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The General Solution and Stability of Nonadecic Functional Equation in Matrix Normed Spaces

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Abstract

In this paper, we present the general solution of a nonadecic functional equation and establish the Ulam-Hyers stability of nonadecic functional equation in matrix normed spaces by using the fixed point method.

Keywords: Hyers-Ulam stability, fixed point, nonadecic functional equation, matrix normed spaces.

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1 Introduction

In 1940, an interesting talk presented by S. M. Ulam [27] triggered the study of stability problems for various functional equations. He raised a question concerning the stability of homomorphism. In the following year, 1941, D. H. Hyers [5] was able to give a partial solution to Ulam's question. The result of Hyers was generalized by Aoki [1] for additive mappings. In 1978, Th. M. Rassias [14] succeeded in extending the result of Hyers theorem by weakening the condition for the Cauchy difference.

The stability phenomenon that was presented by Th. M. Rassias is called the generalized Hyers-Ulam stability. This concept actually means that if one is studying a Hyers-Ulam stable system, one need not have to reach the exact solution, which usually is quite difficult or time consuming. This is quite useful in many applications for example optimization, numerical analysis, biology, life sciences, economics etc., where finding the exact solution is quite difficult.

From 1982-1994, J. M. Rassias (see [16]- [23]) solved the Ulam problem for different mappings and for many Euler-Lagrange type quadratic mappings, by involving a product of different powers of norms. In 1994, a generalization of the Rassias theorem was obtained by Gavruta [4] by replacing the unbounded Cauchy difference by a general control function. A further generalization of the Hyers-Ulam stability for a large class of mapping was obtained by Isac and Th. M. Rassias [6]. They also presented some applications in non-linear analysis, especially in fixed point theory. This terminology may also be applied to the cases of other functional equations [2, 3, 13, 15, 26, 29]. Also, the generalized Hyers-Ulam stability of functional equations and inequalities in matrix normed spaces has been studied by number of authors [7–10, 12, 28].

K. Ravi and B. V. Senthil Kumar [24] discussed the general solution of undecic functional equation

$$\begin{aligned}
&f(x+6y)-11f(x+5y)+55f(x+4y)-165f(x+3y)+330f(x+2y)\\
&\quad -462f(x+y)-462f(x)-330f(x-y)+165f(x-2y)\\
&\qquad -55f(x-3y)+11f(x-4y)-f(x-5y)=39916800f(y)
\end{aligned}$$

and proved the stability of this functional equation in quasi β - normed spaces by applying the fixed point method.

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Very recently, K. Ravi et. al., [25] discussed the general solution of quattuordecic functional equation

$$f(x+7y)-14f(x+6y)+91f(x+5y)-364f(x+4y)+1001f(x+3y)-2002f(x+2y)$$
$$-3003f(x+y)-3432f(x)+3003f(x-y)-2002f(x-2y)+1001f(x-3y)$$
$$-364f(x-4y)+91f(x-5y)-14f(x-6y)+f(x-7y)=87178291200f(y)$$

and its stability in quasi β - normed spaces.

In this paper, we introduce the following new functional equation

$$f(x+10y)-19f(x+9y)+171f(x+8y)-969f(x+7y)+3876f(x+6y)-11628f(x+5y)$$
$$+27132f(x+4y)-50388f(x+3y)+75582f(x+2y)-92378f(x+y)$$
$$+92378f(x)-75582f(x-y)+50388f(x-2y)-27132f(x-3y)$$
$$+11628f(x-4y)-3876f(x-5y)+969f(x-6y)-171f(x-7y)$$
$$+19f(x-8y)-f(x-9y)=19!f(y) \tag{1.1}$$

where $19! = 121645100400000000$, is said to be nonadecic functional equation since the function $f(x) = cx^{19}$ is its solution. In this paper, we determine the general solution of the functional equation (1.1) and we also prove the Ulam-Hyers stability of the functional equation (1.1) in matrix normed spaces by using fixed point approach.

2 General Solution of Nonadecic Functional Equation (1.1)

In this section, we present the general solution of nonadecic functional equation (1.1). For this, let us consider $\mathcal{A}$ and $\mathcal{B}$ be real vector spaces.

Theorem 2.1. *If $f : \mathcal{A} \to \mathcal{B}$ be a mapping satisfying (1.1) for all $x, y \in \mathcal{A}$, then f is nonadecic.*

Proof. Letting $x = y = 0$ in (1.1), one gets $f(0) = 0$. Replacing $x = 0, y = x$ and $x = x, y = -x$ in (1.1) and adding the two resulting equations, we get

$$f(-x) = -f(x)$$

Hence, f is an odd mapping. Replacing $x = 0, y = 2x$ and $x = 10x, y = x$ in (1.1) and subtracting the two resulting equations, we get

$$19f(19x)-189f(18x)+969f(17x)-3724f(16x)+11628f(15x)-27930f(14x)$$
$$+50388f(13x)-72675f(12x)+92378f(11x)-100130f(10x)$$
$$+75582f(9x)-34884f(8x)+27132f(7x)-34884f(6x)+3876f(5x)$$
$$+24225f(4x)+171f(3x)-(16815+19!)f(2x)+19!f(x)=0 \tag{2.2}$$

for all $x \in \mathcal{A}$. Replacing (x, y) by $(9x, x)$ in (1.1), we obtain that

$$f(19x)-19f(18x)+171f(17x)-969f(16x)+3876f(15x)-11628f(14x)$$
$$+27132f(13x)-50388f(12x)+75582f(11x)-92378f(10x)$$
$$+92378f(9x)-75582f(8x)+50388f(7x)-27132f(6x)+11628f(5x)$$
$$-3876f(4x)+969f(3x)-171f(2x)+(19-19!)f(x)=0 \tag{2.3}$$

for all $x \in \mathcal{A}$. Multiplying (2.3) by 19, and then subtracting (2.2) from the resulting equation, we get

$$172f(18x)-2280f(17x)+14687f(16x)-62016f(15x)+193002f(14x)-18240f(3x)$$
$$-465120f(13x)+884697f(12x)-1343680f(11x)+1655052f(10x)$$
$$-1679600f(9x)+1401174f(8x)-930240f(7x)+480624f(6x)$$
$$-217056f(5x)+97869f(4x)+(13566-19!)f(2x)+20(19!)f(x)=0 \tag{2.4}$$

for all $x \in \mathcal{A}$. Replacing (x, y) by $(8x, x)$ in (1.1), we have

$$f(18x)-19f(17x)+171f(16x)-969f(15x)+3876f(14x)-11628f(13x)$$
$$+27132f(12x)-50388f(11x)+75582f(10x)-92378f(9x)$$
$$+92378f(8x)-75582f(7x)+50388f(6x)-27132f(5x)+11628f(4x)$$
$$-3876f(3x)+969f(2x)-(170+19!)f(x)=0 \tag{2.5}$$

