/λ^2	:	Constant failure rate of hardware/software
α_0	:	Constant rate of Maximum Operation Time
Pm/PM	:	The unit is under preventive Maintenance/under preventive maintenance continuously from previous state
WPm/WPM	:	The unit is waiting for preventive Maintenance/waiting for preventive maintenance continuously from previous state
$HFur/HFUR$	:	The hardware is failed and is under repair/under repair continuously from previous state
$HFwr/HFWR$	:	The hardware is failed and is waiting for repair/waiting for repair continuously from previous state
$SFurp/SFURP$	:	The software is failed and is under up-gradation/under up-gradation continuously from previous state
$SFwrp/SFWRP$	:	The software is failed and is waiting for up-gradation/waiting for up-gradation continuously from previous state
$h(t)/H(t)$	:	pdf/cdf of software up-gradation time
$g(t)/G(t)$	:	pdf/cdf of repair time of the hardware
$f(t)/F(t)$	:	pdf/cdf of the time for preventive maintenance of the unit
$q^{ij}(t)/Q^{ij}(t)$	:	pdf/cdf of passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in $(0,t]$
pdf/cdf	:	Probability density function/ Cumulative density function
$q^{ij\cdot kr}(t)/Q^{ij\cdot kr}(t)$	:	pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k, r once in $(0,t]$
$\mu_i(t)$	:	Probability that the system up initially in state $S_i \in E$ is up at time t without visiting to any regenerative state
$W_i(t)$	:	Probability that the server is busy in the state S_i up to time 't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.
m_{ij}	:	Contribution to mean sojourn time (μ_i) in state S_i when system transit directly to state S_j so that $\mu_i = \sum_j m_{ij}$ and $m^{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*'}(0)$?
S/©	:	Symbol for Laplace-Stieltjes convolution/Laplace convolution

Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)dt \tag{1.1}$$

as

$$\left.\begin{aligned}
&p^{01} = \frac{\int_0 \alpha}{A}, && p^{02} = \frac{\int_1 a\lambda}{A},\\
&p^{03} = \frac{\int_2 b\lambda}{A}, && p^{10} = f^*(A),\\
&p^{1.10} = \frac{\int_1 a\lambda}{A}[1 - f^*(A)] = p^{12.10}, && p^{1.12} = \frac{\int_2 b\lambda}{A}[1 - f^*(A)] = p^{13.12},\\
&p^{1.4} = \frac{\int_0 \alpha}{A}[1 - f^*(A)] = p^{11.4}, && p^{20} = g^*(A),\\
&p^{29} = \frac{\int_0 \alpha}{A}[1 - g^*(A)] = p^{21.9}, && p^{27} = \frac{\int_2 b\lambda}{A}[1 - g^*(A)] = p^{23.7},\\
&p^{28} = \frac{\int_1 a\lambda}{A}[1 - g^*(A)] = p^{22.8}, && p^{30} = h^*(A),\\
&p^{35} = \frac{\int_1 a\lambda}{A}[1 - h^*(A)] = p^{32.5}, && p^{3.11} = \frac{\int_0 \alpha}{A}[1 - h^*(A)] = p^{3,1.11},\\
&p^{41} = f^*(s), && p^{36} = \frac{\int_2 b\lambda}{A}[1 - h^*(A)] = p^{33.6},\\
&p^{52} = h^*(s), && p^{63} = h^*(s),\\
&p^{73} = g^*(s) = p^{82} = p^{91}, && p^{10.2} = f^*(s),\\
&p^{11.1} = h^*(s), && p^{12.3} = f^*(s)
\end{aligned}\right\} \tag{1.2}$$

where $A = \int_1 a\lambda + \int_2 b\lambda + \int_0 \alpha$.

It can be easily verified that

$$\left.\begin{aligned}
p^{01} + p^{02} + p^{03} &= p^{10} + p^{14} + p^{1.10} + p^{1.12} = p^{20} + p^{27} + p^{29} + p^{28}\\
&= p^{30} + p^{35} + p^{3.11} + p^{36} = p^{41} = p^{52} = p^{63} = p^{73}\\
&= p^{82} = p^{91} = p^{10.2} = p^{11.1} = p^{12.3}\\
&= p^{10} + p^{12.10} + p^{11.4} + p^{13.12} = p^{20} + p^{21.9} + p^{22,8} + p^{23.7}\\
&= p^{30} + p^{31.11} + p^{32.5} + p^{33.6} = 1
\end{aligned}\right\} \tag{1.3}$$

