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Edge pair sum labeling of butterfly graph with shell order

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Abstract

The concept of an edge pair sum labeling was introduced in [3]. Let $G(p,q)$ be a graph. An injective map $f : E(G) \to \{\pm 1, \pm 2, \cdots, \pm q\}$is said to be an edge pair sum labeling if the induced vertex function $f^* : V(G) \to Z - \{0\}$ defined by $f^*(v) = \sum_{e \in E_v} f(e)$ is one- one where E_v denotes the set of edges in G that are incident with a vertex v and $f^*(V(G))$ is either of the form $\left\{\pm k_1, \pm k_2, \cdots, \pm k_{\frac{p}{2}}\right\}$ or$\left\{\pm k_1, \pm k_2, \cdots, \pm k_{\frac{p-1}{2}}\right\} \bigcup \left\{\pm k_{\frac{p+1}{2}}\right\}$ according as p is even or odd. A graph with an edge pair sum labeling is called an edge pair sum graph. In this paper we prove that the shell graph and butterfly graph with shell order are edge pair sum graphs.

Keywords: Edge pair sum labeling, edge pair sum graph, shell graph, butterfly graph.

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1 Introduction

Throughout this paper we consider finite, simple and undirected graph $G = (V(G), E(G))$ with p vertices and q edges. G is also called a (p,q) graph. We follow the basic notations and terminologies of graph theory as in [2]. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. There are several types of labeling and for a dynamic survey of various graph labeling problems with extensive bibliography we refer to Gallian [1]. Ponraj et.al introduced the concept of pair sum labeling in [13]. An injective map $f : V(G) \to \{\pm 1, \pm 2, \ldots, \pm p\}$ is said to be a pair sum labeling of a graph $G(p,q)$ if the induced edge function $f_e : E(G) \to Z - \{0\}$ defined by $f_e(uv) = f(u) + f(v)$ is one-one and $f_e(E(G))$ is either of the form $\left\{\pm k_1, \pm k_2, \ldots, \pm k_{\frac{q}{2}}\right\}$ or $\left\{\pm k_1, \pm k_2, \ldots, \pm k_{\frac{q-1}{2}}\right\} \cup \left\{\pm k_{\frac{q+1}{2}}\right\}$ according as q is even or odd. Analogous to pair sum labeling we define a new labeling called edge pair sum labeling [3] and we [4-12] establish that the path, cycle, star graph, $P_m \bigcup K_{1,n}$, $C_n \bigcup K_m^c$ if n is even, triangular snake, star graph, bistar, complete bipartite graphs, $k_{1,n}$, jelly fish, Y-tree, theta graph, spider graphs, ladder graph, $WT(n : k)$, subdivision of spokes in wheels, N quadrilateral graph, wheel graph, double triangular snake, flower graph, one point union of cycles, the perfect binary tree, shadow graph, total graph and P_n^2 are edge pair sum graphs. In this paper we prove that the shell graph and butterfly graph with shell order are edge pair sum graphs.

We use the following definitions in the subsequent sequel.

Definition 1.1. *A shell S_n is the graph obtained by taking (n-3)concurrent cords in a cycle C_n. The vertex at which all the chords are concurrent is called the apex. The shell is also called fan f_{n-1}.*

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Definition 1.2. *A multiple shell is a collection of edge disjoint shells that have their apex in common. Hence a double shell consists of two edge disjoint shells with a common apex.*

Definition 1.3. *A bow graph is a double shell in which each shell has any order.*

Definition 1.4. *A butterfly graph is a bow graph with exactly two pendant edges at the apex.*