for all $x \in \mathcal{A}$. Multiplying (2.5) by 172, and then subtracting (2.4) from the resulting equation , we get

$$\begin{aligned}&988f(17x) - 14725f(16x) + 104652f(15x) - 473670f(14x) + 1534896f(13x)\\&\quad +7323056f(11x) - 11345052f(10x) + 14209416f(9x) - 14487842f(8x)\\&\quad +12069864f(7x) - 8186112f(6x) + 4449648f(5x) - 1902147f(4x)\\&\quad + 648432f(3x) - 3782007f(12x) - (180234 + 19!)f(2x) + 192(19!)f(x) = 0 \end{aligned} \tag{2.6}$$

for all $x \in \mathcal{A}$. Replacing (x, y) by $(7x, x)$ in (1.1), it follows that

$$\begin{aligned}&f(17x) - 19f(16x) + 171f(15x) - 969f(14x) + 3876f(13x) - 11628f(12x)\\&\quad +27132f(11x) - 50388f(10x) + 75582f(9x) - 92378f(8x)\\&\quad +92378f(7x) - 75582f(6x) + 50388f(5x) - 27132f(4x) + 11628f(3x)\\&\quad - 3875f(2x) + (950 - 19!)f(x) = 0 \end{aligned} \tag{2.7}$$

for all $x \in \mathcal{A}$. Multiplying (2.7) by 988, and then subtracting (2.6) from the resulting equation, we get

$$\begin{aligned}&4047f(16x) - 64296f(15x) + 483702f(14x) - 2294592f(13x) + 7706457f(12x)\\&\quad +38438292f(10x) - 60465600f(9x) + 76781622f(8x) - 79199600f(7x)\\&\quad +66488904f(6x) - 45333696f(5x) + 24904269f(4x) - 10840032f(3x)\\&\quad - 19483360f(11x) + (3648266 - 19!)f(2x) + 1180(19!)f(x) = 0 \end{aligned} \tag{2.8}$$

for all $x \in \mathcal{A}$. Replacing (x, y) by $(6x, x)$ in (1.1), we have

$$\begin{aligned}&f(16x) - 19f(15x) + 171f(14x) - 969f(13x) + 3876f(12x) - 11628f(11x)\\&\quad +27132f(10x) - 50388f(9x) + 75582f(8x) - 92378f(7x) + 92378f(6x)\\&\quad - 75582f(5x) + 50388f(4x) - 27131f(3x) + 11609f(2x) - (3705 + 19!)f(x) = 0 \end{aligned} \tag{2.9}$$

for all $x \in \mathcal{A}$. Multiplying (2.9) by 4047, and then subtracting (2.8) from the resulting equation, we arrive at

$$\begin{aligned}&12597f(15x) - 208335f(14x) + 1626951f(13x) - 7979715f(12x) + 27575156f(11x)\\&\quad +143454636f(9x) - 229098732f(8x) + 294654166f(7x) - 307364862f(6x)\\&\quad +260546658f(5x) - 179015967f(4x) + 98959125f(3x)\\&\quad - 71364912f(10x) - (43333357 + 19!)f(2x) + 5227(19!)f(x) = 0 \end{aligned} \tag{2.10}$$

for all $x \in \mathcal{A}$. Replacing (x, y) by $(5x, x)$ in (1.1), we obtain

$$\begin{aligned}&f(15x) - 19f(14x) + 171f(13x) - 969f(12x) + 3876f(11x) - 11628f(10x)\\&\quad +27132f(9x) - 50388f(8x) + 75582f(7x) - 92378f(6x) + 92378f(5x)\\&\quad - 75581f(4x) + 50369f(3x) - 26961f(2x) + (10659 - 19!)f(x) = 0 \end{aligned} \tag{2.11}$$

for all $x \in \mathcal{A}$. Multiplying (2.11) by 12597, and then subtracting (2.10) from the resulting equation, we arrive at

$$\begin{aligned}&31008f(14x) - 527136f(13x) + 4226778f(12x) - 21250816f(11x)\\&\quad +75113004f(10x) - 198327168f(9x) + 405638904f(8x) - 657452288f(7x)\\&\quad +856320804f(6x) - 903139008f(5x) + 773077890f(4x) - 535539168f(3x)\\&\quad + (296294360 - 19!)f(2x) + 17824(19!)f(x) = 0 \end{aligned} \tag{2.12}$$

for all $x \in \mathcal{A}$. Replacing (x, y) by $(4x, x)$ in (1.1), we get

$$\begin{aligned}&f(14x) - 19f(13x) + 171f(12x) - 969f(11x) + 3876f(10x) - 11628f(9x)\\&\quad +27132f(8x) - 50388f(7x) + 75582f(6x) - 92377f(5x)\\&\quad + 92359f(4x) - 75411f(3x) + 49419f(2x) - (23256 + 19!)f(x) = 0 \end{aligned} \tag{2.13}$$

for all $x \in \mathcal{A}$. Multiplying (2.13) by 31008, and then subtracting (2.12) from the resulting equation, we obtain

$$\begin{aligned}&62016f(13x) - 1075590f(12x) + 8795936f(11x) - 45074004f(10x)\\&\quad +162233856f(9x) - 435670152f(8x) + 904978816f(7x) + 1802805120f(3x)\\&\quad -1487325852f(6x) + 1961287008f(5x) - 2090789982f(4x) + 48832(19!)f(x)\\&\quad - (1236089992 + 19!)f(2x) = 0 \end{aligned} \tag{2.14}$$

for all $x \in \mathcal{A}$. Replacing (x, y) by $(3x, x)$ in (1.1), we have

$$\begin{aligned} &f(13x) - 19f(12x) + 171f(11x) - 969f(10x) + 3876f(9x) - 11628f(8x) \\ &\quad +27132f(7x) - 50387f(6x) + 75563f(5x) - 92207f(4x) \\ &\quad + 91409f(3x) - 71706f(2x) + (38760 - 19!)f(x) = 0 \end{aligned} \tag{2.15}$$

for all $x \in \mathcal{A}$. Multiplying (2.15) by 62016, and then subtracting (2.14) from the resulting equation, we obtain

$$\begin{aligned} &102714f(12x) - 1808800f(11x) + 15019500f(10x) - 78140160f(9x) + 285451896f(8x) \\ &\quad -777639296f(7x) + 1637474340f(6x) - 2724828000f(5x) + 3627519330f(4x) \\ &\quad - 3866015424f(3x) + (3210829304 - 19!)f(2x) + 110848(19!)f(x) = 0 \end{aligned} \tag{2.16}$$

for all $x \in \mathcal{A}$. Replacing (x, y) by $(2x, x)$ in (1.1), it follows that