The mean sojourn times (μ_i) is the state S_i are μ

$$\left.\begin{aligned}
&\mu_0 = \frac{1}{\int_1 a\lambda + \int_2 b\lambda + \int_0 \alpha}, && \mu_1 = \frac{1}{\int_1 a\lambda + \int_2 b\lambda + \int_0 \alpha + \alpha},\\
&\mu_2 = \frac{1}{\int_1 a\lambda + \int_2 b\lambda + \int_0 \alpha + \theta}, && \mu_3 = \frac{1}{\int_1 a\lambda + \int_2 b\lambda + \int_0 \alpha + \int \beta},\\
&\mu_1' = \frac{1}{\alpha}, && \mu_2' = \frac{1}{\theta}, \quad \mu_3' = \frac{1}{\beta}
\end{aligned}\right\} \tag{1.4}$$

The states S_0, S_1, S_2 and S_3 are regenerative states while S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11} and S_{12}, are non-regenerative states. Thus $E = \{S_0, S_1, S_2, S_3\}$. The possible transition between states along with transition rates for the model is shown in Fig. **??**.

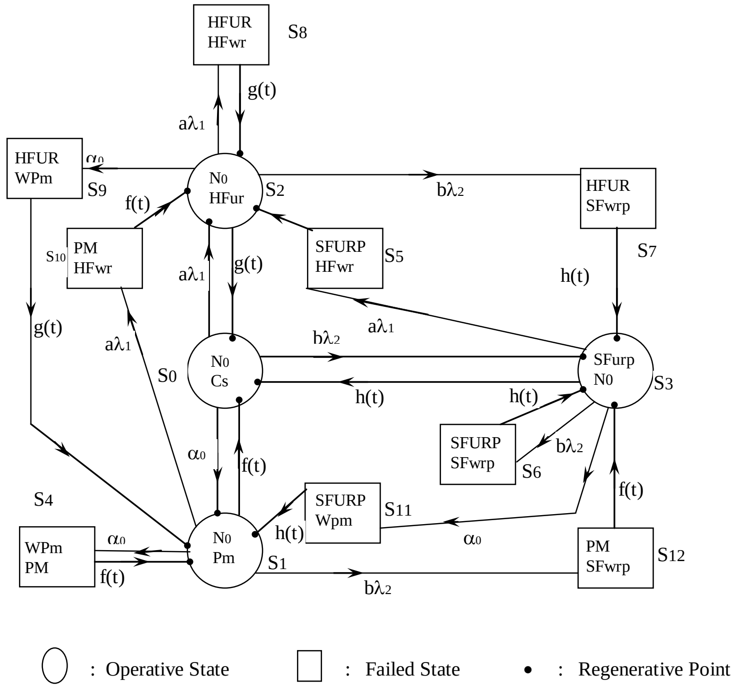

Figure 1:

Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from the regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

$$\varphi_i(t) = \sum_j Q_{i,j}(t) + \circledR \varphi_j(t) + \sum_k Q_{i,j}(t), \tag{1.5}$$

where j is an un-failed regenerative state to which the given regenerative state i can transit and k is a failed state to which the state i can transit directly. Taking LST of above relation (1.5) and solving for $\tilde{\phi}_0(s)$. We have

$$R^*(s) = \frac{1 - \tilde{\phi}_0(s)}{s} \tag{1.6}$$

The reliability of the system model can be obtained by taking Laplace inverse transform of (1.6).

The mean time to system failure (MTSF) is given by

$$MTSF = \lim_{s \to 0} \frac{1 - \widetilde{\phi}_0(s)}{s} = \frac{N_1}{D_1} \tag{1.7}$$

where $N^1 = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + +p_{03}\mu_3$ and $D^1 = 1 - p_{01}p_{10} - p_{02}p_{20} - p_{03}p_{30}$.