2 Main results

Theorem 2.1. *The shell graph S_n is an edge pair sum graph.*

Proof. Let G be a shell graph.
Define $V(G) = \{u, v_i : 1 \leq i \leq (n-1)\}$ and
$E(G) = \{e_i^{'} = uv_i : 1 \leq i \leq (n-1); e_i = v_iv_{i+1} : 1 \leq i \leq (n-2)\}$ are the vertices and edges of the graph G.
Define an edge pair sum labeling $f : E(G) \to \{\pm1, \pm2, \pm3, ..., \pm(2n-3)\}$ by considering two cases.
Case(i). n is even and $n \geq 6$.
Define $f(e_{\frac{n-2}{2}}) = 2$, $f(e_{\frac{n}{2}}) = 1$,
for $1 \leq i \leq \frac{n-4}{2}$ $f(e_i) = -(n+1-2i)$, $f(e_{\frac{n}{2}+i}) = (3+2i)$,
$f(e_i^{'}) = -(2n-3-2i)$ and $f(e^{'}_{\frac{n+2}{2}+i}) = (n-1+2i)$,
$f(e^{'}_{\frac{n-2}{2}}) = -4 = -f(e^{'}_{\frac{n}{2}})$ and $f(e_{\frac{n+2}{2}}) = -3$.
For each edge label f, the induced vertex label f^* is calculated as follows:
$f^*(u) = -3 = -f^*(v_{\frac{n+2}{2}})$, $f^*(v_{\frac{n-2}{2}}) = -7 = -f^*(v_{\frac{n}{2}})$, $f^*(v_1) = -(3n-6) = -f^*(v_{n-1})$,
for $1 \leq i \leq (\frac{n-6}{2})$ $f^*(v_{1+i}) = (-4n+5+6i)$ and $f^*(v_{\frac{n+2}{2}+i}) = (n+7+6i)$.
$f^*(V(G)) = \{\pm3, \pm7, \pm(3n-6), \pm(n+13), \pm(n+19), \pm(n+25), ..., \pm(4n-11)\}$.
Hence f is an edge pair sum labeling.
The example for the edge pair sum graph labeling of S_6 is shown in Figure 1.

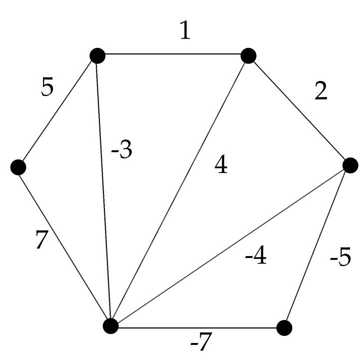


Figure 1

Case(ii). n is odd and $n \geq 6$.
Subcase (a). $n = 3$.
If $n = 3$ the shell graph becomes C_3 and proved that C_3 admit edge pair sum labeling [3].
Subcase (b). $n = 5$.
Define $f(e_1) = -2$, $f(e_2) = -1$, $f(e_3) = 3$, $f(e_1^{'}) = -7 = -f(e_3^{'})$, $f(e_2^{'}) = 4$ and $f(e_4^{'}) = -5$.
For each edge label f, the induced vertex label f^* is calculated as follows:
$f^*(u) = -1 = -f^*(v_2)$, $f^*(v_1) = -9 = -f^*(v_3)$ and $f^*(v_4) = -2$.
Then $f^*(V(G)) = \{\pm1, \pm9\} \bigcup \{-2\}$. Hence f is an edge pair sum labeling.
Subcase (c). $n \geq 7$.
Define $f(e_{\frac{n-1}{2}}) = -1$, $f(e_{\frac{n-3}{2}}) = -2$, $f(e_{\frac{n+1}{2}}) = 3$,
for $1 \leq i \leq \frac{n-5}{2}$ $f(e_i) = (n-2i)$, $f(e_{\frac{n+1}{2}+i}) = -(3+2i)$, $f(e_i^{'}) = (2n-3-2i)$ and $f(e^{'}_{\frac{n+3}{2}+i}) = -(n+2i)$,
$f(e^{'}_{\frac{n-1}{2}}) = -6 = -f(e^{'}_{\frac{n-3}{2}})$, $f(e^{'}_{\frac{n+1}{2}}) = 4$ and $f(e^{'}_{\frac{n+3}{2}}) = -10$.

For each edge label f, the induced vertex label f^* is calculated as follows:
$f^*(u) = -6 = -f^*(v_{\frac{n+1}{2}}), f^*(v_{\frac{n+3}{2}}) = -12, f^*(v_{\frac{n-3}{2}}) = 9 = -f^*(v_{\frac{n-1}{2}}), f^*(v_1) = (3n-7) = -f^*(v_{n-1}),$
for $1 \leq i \leq \frac{n-7}{2}$ $f^*(v_{1+i}) = (4n-7-6i)$ and $f^*(v_{\frac{n+3}{2}+i}) = -(n+8+6i)$. From the above vertex labeling we get $f^*(V(G)) = \{\pm 6, \pm 9, \pm(3n-7), \pm(n+14), \pm(n+20), \pm(n+26), ..., \pm(4n-13)\} \bigcup \{-12\}$.
Hence f is an edge pair sum labeling.
The examples for the edge pair sum graph labeling of S_5 and S_7 are shown in Figures 2 and 3 respectively.