$$\begin{aligned} &f(12x) - 19f(11x) + 171f(10x) - 969f(9x) + 3876f(8x) - 11627f(7x) + 27113f(6x) \\ &\quad - 50217f(5x) + 74613f(4x) - 88502f(3x) + 80750f(2x) - (48450 - 19!)f(x) = 0 \end{aligned} \tag{2.17}$$

for all $x \in \mathcal{A}$. Multiplying (2.17) by 102714, and then subtracting (2.16) from the resulting equation, we obtain

$$\begin{aligned} &142766f(11x) - 2544594f(10x) + 21389706f(9x) - 112667568f(8x) + 416616382f(7x) \\ &\quad -1147410342f(6x) + 2433160938f(5x) - 4036280352f(4x) + 5224379004f(3x) \\ &\quad - (5083326196 + 19!)f(2x) + 213562(19!)f(x) = 0 \end{aligned} \tag{2.18}$$

for all $x \in \mathcal{A}$. Replacing (x, y) by (x, x) in (1.1), we get

$$\begin{aligned} &f(11x) - 19f(10x) + 171f(9x) - 968f(8x) + 3857f(7x) - 11457f(6x) + 26163f(5x) \\ &\quad - 46512f(4x) + 63954f(3x) - 65246f(2x) + (41990 - 19!)f(x) = 0 \end{aligned} \tag{2.19}$$

for all $x \in \mathcal{A}$. Multiplying (2.19) by 142766, and then subtracting (2.18) from the resulting equation, we obtain

$$\begin{aligned} &167960f(10x) - 3023280f(9x) + 25529920f(8x) - 134032080f(7x) \\ &\quad +488259720f(6x) - 1302025920f(5x) + 2604051840f(4x) \\ &\quad - 3906077760f(3x) + (4231584240 - 19!)f(2x) + 356328(19!)f(x) = 0 \end{aligned} \tag{2.20}$$

for all $x \in \mathcal{A}$. Replacing (x, y) by $(0, x)$ in (1.1), we obtain that

$$\begin{aligned} &f(10x) - 18f(9x) + 152f(8x) - 798f(7x) + 2907f(6x) - 7752f(5x) \\ &\quad + 15504f(4x) - 23256f(3x) + 25194f(2x) - (16796 + 19!)f(x) = 0 \end{aligned} \tag{2.21}$$

for all $x \in \mathcal{A}$. Multiplying (2.21) by 167960, and then subtracting (2.20) from the resulting equation, we can obtain that

$$f(2x) = 2^{19}f(x)$$

for all $x \in \mathcal{A}$. Hence $f : \mathcal{A} \to \mathcal{B}$ is a nonadecic mapping. This completes the proof. □

3 Ulam-Hyers Stability of Nonadecic Functional Equation (1.1)

In this section, We will investigate the Ulam-Hyers stability for the functional equation (1.1) in matrix normed spaces by using the fixed point method.

Throughout this section, let us consider $(X, \|.\|_n)$ be a matrix normed space, $(Y, \|.\|_n)$ be a matrix Banach space and let n be a fixed non-negative integer.

For a mapping $f : X \to Y$, define $\mathcal{G}f : X^2 \to Y$ and $\mathcal{G}f_n : M_n(X^2) \to M_n(Y)$ by,

$$\begin{aligned} \mathcal{G}f(a, b) = {} & f(a + 10b) - 19f(a + 9b) + 171f(a + 8b) - 969f(a + 7b) + 3876f(a + 6b) \\ & - 11628f(a + 5b) + 27132f(a + 4b) - 50388f(a + 3b) - f(a - 9b) \\ & + 75582f(a + 2b) - 92378f(a + b) + 92378f(a) - 75582f(a - b) \\ & + 50388f(a - 2b) - 27132f(a - 3b) + 11628f(a - 4b) - 19!f(b) \\ & - 3876f(a - 5b) + 969f(a - 6b) - 171f(a - 7b) + 19f(a - 8b), \end{aligned}$$

$$\begin{aligned}\mathcal{G}f_n(x_{ij},y_{ij}) = {} & f_n(x_{ij}+10y_{ij}) - 19f_n(x_{ij}+9y_{ij}) + 171f_n(x_{ij}+8y_{ij}) - 969f_n(x_{ij}+7y_{ij})\\ & + 3876f_n(x_{ij}+6y_{ij}) - 11628f_n(x_{ij}+5y_{ij}) + 27132f_n(x_{ij}+4y_{ij})\\ & - 50388f_n(x_{ij}+3y_{ij}) + 75582f_n(x_{ij}+2y_{ij}) - 92378f_n(x_{ij}+y_{ij})\\ & + 92378f_n(x_{ij}) - 75582f_n(x_{ij}-y_{ij}) + 50388f_n(x_{ij}-2y_{ij})\\ & - 27132f_n(x_{ij}-3y_{ij}) + 11628f_n(x_{ij}-4y_{ij}) - 3876f_n(x_{ij}-5y_{ij})\\ & + 969f_n(x_{ij}-6y_{ij}) - 171f_n(x_{ij}-7y_{ij}) + 19f_n(x_{ij}-8y_{ij})\\ & - f_n(x_{ij}-9y_{ij}) - 19!f_n(y_{ij})\end{aligned}$$

for all $a,b \in X$ and all $x = [x_{ij}], y = [y_{ij}] \in M_n(X)$.

Theorem 3.2. *Let $l = \pm 1$ be fixed and $\psi : X^2 \to [0,\infty)$ be a function such that there exists a $\eta < 19$ with*

$$\psi(a,b) \le 2^{19l}\eta\psi(\frac{a}{2^l},\frac{b}{2^l}) \qquad \forall\, a,b \in X. \tag{3.22}$$

Let $f : X \to Y$ be a mapping satisfying

$$\left\|\mathcal{G}f_n([x_{ij}],[y_{ij}])\right\| \le \sum_{i,j=1}^{n} \psi(x_{ij},y_{ij}) \qquad \forall\, x = [x_{ij}], y = [y_{ij}] \in M_n(X). \tag{3.23}$$

Then there exists a unique nonadecic mapping $\mathcal{N}_{\mathcal{D}} : X \to Y$ such that

$$\left\|f_n([x_{ij}]) - \mathcal{N}_{\mathcal{D}n}([y_{ij}])\right\|_n \le \sum_{i,j=1}^{n} \frac{\eta^{\frac{1-l}{2}}}{2^{19}(1-\eta)}\overline{\psi}(x_{ij}) \qquad \forall\, x = [x_{ij}] \in M_n(X), \tag{3.24}$$

where $$\begin{aligned}\overline{\psi}(x_{ij}) = \frac{1}{19!}[&\psi(0,2x_{ij}) + \psi(10x_{ij},x_{ij}) + 19\psi(9x_{ij},x_{ij}) + 172\psi(8x_{ij},x_{ij})\\ &+988\psi(7x_{ij},x_{ij}) + 4047\psi(6x_{ij},x_{ij}) + 12597\psi(5x_{ij},x_{ij})\\ &+31008\psi(4x_{ij},x_{ij}) + 62016\psi(3x_{ij},x_{ij}) + 102714\psi(2x_{ij},x_{ij})\\ &+142766\psi(x_{ij},x_{ij}) + 167960\psi(0,x_{ij})]\end{aligned}$$