2 Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state i at $t = 0$. The recursive relations for $A_i(t)$ are given as

$$A_i(t) = M_i(t) + \sum_j a_{ij}^{(n)}(t) \copyright A_j(t) \tag{2.8}$$

where j is any successive regenerative state to which the regenerative state i can transit through n transitions. $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ up at time t without visiting to any other regenerative state, we have is

$$M_0(t) = e^{-(a\lambda_1+b\lambda_2+\alpha_0)^t}, \quad , M_1(t) = e^{-(a\lambda_1+b\lambda_2+\alpha_0)^t}\overline{F(t)},$$
$$M_2(t) = e^{-(a\lambda_1+b\lambda_2+\alpha_0)^t}\overline{G(t)}, \quad , M_3(t) = e^{-(a\lambda_1+b\lambda_2+\alpha_0)^t}\overline{H(t)}, \tag{2.9}$$

Taking LT of above relations (2.8) and solving for $A_0^*(s)$. The steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2}, \tag{2.10}$$

where

$$\begin{aligned}N_2 = {} & \mu_0[(1-p_{11.4})\{(1-p_{22.8})(1-p_{33.6}) - p_{23.7}p_{32.5}\} - p_{12.10}\{(1-p_{33.6})p_{21.9} + p_{31.11}p_{23.7}\} \\ & - p_{13.12}\{p_{21.9}p_{32.5} + (1-p_{22.8})p_{33.6}\}] + \mu_1[p_{01}\{(1-p_{22.8})(1-p_{33.6}) - p_{23.7}p_{32.5}\} \\ & + p_{02}\{(1-p_{33.6})p_{21.9} + p_{31.11}p_{23.7}\} + p_{03}\{p_{21.9}p_{32.5} + (1-p_{22.8})p_{33.6}\}] \\ & + \mu_2[p_{01}\{p_{12.10}(1-p_{33.6}) + p_{13.12}p_{32.5}\} + p_{02}\{(1-p_{33.6})(1-p_{11.4}) - p_{31.11}p_{13.12}\} \\ & + p_{03}\{p_{31.11}p_{12.10} + (1-p_{11.4})p_{32.5}\}] + \mu_3[p_{01}\{p_{12.10}p_{23.7} + p_{13.12}(1-p_{22.8})\} \\ & + p_{02}\{(1-p_{11.4})p_{23.7} + p_{21.9}p_{13.12}\} + p_{03}\{(1-p_{22.8})(1-p_{11.14}) - p_{21.9}p_{12.10}\}]\end{aligned}$$

and

$$\begin{aligned}D_2 = {} & \mu_0[(1-p_{11.4})\{(1-p_{22.8})(1-p_{33.6}) - p_{23.7}p_{32.5}\} - p_{12.10}\{(1-p_{33.6})p_{21.9} + p_{31.11}p_{23.7}\} \\ & - p_{13.12}\{p_{21.9}p_{32.5} + (1-p_{22.8})p_{33.6}\}] + [p_{01}\{(1-p_{22.8})(1-p_{33.6}) - p_{23.7}p_{32.5}\} \\ & + p_{02}\{(1-p_{33.6})p_{21.9} + p_{31.11}p_{23.7}\} + p_{03}\{p_{21.9}p_{32.5} + (1-p_{22.8})p_{33.6}\}] \\ & + [p_{01}\{p_{12.10}(1-p_{33.6}) + p_{13.12}p_{32.5}\} + p_{02}\{(1-p_{33.6})(1-p_{11.4}) - p_{31.11}p_{13.12}\} \\ & + p_{03}\{p_{31.11}p_{12.10} + (1-p_{11.4})p_{32.5}\}] + [p_{01}\{p_{12.10}p_{23.7} + p_{13.12}(1-p_{22.8})\} \\ & + p_{02}\{(1-p_{11.4})p_{23.7} + p_{21.9}p_{13.12}\} + p_{03}\{(1-p_{22.8})(1-p_{11.14}) - p_{21.9}p_{12.10}\}]\end{aligned}$$

Busy Period Analysis for Server

Let $B_i^P(t)$, $B_i^R(t)$ and $B_i^S(t)$ be the probability that the server is busy in preventive maintenance, hardware repair and software up-gradation of the system (unit) at an instant 't' given that the system entered state i at $t = 0$. The recursive relations for $B_i^P(t)$, $B_i^R(t)$ and $B_i^S(t)$ are as follows:

$$B_i^P(t) = W_i(t) + \sum_j q_{ij}^{(n)}(t) © B_J^p(t) \tag{2.11}$$

$$B_i^R(t) = W_i(t) + \sum_j q_{ij}^{(n)}(t) © B_J^R(t) \tag{2.12}$$

$$B_i^S(t) = W_i(t) + \sum_j q_{ij}^{(n)}(t) © B_J^S(t) \tag{2.13}$$

Where j is any successive regenerative state to which the regenerative state i can transit through n transitions. $W_i(t)$ be the probability that the server is busy in state S_i due to PM, h/w repair and s/w up-gradation of the system up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states. Taking LT of above relations (2.11) to (2.13) and solving for $B_0^{*P}(s)$, $B_0^{*R}(s)$ and $B_0^{*S}(s)$. The time for which server is busy due to preventive maintenance, h/w repair and s/w up-gradation respectively is given by