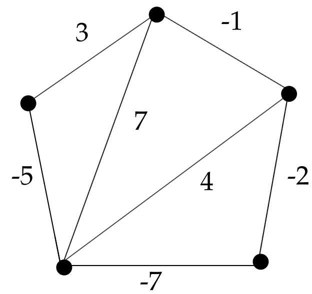


Figure 2

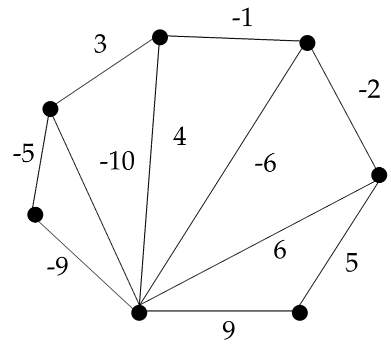


Figure 3

□

Theorem 2.2. *The butterfly graph with shell order m and m (order excludes the apex) is an edge pair sum graph.*

Proof. Let G be a butterfly graph with shells of order m and m excluding the apex.
Define $V(G) = \{w_0, w_1, w_2, v_i, u_i : 1 \leq i \leq m\}$ and
$E(G) = \{e_1' = w_0w_1, e_2' = w_0w_2, e_i = w_0u_i$ and $e_{2m-1+i} = w_0v_i : 1 \leq i \leq m, e_{m+i} = u_iu_{i+1}$
and $e_{3m-1+i} = v_iv_{i+1} : 1 \leq i \leq (m-1)\}$ are the vertices and edges of the graph G.
Define an edge labeling $f : E(G) \rightarrow \{\pm 1, \pm 2, \pm 3, ..., \pm 4m\}$.
Define $f(e_1') = 1, f(e_2') = -2$,
for $1 \leq i \leq m$ $f(e_i) = (2+2i) = -f(e_{2m-1+i})$ and
for $1 \leq i \leq (m-1)$ $f(e_{m+i}) = (2i+1) = -f(e_{3m-1+i})$.
For each edge label f, the induced vertex label f^* is calculated as follows:
$f^*(w_0) = -1 = -f^*(w_1), f^*(w_2) = -2, f^*(u_1) = 7 = -f^*(v_1)$,
for $1 \leq i \leq (m-2)$ $f^*(u_{1+i}) = (8+6i) = -f^*(v_{1+i})$ and
$f^*(u_m) = (4m+1) = -f^*(v_m)$.
Then we get $f^*(V(G)) = \{\pm 1, \pm 7, \pm(4m+1), \pm 14, \pm 20, \pm 26, ..., \pm(6m-4)\} \bigcup \{-2\}$.
Hence f is an edge pair sum labeling.
The example for the edge pair sum graph labeling of graph G with $m = 5$ is shown in Figure 4.

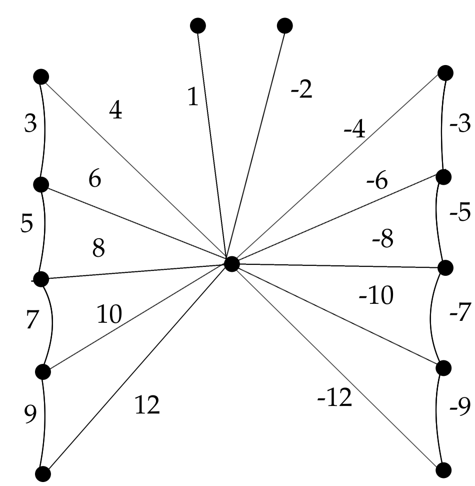


Figure 4

□

Theorem 2.3. *The butterfly graph with shell order m and 2m (order excludes the apex) is an edge pair sum graph for m is even.*