Proof. Substituting $n = 1$ in (3.23), we obtain

$$\|\mathcal{G}f(a,b)\| \le \psi(a,b) \tag{3.25}$$

Replacing (a,b) by $(0,2a)$ in (3.25), we get

$$\begin{aligned}&\|f(20a) - 18f(18a) + 152f(16a) - 798f(14a) + 2907f(12a) - 7752f(10a)\\ &+15504f(8a) - 23256f(6a) + 25194f(4a) - (16796+19!)f(2a)\| \le \psi(0,2a)\end{aligned} \tag{3.26}$$

for all $a \in X$. Replacing (a,b) by $(10a,a)$ in (3.25), we obtain

$$\begin{aligned}&\|f(20a) - 19f(19a) + 171f(18a) - 969f(17a) + 3876f(16a) - 11628f(15a)\\ &+27132f(14a) - 50388f(13a) + 75582f(12a) - 92378f(11a)\\ &+92378f(10a) - 75582f(9a) + 50388f(8a) - 27132f(7a) + 11628f(6a)\\ &-3876f(5a) + 969f(4a) - 171f(3a) + 19f(2a) - (1+19!)f(a)\| \le \psi(10a,a)\end{aligned} \tag{3.27}$$

for all $a \in X$. Combining (3.26) and (3.27), we arrive at

$$\begin{aligned}&\|19f(19a) - 189f(18a) + 969f(17a) - 3724f(16a) + 11628f(15a) - 27930f(14a)\\ &+50388f(13a) - 72675f(12a) + 92378f(11a) - 100130f(10a)\\ &+75582f(9a) - 34884f(8a) + 27132f(7a) - 34884f(6a) + 3876f(5a)\\ &+24225f(4a) + 171f(3a) - (16815+19!)f(2a) + 19!f(a)\| \le \psi(0,2a) + \psi(10a,a)\end{aligned} \tag{3.28}$$

for all $a \in X$. Replacing (a,b) by $(9a,a)$ in (3.25), we obtain

$$\begin{aligned}&\|f(19a) - 19f(18a) + 171f(17a) - 969f(16a) + 3876f(15a) - 11628f(14a)\\ &+27132f(13a) - 50388f(12a) + 75582f(11a) - 92378f(10a)\\ &+92378f(9a) - 75582f(8a) + 50388f(7a) - 27132f(6a) + 11628f(5a)\\ &-3876f(4a) + 969f(3a) - 171f(2a) + (19-19!)f(a)\| \le \psi(9a,a)\end{aligned} \tag{3.29}$$

$\forall\ a \in X$. Multiplying (3.29) by 19, and combining the resulting inequality with (3.28), we get

$$\begin{aligned}\|172f(18a) - 2280f(17a) + 14687f(16a) - 62016f(15a) + 193002f(14a)\\ -465120f(13a) + 884697f(12a) - 1343680f(11a) + 1655052f(10a)\\ -1679600f(9a) + 1401174f(8a) - 930240f(7a) + 480624f(6a)\\ -217056f(5a) + 97869f(4a) - 18240f(3a) + (13566 - 19!)f(2a)\\ +20(19!)f(a)\| \leq \psi(0,2a) + \psi(10a,a) + 19\psi(9a,a)\end{aligned} \tag{3.30}$$

for all $a \in X$. Replacing (a,b) by $(8a,a)$ in (3.25), we obtain

$$\begin{aligned}\|f(18a) - 19f(17a) + 171f(16a) - 969f(15a) + 3876f(14a) - 11628f(13a)\\ +27132f(12a) - 50388f(11a) + 75582f(10a) - 92378f(9a)\\ +92378f(8a) - 75582f(7a) + 50388f(6a) - 27132f(5a) + 11628f(4a)\\ -3876f(3a) + 969f(2a) - (170 + 19!)f(a)\| \leq \psi(8a,a)\end{aligned} \tag{3.31}$$

$\forall\ a \in X$. Multiplying (3.31) by 172, and combining the resulting inequality with (3.30), we get

$$\begin{aligned}\|988f(17a) - 14725f(16a) + 104652f(15a) - 473670f(14a) + 1534896f(13a)\\ +7323056f(11a) - 11345052f(10a) + 14209416f(9a) - 14487842f(8a)\\ +12069864f(7a) - 8186112f(6a) + 4449648f(5a) - 1902147f(4a)\\ +648432f(3a) - 3782007f(12a) - (180234 + 19!)f(2a)\\ +192(19!)f(a)\| \leq \psi(0,2a) + \psi(10a,a) + 19\psi(9a,a) + 172\psi(8a,a)\end{aligned} \tag{3.32}$$

for all $a \in X$. Replacing (a,b) by $(7a,a)$ in (3.25), we get

$$\begin{aligned}\|f(17a) - 19f(16a) + 171f(15a) - 969f(14a) + 3876f(13a) - 11628f(12a)\\ +27132f(11a) - 50388f(10a) + 75582f(9a) - 92378f(8a)\\ +92378f(7a) - 75582f(6a) + 50388f(5a) - 27132f(4a) + 11628f(3a)\\ -3875f(2a) + (950 - 19!)f(a)\| \leq \psi(7a,a)\end{aligned} \tag{3.33}$$

$\forall\ a \in X$. Multiplying (3.33) by 988, and combining the resulting inequality with (3.32), we get

$$\begin{aligned}\|4047f(16a) - 64296f(15a) + 483702f(14a) - 2294592f(13a) + 7706457f(12a)\\ +38438292f(10a) - 60465600f(9a) + 76781622f(8a) - 79199600f(7a)\\ +66488904f(6a) - 45333696f(5a) + 24904269f(4a) - 10840032f(3a)\\ -19483360f(11a) + (3648266 - 19!)f(2a)\\ +1180(19!)f(a)\| \leq \psi(0,2a) + \psi(10a,a) + 19\psi(9a,a) + 172\psi(8a,a) + 988\psi(7a,a)\end{aligned} \tag{3.34}$$

for all $a \in X$. Replacing (a,b) by $(6a,a)$ in (3.25), we get

$$\begin{aligned}\|f(16a) - 19f(15a) + 171f(14a) - 969f(13a) + 3876f(12a) - 11628f(11a)\\ +27132f(10a) - 50388f(9a) + 75582f(8a) - 92378f(7a)\\ +92378f(6a) - 75582f(5a) + 50388f(4a) - 27131f(3a) + 11609f(2a)\\ -(3705 + 19!)f(a)\| \leq \psi(6a,a)\end{aligned} \tag{3.35}$$