$$B_0^p = \lim_{s\to 0} sB_0^{*P}(s) = \frac{N_3^p}{D_2}, \ B_0^R = \lim_{s\to 0} sB_0^{*R}(s) = \frac{N_3^R}{D_2} \text{ and } B_0^S = \lim_{s\to 0} sB_0^{*S}(s) = \frac{N_3^S}{D_2},$$

where

$$N_3^P = W_1^*(0)[p_{01}\{(1-p_{22.8})(1-p_{33.6})-p_{23.7}p_{32.5}\}+p_{02}\{(1-p_{33.6})p_{21.9}+p_{31.11}p_{23.7}\}$$
$$+p_{03}\{p_{21.9}p_{32.5}+(1-p_{22.8})p_{33.6}\}] \quad (2.14)$$
$$N_3^P = W_2^*(0)[p_{01}\{p_{12.10}(1-p_{33.6})+p_{13.12}p_{32.5}\}+p_{02}\{(1-p_{33.6})(1-p_{11.4})-p_{31.11}p_{13.12}\}$$
$$+p_{03}\{p_{31.11}p_{12.10}+(1-p_{11.4})p_{32.5}\}] \quad (2.15)$$
$$N_3^S = W_2^*(0)[p_{01}\{p_{12.10}p_{23.7}+p_{13.12}(1-p_{22.8})\}+p_{02}\{(1-p_{11.4})p_{23.7}+p_{21.9}p_{13.12}\}$$
$$+p_{03}\{(1-p_{22.8})(1-p_{11.14})-p_{21.9}p_{12.10}\}] \quad (2.16)$$

Expected Number of S/w Up-gradations

Let $R_i^S(t)$ be the expected number of software up-gradations by the server in $(0,t]$ given that the system entered the regenerative state i at $t=0$. The recursive relations for $R_i^S(t)$ are given as

$$R_i^S(t) = \sum_j Q_{i,j}^{(n)}(t)Ⓡ[\delta_j + R_j^S(t)]\,. \quad (2.17)$$

Where j is any regenerative state to which the given regenerative state i transits and $\delta_j = 1$, if j is the regenerative state where the server does job afresh, otherwise $\delta_j = 0$.

Taking LST of relations (2.17) and solving for $\tilde{R}_0^S(s)$. The expected numbers of s/w up-gradations per unit time are given by

$$R_0^S(\infty) = \lim_{s\to 0}\tilde{R}_0^S = \frac{N_4^S}{D_2}\,. \quad (2.18)$$

Where D_2 is already mentioned.

$$N_4^S = [p_{01}\{p_{12.10}p_{23.7}+p_{13.12}(1-p_{22.8})\}+p_{02}\{(1-p_{11.4})p_{23.7}+p_{21.9}p_{13.12}\}$$
$$+p_{03}\{(1-p_{22.8})(1-p_{11.14})-p_{21.9}p_{12.10}\}]$$

Expected Number of Visits by the Server

Let $N_i(t)$ be the expected number of visits by the server in $(0,t]$ given that the system entered the regenerative state i at $t=0$. The recursive relations for $N_i(t)$ are given as

$$N_i(t) = \sum_j Q_{i,j}^{(n)}(t)Ⓡ[\delta_j + N_j(t)] \quad (2.19)$$

Where j is any regenerative state to which the given regenerative state i transits and $\delta_j = 1$, if j is the regenerative state where the server does job afresh, otherwise $\delta_j = 0$. Taking LST of relation (2.19) and solving for $\tilde{N}_0(s)$. The expected number of visit per unit time by the $\tilde{N}_0(s)$ server are given by

$$N_0(\infty) = \lim_{s\to 0} s\tilde{N}_0(s) = \frac{N_2}{D_2}, \quad (2.20)$$

where

$$N_5 = [(1-p_{11.4})\{(1-p_{22.8})(1-p_{33.6})-p_{23.7}p_{32.5}\}-p_{12.10}\{(1-p_{33.6})p_{21.9}+p_{31.11}p_{23.7}\}$$
$$-p_{13.12}\{p_{21.9}p_{32.5}+(1-p_{22.8})p_{33.6}\}]$$