Proof. Let G be a butterfly graph with shell order m and $2m$ excluding the apex.
Define $V(G) = \{w_0, w_1, w_2, v_i : 1 \leq i \leq m, u_i : 1 \leq i \leq 2m\}$ and
$E(G) = \{e_1^{'} = w_0w_1, e_2^{'} = w_0w_2, e_i = w_0u_i : 1 \leq i \leq 2m, e_{2m+i} = u_iu_{1+i} : 1 \leq i \leq (2m-1)$,
$e_{4m-1+i} = w_0v_i : 1 \leq i \leq m, e_{5m-1+i} = v_iv_{i+1} : 1 \leq i \leq (m-1)\}$ are the vertices and edges of the graph G.
Define an edge labeling $f : E(G) \rightarrow \{\pm1, \pm2, \pm3, ..., \pm6m\}$ by considering two cases.
Case(i). m = 2.
Define $f(e_1) = 3 = -f(e_4)$, $f(e_2) = 5 = -f(e_3)$, $f(e_5) = 2 = -f(e_7)$, $f(e_6) = 1$,
$f(e_8) = 7 = -f(e_9)$, $f(e_{10}) = 9$, $f(e_1^{'}) = -8$ and $f(e_2^{'}) = 6$.
For each edge label f, the induced vertex label f^* is calculated as follows:
$f^*(w_1) = -8 = -f^*(u_2)$, $f^*(w_2) = 6 = -f^*(u_3)$, $f^*(w_0) = -2 = -f^*(v_2)$,
$f^*(u_1) = 5 = -f^*(u_4)$ and $f^*(v_1) = 16$.
Therefore we get $f^*(V(G)) = \{\pm2, \pm5, \pm6, \pm8\} \bigcup \{16\}$. Hence f is an edge pair sum labeling.
Case(ii). $m \geq 4$.
Define $f(e_1^{'}) = -(5m-2)$, $f(e_2^{'}) = (5m-4)$, $f(e_{3m}) = 1$,
for $1 \leq i \leq m$ $f(e_i) = (2i+1)$ and $f(e_{m+i}) = -(2m+3-2i)$,
for $1 \leq i \leq (m-1)$ $f(e_{2m+i}) = (m-2+2i)$ and $f(e_{3m+i}) = -(3m-2-2i)$,
for $1 \leq i \leq \frac{m}{2}$ $f(e_{4m-1+i}) = (2m+1+2i)$ and $f(e_{\frac{9m}{2}-1+i}) = -(3m+3-2i)$,
for $1 \leq i \leq \frac{m}{2} - 1$ $f(e_{5m-1+i}) = 2i$ and $f(e_{\frac{11m}{2}-1+i}) = -(m-2i)$ and $f(e_{\frac{11m}{2}-1}) = (4m+1)$.
For each edge label f, the induced vertex label f^* is calculated as follows:
$f^*(w_1) = -(5m-2) = -f^*(u_m)$, $f^*(w_2) = (5m-4) = -f^*(u_{m+1})$, $f^*(w_0) = -2 = -f^*(v_{\frac{m}{2}+1})$,
$f^*(u_1) = (m+3) = -f^*(u_{2m})$,
for $1 \leq i \leq (m-2)$ $f^*(u_{1+i}) = (2m+1+6i)$ and $f^*(u_{m+1+i}) = -(8m-5-6i)$,
for $1 \leq i \leq \frac{m-4}{2}$ $f^*(v_{1+i}) = (2m+5+6i)$ and $f^*(v_{\frac{m}{2}+1+i}) = -(5m-1-6i)$,
$f^*(v_1) = (2m+5) = -f^*(v_m)$ and $f^*(v_{\frac{m}{2}}) = 8m$.
Then $f^*(V(G)) = \{\pm2, \pm(2m+5), \pm(5m-2), \pm(5m-4), \pm(m+3), \pm(2m+7), \pm(2m+13), \pm(2m+19), ..., \pm(8m-11), \pm(2m+11), \pm(2m+17), \pm(2m+23), ..., \pm(5m-7)\} \bigcup \{8m\}$.
Hence f is an edge pair sum labeling.
The example for the edge pair sum graph labeling of graph G with $m = 4$ is shown in Figure 5.