$\forall\ a \in X$. Multiplying (3.35) by 4047, and combining the resulting inequality with (3.34), we arrive at

$$\begin{aligned}\|12597f(15a) - 208335f(14a) + 1626951f(13a) - 7979715f(12a) + 27575156f(11a)\\ +143454636f(9a) - 229098732f(8a) + 294654166f(7a) - 307364862f(6a)\\ +260546658f(5a) - 179015967f(4a) + 98959125f(3a)\\ -71364912f(10a) - (43333357 + 19!)f(2a) + 5227(19!)f(a)\|\\ \leq \psi(0,2a) + \psi(10a,a) + 19\psi(9a,a) + 172\psi(8a,a) + 988\psi(7a,a) + 4047\psi(6a,a)\end{aligned} \tag{3.36}$$

for all $a \in X$. Replacing (a,b) by $(5a,a)$ in (3.25), we obtain

$$\begin{aligned}\|f(15a) - 19f(14a) + 171f(13a) - 969f(12a) + 3876f(11a) - 11628f(10a)\\ +27132f(9a) - 50388f(8a) + 75582f(7a) - 92378f(6a)\\ +92378f(5a) - 75581f(4a) + 50369f(3a) - 26961f(2a)\\ +(10659 - 19!)f(a)\| \leq \psi(5a,a)\end{aligned} \tag{3.37}$$

$\forall\ a \in X$. Multiplying (3.37) by 12597, and combining the resulting inequality with (3.36), we arrive at

$$\begin{aligned}\|&31008f(14a) - 527136f(13a) + 4226778f(12a) - 21250816f(11a)\\ &+75113004f(10a) - 198327168f(9a) + 405638904f(8a) - 657452288f(7a)\\ &+856320804f(6a) - 903139008f(5a) + 773077890f(4a) - 535539168f(3a)\\ &+(296294360 - 19!)f(2a) + 17824(19!)f(a)\| \leq \psi(0,2a) + \psi(10a,a)\\ &+ 19\psi(9a,a) + 172\psi(8a,a) + 988\psi(7a,a) + 4047\psi(6a,a) + 12597\psi(5a,a) \end{aligned} \tag{3.38}$$

for all $a \in X$. Replacing (a,b) by $(4a,a)$ in (3.25), we get

$$\begin{aligned}\|&f(14a) - 19f(13a) + 171f(12a) - 969f(11a) + 3876f(10a) - 11628f(9a)\\ &+27132f(8a) - 50388f(7a) + 75582f(6a) - 92377f(5a)\\ &+92359f(4a) - 75411f(3a) + 49419f(2a) - (23256 + 19!)f(a)\| \leq \psi(4a,a) \end{aligned} \tag{3.39}$$

$\forall\ a \in X$. Multiplying (3.39) by 31008, and combining the resulting inequality with (3.38), we obtain

$$\begin{aligned}\|&62016f(13a) - 1075590f(12a) + 8795936f(11a) - 45074004f(10a)\\ &+162233856f(9a) - 435670152f(8a) + 904978816f(7a) + 1802805120f(3a)\\ &-1487325852f(6a) + 1961287008f(5a) - 2090789982f(4a) + 48832(19!)f(a)\\ &-(1236089992 + 19!)f(2a)\| \leq \psi(0,2a) + \psi(10a,a) + 19\psi(9a,a) + 172\psi(8a,a)\\ &+ 988\psi(7a,a) + 4047\psi(6a,a) + 12597\psi(5a,a) + 31008\psi(4a,a) \end{aligned} \tag{3.40}$$

for all $a \in X$. Replacing (a,b) by $(3a,a)$ in (3.25), we get

$$\begin{aligned}\|&f(13a) - 19f(12a) + 171f(11a) - 969f(10a) + 3876f(9a) - 11628f(8a)\\ &+27132f(7a) - 50387f(6a) + 75563f(5a) - 92207f(4a)\\ &+91409f(3a) - 71706f(2a) + (38760 - 19!)f(a)\| \leq \psi(3a,a) \end{aligned} \tag{3.41}$$

$\forall\ a \in X$. Multiplying (3.53) by 62016, and combining the resulting inequality with (3.40), we obtain

$$\begin{aligned}\|&102714f(12a) - 1808800f(11a) + 15019500f(10a) - 78140160f(9a)\\ &+285451896f(8a) - 2724828000f(5a) + 3627519330f(4a) - 3866015424f(3a)\\ &-777639296f(7a) + 1637474340f(6a) + (3210829304 - 19!)f(2a)\\ &+110848(19!)f(a)\| \leq \psi(0,2a) + \psi(10a,a) + 19\psi(9a,a) + 12597\psi(5a,a)\\ &+ 172\psi(8a,a) + 988\psi(7a,a) + 4047\psi(6a,a) + 31008\psi(4a,a) + 62016\psi(3a,a) \end{aligned} \tag{3.42}$$

for all $a \in X$. Replacing (a,b) by $(2a,a)$ in (3.25), we get

$$\begin{aligned}\|&f(12a) - 19f(11a) + 171f(10a) - 969f(9a) + 3876f(8a) - 11627f(7a)\\ &+27113f(6a) - 50217f(5a) + 74613f(4a) - 88502f(3a)\\ &+80750f(2a) - (48450 - 19!)f(a)\| \leq \psi(2a,a) \end{aligned} \tag{3.43}$$

for all $a \in X$. Multiplying (3.43) by 102714, and combining the resulting inequality with (3.42), we obtain

$$\begin{aligned}\|&142766f(11a) - 2544594f(10a) + 21389706f(9a) - 112667568f(8a)\\ &+416616382f(7a) - 1147410342f(6a) + 2433160938f(5a)\\ &-4036280352f(4a) + 5224379004f(3a) - (5083326196 + 19!)f(2a)\\ &+213562(19!)f(a)\| \leq \psi(0,2a) + \psi(10a,a) + 19\psi(9a,a) + 12597\psi(5a,a)\\ &+172\psi(8a,a) + 988\psi(7a,a) + 4047\psi(6a,a)\\ &+ 31008\psi(4a,a) + 62016\psi(3a,a) + 102714\psi(2a,a) \end{aligned} \tag{3.44}$$