Profit Analysis

The profit incurred to the system model in steady state can be obtained as

$$P = K_0A_0 - K_1B_0^P - K_2B_0^R - K_3B_0^S - K_4B_0^S - K_5N_0 \quad (2.21)$$

K_0 = Revenue per unit up-time of the system
K_1 = Cost per unit time for which server is busy due preventive maintenance
K_2 = Cost per unit time for which server is busy due to hardware failure
K_3 = Cost per unit time for which server is busy due to software up-gradation
K_4 = Cost per unit time s/w up-gradation
K_5 = Cost per unit time visit by the server

3 Conclusion

By considering a particular case $g(t) = \theta e^{-\theta t}$, $h(t) = \beta e^{-\beta t}$ and $f(t) = \alpha e^{-\alpha t}$, the numerical results some reliability measures are obtained for the system under study. The graphs for mean time to system failure (MTSF), availability and profit are drawn with respect to preventive maintenance (α) rate for fixed values of parameters as shown respectively in Figures 4, **??** and **??**. It is revealed that MTSF, Availability and profit increase with the increase of PM rate (α) and h/w repair rate (θ). But the value of these measures decrease with the increase of maximum operation time (α_0). Thus finally it is concluded that a system in which chances of h/w failure are high can be made reliable and economical to use

(i) By taking one more unit in cold standby.

(ii) By conducting PM of the system after a specific period of time.

(iii) By increasing h/w repair rate in case preventive maintenance of the system is not conducted after a maximum operation time.

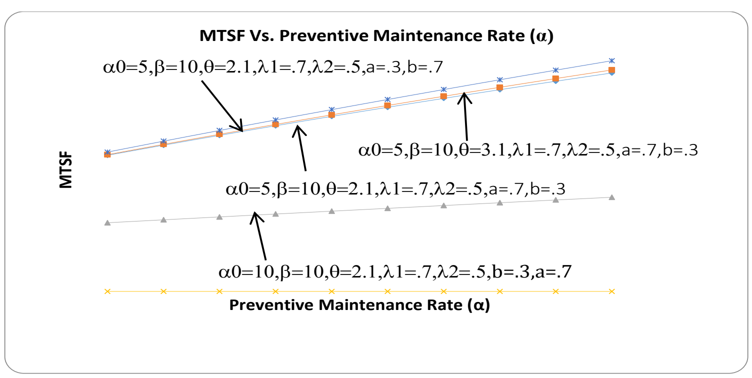

Figure 2:

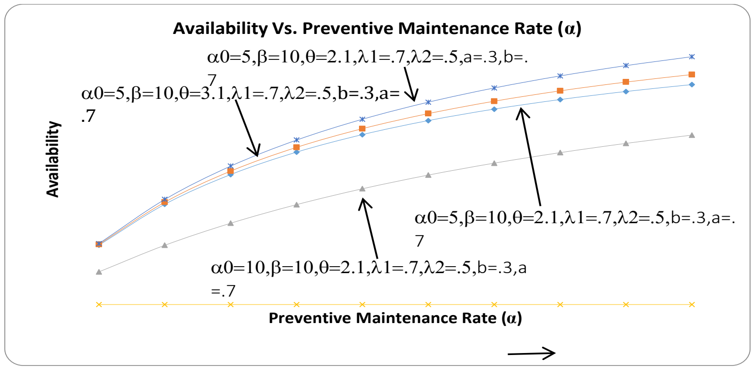

Figure 3:

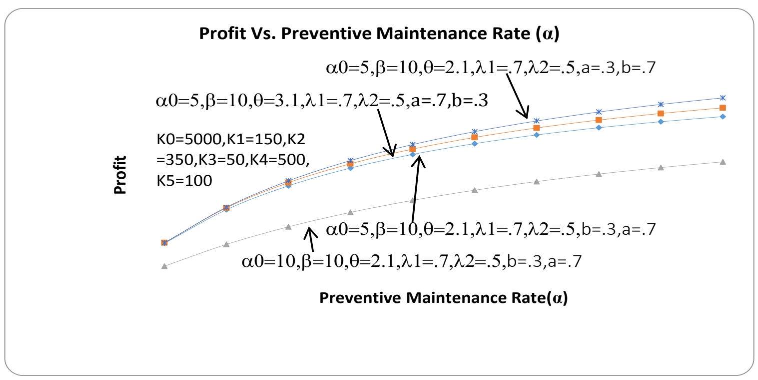

Figure 4:

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