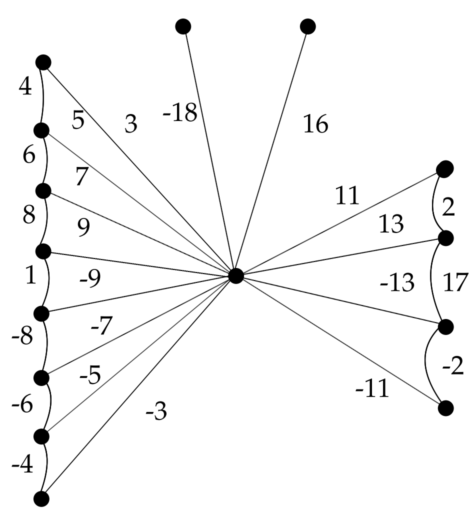
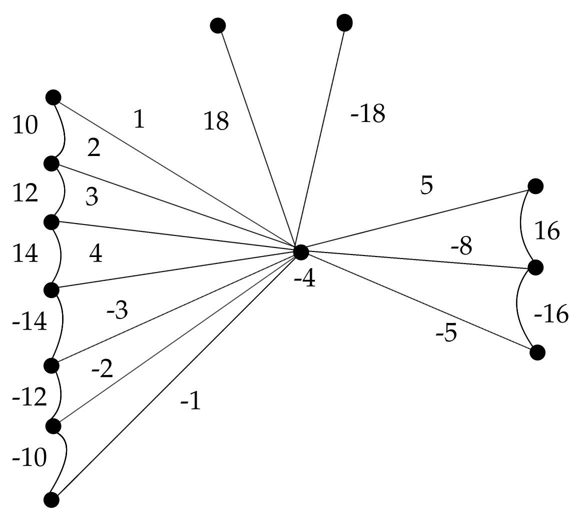


Figure 5

□

Theorem 2.4. *The butterfly graph with shell order m and (2m+1) (order excludes the apex) is an edge pair sum graph for m is odd.*

Proof. Let G be a butterfly graph with shells of order m and $2m+1$ excluding the apex.
Define $V(G) = \{w_0, w_1, w_2, v_i : 1 \leq i \leq m, u_i : 1 \leq i \leq (2m+1)\}$ and
$E(G) = \{e_1^{'} = w_0w_1, e_2^{'} = w_0w_2, e_i = w_0u_i : 1 \leq i \leq (2m+1), e_{2m+1+i} = u_iu_{1+i} : 1 \leq i \leq 2m,$
$e_{4m+1+i} = w_0v_i : 1 \leq i \leq m, e_{5m+1+i} = v_iv_{i+1} : 1 \leq i \leq (m-1)\}$ are the vertices and edges of the graph G.
Define an edge labeling $f : E(G) \to \{\pm1, \pm2, \pm3, ..., \pm(6m+2)\}$.
Define $f(e_1^{'}) = (5m+3) = -f(e_2^{'})$,
for $1 \leq i \leq (m+1)$ $f(e_i) = i$,
for $1 \leq i \leq m$ $f(e_{m+1+i}) = -(m+1-i)$, $f(e_{2m+1+i}) = 2(m+1+i)$ and $f(e_{3m+1+i}) = -(4m+4-2i)$,
for $1 \leq i \leq \frac{m-1}{2}$ $f(e_{4m+1+i}) = (m+1+i)$ $f(e_{\frac{9m+3}{2}+i}) = -(\frac{3m+3}{2} - i)$,
$f(e_{5m+1+i}) = 2(2m+1+i)$ and $f(e_{\frac{11m+1}{2}+i}) = -(5m+3-2i)$ and
$f(e_{\frac{9m+3}{2}}) = -2(m+1)$.
For each edge label f, the induced vertex label f^* is calculated as follows:
$f^*(w_1) = (5m+3) = -f^*(w_2), f^*(w_0) = -(m+1) = -f^*(u_{m+1}), f^*(u_1) = (2m+5) = -f^*(u_{2m+1}),$
for $1 \leq i \leq (m-1)$ $f^*(u_{1+i}) = (4m+7+5i)$ and $f^*(u_{m+1+i}) = -(9m+7-5i)$,
for $1 \leq i \leq \frac{m-3}{2}$ $f^*(v_{1+i}) = (9m+8+5i)$ and $f^*(v_{\frac{m+1}{2}+i}) = -(\frac{23m+11}{2} - 5i)$,
$f^*(v_1) = (5m+6) = -f^*(v_m)$ and $f^*(v_{\frac{m+1}{2}}) = -2(m+1)$.
Then $f^*(V(G)) = \{\pm(5m+3), \pm(m+1), \pm(2m+5), \pm(5m+6), \pm(4m+12), \pm(4m+17), \pm(4m+22), ..., \pm(9m+2), \pm(9m+13), \pm(9m+18), \pm(9m+23), ..., \pm(\frac{23m+1}{2})\} \bigcup \{-2(m+1)\}$.
Hence f is an edge pair sum labeling.
The example for the edge pair sum graph labeling of graph G with $m = 3$ is shown in Figure 6.



□

Figure 6

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