for all $a \in X$. Replacing (a,b) by (a,a) in (3.25), we get

$$\begin{aligned}\|&f(11a) - 19f(10a) + 171f(9a) - 968f(8a) + 3857f(7a) - 11457f(6a)\\ &+26163f(5a) - 46512f(4a) + 63954f(3a) - 65246f(2a)\\ &+(41990 - 19!)f(a)\| \leq \psi(a,a) \end{aligned} \tag{3.45}$$

for all $a \in X$. Multiplying (3.45) by 142766, and combining the resulting inequality with (3.44), we get

$$\begin{aligned}&\|167960f(10a) - 3023280f(9a) + 25529920f(8a) - 134032080f(7a)\\ &\quad +488259720f(6a) - 1302025920f(5a) + 2604051840f(4a)\\ &\quad -3906077760f(3a) + (4231584240 - 19!)f(2a) + 356328(19!)f(a)\|\\ &\quad \leq \psi(0,2a) + \psi(10a,a) + 19\psi(9a,a) + 12597\psi(5a,a) + 172\psi(8a,a) + 988\psi(7a,a)\\ &\qquad + 4047\psi(6a,a) + 31008\psi(4a,a) + 62016\psi(3a,a) + 102714\psi(2a,a) + 142766\psi(a,a) \end{aligned} \tag{3.46}$$

for all $a \in X$. Replacing (a,b) by $(0,a)$ in (3.25), we get

$$\begin{aligned}&\|f(10a) - 18f(9a) + 152f(8a) - 798f(7a) + 2907f(6a) - 7752f(5a)\\ &\quad +15504f(4a) - 23256f(3a) + 25194f(2a) - (16796 + 19!)f(a)\| \leq \psi(0,a)\end{aligned} \tag{3.47}$$

for all $a \in X$. Multiplying (3.47) by 167960, and combining the resulting inequality with (3.46), we obtain

$$\begin{aligned}\|-19!f(2a) + 524288(19!)f(a)\| &\leq \psi(0,2a) + \psi(10a,a) + 19\psi(9a,a) + 12597\psi(5a,a)\\ &\quad +172\psi(8a,a) + 988\psi(7a,a) + 4047\psi(6a,a) + 31008\psi(4a,a)\\ &\quad + 62016\psi(3a,a) + 102714\psi(2a,a) + 142766\psi(a,a) + 167960\psi(0,a)\end{aligned} \tag{3.48}$$

for all $a \in X$. From (3.48), we can obtain

$$\begin{aligned}\left\|-f(2a) + 2^{19}f(a)\right\| \leq \frac{1}{19!}\,&[\psi(0,2a) + \psi(10a,a) + 19\psi(9a,a) + 172\psi(8a,a)\\ &+988\psi(7a,a) + 4047\psi(6a,a) + 12597\psi(5a,a)\\ &+31008\psi(4a,a) + 62016\psi(3a,a) + 102714\psi(2a,a)\\ &+142766\psi(a,a) + 167960\psi(0,a)]\end{aligned} \tag{3.49}$$

Therefore,

$$\left\|f(2a) - 2^{19}f(a)\right\| \leq \overline{\psi}(a) \qquad \forall\, a \in X. \tag{3.50}$$

Thus

$$\left\|f(a) - \frac{1}{2^{19l}}f(2^l a)\right\| \leq \frac{\eta^{\left(\frac{1-l}{2}\right)}}{2^{19}}\overline{\psi}(a) \qquad \forall\, a \in X. \tag{3.51}$$

We consider the set $\mathcal{M} = \{f : X \to Y\}$ and introduce the generalized metric ρ on $\mathcal{M}$ as follows:

$$\rho(f,g) = \inf\left\{\mu \in \mathbb{R}_+ : \|f(a) - g(a)\| \leq \mu\overline{\psi}(a), \forall a \in X\right\},$$

It is easy to check that $(\mathcal{M},\rho)$ is a complete generalized metric (see also [11]). Define the mapping $\mathcal{P} : \mathcal{M} \to \mathcal{M}$ by

$$\mathcal{P}f(a) = \frac{1}{2^{19l}}f(2^l a) \qquad \forall\, f \in \mathcal{M} \text{ and } a \in X.$$

Let $f,g \in \mathcal{M}$ and ν be an arbitrary constant with $\rho(f,g) = \nu$. Then

$$\|f(a) - g(a)\| \leq \nu\overline{\psi}(a) \qquad \text{for all } a \in X.$$

Utilizing (3.22), we find that

$$\|\mathcal{P}f(a) - \mathcal{P}g(a)\| = \left\|\frac{1}{2^{19l}}f(2^l a) - \frac{1}{2^{19l}}g(2^l a)\right\| \leq \eta\nu\overline{\psi}(a) \quad \text{for all } a \in X.$$

Hence it holds that $\rho(\mathcal{P}f,\mathcal{P}g) \leq \eta\nu$, that is, $\rho(\mathcal{P}f,\mathcal{P}g) \leq \eta\rho(f,g)$ for all $f,g \in \mathcal{M}$.

It follows from (3.51) that $\rho(f,\mathcal{P}f) \leq \dfrac{\eta^{\left(\frac{1-l}{2}\right)}}{2^{19}}$.

Therefore according to Theorem 2.2 in [3], there exists a mapping $\mathcal{N}_{\mathcal{D}} : X \to Y$ which satisfying:

1. $\mathcal{N}_{\mathcal{D}}$ is a unique fixed point of $\mathcal{P}$ in the set $\mathcal{S} = \{g \in \mathcal{M} : \rho(f,g) < \infty\}$, which is satisfied

$$\mathcal{N}_{\mathcal{D}}(2^l a) = 2^{19l}\mathcal{N}_{\mathcal{D}}(a) \qquad \forall\, a \in X. \tag{3.52}$$

In other words, there exists a μ satisfying

$$\|f(a) - g(a)\| \leq \mu\overline{\psi}(a)\forall\, a \in X.$$

2. $\rho(\mathcal{P}^k f, \mathcal{N}_{\mathcal{D}}) \to 0$ as $k \to \infty$. This implies that

$$\lim_{k\to\infty} \frac{1}{2^{19kl}} f(2^{kl}a) = \mathcal{N}_{\mathcal{D}}(a) \qquad \forall\, a \in X.$$

3. $\rho(f, \mathcal{N}_{\mathcal{D}}) \leq \dfrac{1}{1-\eta}\rho(f, \mathcal{P}f)$, which implies the inequality $\rho(f, \mathcal{N}_{\mathcal{D}}) \leq \dfrac{\eta^{\frac{1-l}{2}}}{2^{19}(1-\eta)}$.

$$\text{So} \quad \|f(a) - \mathcal{N}_{\mathcal{D}}(a)\| \leq \frac{\eta^{\frac{1-l}{2}}}{2^{19}(1-\eta)}\overline{\psi}(a) \qquad \forall\, a \in X. \tag{3.53}$$

It follows from (3.22) and (3.23) that

$$\begin{aligned}\|\mathcal{G}\mathcal{N}_{\mathcal{D}}(a,b)\| &= \lim_{k\to\infty} \frac{1}{2^{19kl}} \left\|\mathcal{G}f(2^{kl}a, 2^{kl}b)\right\| \\ &\leq \lim_{k\to\infty} \frac{1}{2^{19kl}} \psi(2^{kl}a, 2^{kl}b) \leq \lim_{k\to\infty} \frac{2^{kl}\eta^k}{2^{19kl}} \psi(a,b) = 0\end{aligned}$$

for all $a, b \in X$. Hence

$$\begin{aligned}&\mathcal{N}_{\mathcal{D}}(a+10b) - 19\mathcal{N}_{\mathcal{D}}(a+9b) + 171\mathcal{N}_{\mathcal{D}}(a+8b) - 969\mathcal{N}_{\mathcal{D}}(a+7b) + 3876\mathcal{N}_{\mathcal{D}}(a+6b)\\ &\quad + 27132\mathcal{N}_{\mathcal{D}}(a+4b) - 50388\mathcal{N}_{\mathcal{D}}(a+3b) + 75582\mathcal{N}_{\mathcal{D}}(a+2b) - 92378\mathcal{N}_{\mathcal{D}}(a+b)\\ &\quad + 92378\mathcal{N}_{\mathcal{D}}(a) - 75582\mathcal{N}_{\mathcal{D}}(a-b) + 50388\mathcal{N}_{\mathcal{D}}(a-2b) - 27132\mathcal{N}_{\mathcal{D}}(a-3b)\\ &\quad + 11628\mathcal{N}_{\mathcal{D}}(a-4b) - 3876\mathcal{N}_{\mathcal{D}}(a-5b) + 969\mathcal{N}_{\mathcal{D}}(a-6b) - 171\mathcal{N}_{\mathcal{D}}(a-7b)\\ &\qquad - 11628\mathcal{N}_{\mathcal{D}}(a+5b) + 19\mathcal{N}_{\mathcal{D}}(a-8b) - \mathcal{N}_{\mathcal{D}}(a-9b) = 19!\mathcal{N}_{\mathcal{D}}(b)\end{aligned}$$

Therefore, the mapping $\mathcal{N}_{\mathcal{D}} : X \to Y$ is nonadecic mapping.
By Lemma 2.1 in [9] and (3.53),

$$\begin{aligned}\left\|f_n([x_{ij}]) - \mathcal{N}_{\mathcal{D}n}([x_{ij}])\right\| &\leq \sum_{i,j=1}^{n} \left\|f(x_{ij}) - \mathcal{N}_{\mathcal{D}}(x_{ij})\right\| \\ &\leq \sum_{i,j=1}^{n} \frac{\eta^{\frac{1-l}{2}}}{2^{19}(1-\eta)}\overline{\psi}(x_{ij}) \qquad \forall\, x = [x_{ij}] \in M_n(X),\end{aligned}$$

where $\overline{\psi}(x_{ij}) = \dfrac{1}{19!}[\psi(0, 2x_{ij}) + \psi(10x_{ij}, x_{ij}) + 19\psi(9x_{ij}, x_{ij}) + 172\psi(8x_{ij}, x_{ij})$
$+988\psi(7x_{ij}, x_{ij}) + 4047\psi(6x_{ij}, x_{ij}) + 12597\psi(5x_{ij}, x_{ij})$
$+31008\psi(4x_{ij}, x_{ij}) + 62016\psi(3x_{ij}, x_{ij}) + 102714\psi(2x_{ij}, x_{ij})$
$+142766\psi(x_{ij}, x_{ij}) + 167960\psi(0, x_{ij})]$,

Thus $\mathcal{N}_{\mathcal{D}} : X \to Y$ is a unique nonadecic mapping satisfying (3.24). □

Corollary 3.1. *Let $l = \pm 1$ be fixed and let t, ϵ be positive real numbers with $t \neq 19$. Let $f : X \to Y$ be a mapping such that*

$$\left\|\mathcal{G}f_n([x_{ij}], [y_{ij}])\right\|_n \leq \sum_{i,j=1}^{n} \epsilon(\left\|x_{ij}\right\|^t + \left\|y_{ij}\right\|^t) \qquad \forall\, x = [x_{ij}], y = [y_{ij}] \in M_n(X). \tag{3.54}$$

Then there exists a unique nonadecic mapping $\mathcal{N}_{\mathcal{D}} : X \to Y$ such that

$$\left\|f_n([x_{ij}]) - \mathcal{N}_{\mathcal{D}n}([x_{ij}])\right\|_n \leq \sum_{i,j=1}^{n} \frac{\epsilon_s}{l(2^{19} - 2^t)} \left\|x_{ij}\right\|^t \qquad \forall\, x = [x_{ij}] \in M_n(X),$$

where $\epsilon_s = \dfrac{\epsilon}{19!}[667054 + 102715(2^t) + 62016(3^t) + 31008(4^t) + 12597(5^t)$
$+ 4047(6^t) + 988(7^t) + 172(8^t) + 19(9^t) + 10^t]$

Proof. The proof follows from Theorem 3.2 by taking $\psi(a,b) = \epsilon(\|a\|^t + \|b\|^t)$ for all $a, b \in X$. Then we can choose $\eta = 2^{l(t-19)}$, and we can obtain the required result. □

Now we will provide an example to illustrate that the functional equation (1.1) is not stable for $t = 19$ in corollary 3.1.

Example 3.1. *Let $\psi : \mathbb{R} \to \mathbb{R}$ be a function defined by*

$$\psi(x) = \begin{cases} \epsilon x^{19}, & if\ |x| < 1 \\ \epsilon, & otherwise \end{cases}$$

where $\epsilon > 0$ is a constant, and define a function $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \sum_{n=0}^{\infty} \frac{\psi(2^n x)}{2^{19n}}$$

for all $x \in \mathbb{R}$. Then f satisfies the inequality

$$\begin{aligned}\|f(x+10y) &- 19f(x+9y) + 171f(x+8y) - 969f(x+7y) + 3876f(x+6y) \\ &+27132f(x+4y) - 50388f(x+3y) + 75582f(x+2y) - 92378f(x+y) \\ &+92378f(x) - 75582f(x-y) + 50388f(x-2y) - 27132f(x-3y) \\ &+11628f(x-4y) - 3876f(x-5y) + 969f(x-6y) - 171f(x-7y) \\ &\qquad -11628f(x+5y) + 19f(x-8y) - f(x-9y) - 19!f(y)\| \\ &\leq \frac{(121645100400000000)}{524287}(524288)^2\epsilon(|x|^{19} + |y|^{19}) \end{aligned} \tag{3.55}$$

for all $x, y \in \mathbb{R}$. Then there does not exist a nonadecic mapping $\mathcal{N}_{\mathcal{D}} : \mathbb{R} \to \mathbb{R}$ and a constant $\lambda > 0$ such that

$$|f(x) - \mathcal{N}_{\mathcal{D}}(x)| \leq \lambda\, |x|^{19} \qquad \forall\, x \in \mathbb{R}. \tag{3.56}$$

Proof. It is easy to see that f is bounded by $\dfrac{524288\epsilon}{524287}$ on $\mathbb{R}$.
If $|x|^{19} + |y|^{19} = 0$, then (3.55) is trivial.
If $|x|^{19} + |y|^{19} \geq \frac{1}{2^{19}}$, then L.H.S of (3.55) is less than $\dfrac{(121645100400000000)(524288)\epsilon}{524287}$.
Suppose that $0 < |x|^{19} + |y|^{19} < \frac{1}{2^{19}}$, then there exists a non-negative integer k such that

$$\frac{1}{2^{19(k+1)}} \leq |x|^{19} + |y|^{19} < \frac{1}{2^{19k}}, \tag{3.57}$$

so that $2^{19(k-1)}\,|x|^{19} < \frac{1}{2^{19}}, 2^{19(k-1)}\,|y|^{19} < \frac{1}{2^{19}}$, and

$$\begin{aligned}&2^n(x), 2^n(y), 2^n(x+10y), 2^n(x+9y), 2^n(x+8y), 2^n(x+7y), \\ &2^n(x+6y), 2^n(x+5y), 2^n(x+4y), 2^n(x+3y), 2^n(x+7y), 2^n(x+2y), \\ &2^n(x+y), 2^n(x-y), 2^n(x-2y), 2^n(x-3y), 2^n(x-4y), 2^n(x-5y), \\ &\qquad 2^n(x-6y), 2^n(x-7y), 2^n(x-8y), 2^n(x-9y) \in (-1,1)\end{aligned}$$

for all $n = 0, 1, 2, ..., k-1$. Hence

$$\begin{aligned}\psi(2^n(x+10y)) &- 19\psi(2^n(x+9y)) + 171\psi(2^n(x+8y)) - 969\psi(2^n(x+7y)) \\ &+ 3876\psi(2^n(x+6y)) - 11628\psi(2^n(x+5y)) + 27132\psi(2^n(x+4y)) \\ &- 50388\psi(2^n(x+3y)) + 75582\psi(2^n(x+2y)) - 92378\psi(2^n(x+y)) \\ &+ 92378\psi(2^n x) - 75582\psi(2^n(x-y)) + 50388\psi(2^n(x-2y)) \\ &- 27132\psi(2^n(x-3y)) + 11628\psi(2^n(x-4y)) - 3876\psi(2^n(x-5y)) \\ &+ 969\psi(2^n(x-6y)) - 171\psi(2^n(x-7y)) + 19\psi(2^n(x-8y)) \\ &\qquad - \psi(2^n(x-9y)) - 19!\psi(2^n y) = 0\end{aligned}$$

for $n = 0, 1, 2, ..., k-1$. From the definition of f and (3.57), we obtain that

$$\begin{aligned}|f(x+10y) &- 19f(x+9y) + 171f(x+8y) - 969f(x+7y) + 3876f(x+6y) \\ &+27132f(x+4y) - 50388f(x+3y) + 75582f(x+2y) - 92378f(x+y) \\ &+92378f(x) - 75582f(x-y) + 50388f(x-2y) - 27132f(x-3y) \\ &+11628f(x-4y) - 3876f(x-5y) + 969f(x-6y) - 171f(x-7y) \\ &\qquad -11628f(x+5y) + 19f(x-8y) - f(x-9y) - 19!f(y)| \\ \leq \textstyle\sum_{n=0}^{\infty} \frac{1}{2^{19n}}\, &|\psi(2^n(x+10y)) - 19\psi(2^n(x+9y)) + 171\psi(2^n(x+8y)) \\ &+3876\psi(2^n(x+6y)) - 11628\psi(2^n(x+5y)) + 27132\psi(2^n(x+4y)) \\ &-50388\psi(2^n(x+3y)) + 75582\psi(2^n(x+2y)) - 92378\psi(2^n(x+y))\end{aligned}$$

$$+92378\psi(2^n x) - 75582\psi(2^n(x-y)) + 50388\psi(2^n(x-2y))$$
$$-27132\psi(2^n(x-3y)) + 11628\psi(2^n(x-4y)) - 3876\psi(2^n(x-5y))$$
$$+969\psi(2^n(x-6y)) - 171\psi(2^n(x-7y)) + 19\psi(2^n(x-8y))$$
$$-969\psi(2^n(x+7y)) - \psi(2^n(x-9y)) - 19!\psi(2^n y)|$$
$$\leq \sum_{n=k}^{\infty} \frac{(121645100400000000)\epsilon}{2^{19n}} = \frac{(524288)(121645100400000000)\epsilon}{2^{19k}524287}$$
$$\leq \frac{(121645100400000000)}{524287}(524288)^2\epsilon(|x|^{19} + |y|^{19}).$$

Therefore, f satisfies (3.55) for all $x, y \in \mathbb{R}$. Now, we claim that functional equation (1.1) is not stable for $t = 19$ in corollary 3.1. Suppose on the contrary that there exists a nonadecic mapping $\mathcal{N}_{\mathcal{D}} : \mathbb{R} \to \mathbb{R}$ and a constant $\lambda > 0$ satisfying (3.56). Then there exists a constant $c \in \mathbb{R}$ such that $\mathcal{N}_{\mathcal{D}}(x) = cx^{19}$ for any $x \in \mathbb{R}$. Thus we obtain the following inequality

$$|f(x)| \leq (\lambda + |c|)\,|x|^{19} \tag{3.58}$$

Let $m \in \mathbb{N}$ with $m\epsilon > \lambda + |c|$. If $x \in (0, \frac{1}{2^{m-1}})$, then $2^n x \in (0,1)$ for all $n = 0, 1, 2, ..., m-1$, and for this case we get

$$f(x) = \sum_{n=0}^{\infty} \frac{\psi(2^n x)}{2^{19n}} \geq \sum_{n=0}^{m-1} \frac{\epsilon(2^n x)^{19}}{2^{19n}} = m\epsilon x^{19} > (\lambda + |c|)\,|x|^{19}$$

which is a contradiction to (3.58). Therefore the nonadecic functional equation (1.1) is not stable for $t = 19$. □

4 Conclusion

In this investigation, we identified a general solution of nonadecic functional equation and establised the generalized Ulam -Hyers stability of this functional equation in matrix normed spaces by using the fixed point method and also provided an example for non-stability.

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