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On strong domination number of corona related graphs

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Abstract
Let $G=(V(G),E(G))$ be a graph and $uv \in E(G)$ be an edge. A vertex u strongly dominates v if $d_G(u) \geq d_G(v)$. A set $S \subseteq V(G)$ is a *strong dominating set* $(sd-set)$ if every vertex $v \in V(G)-S$ is strongly dominated by some u in S. The minimum cardinality of a strong dominating set is called the strong domination number of G which is denoted by $\gamma_{st}(G)$. We investigate strong domination number of some corona related graphs.

Keywords
Dominating set, domination number, strong dominating set, strong domination number.

AMS Subject Classification
05C69; 05C76.

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Contents

1. Introduction

We consider simple, finite, connected and undirected graph G with vertex set $V(G)$ and edge set $E(G)$. For all standard terminology and notations we follow West [20] while the terms related to the theory of domination in graphs are used in the sense of Haynes *et al*. [7].

Definition 1.1. *A set $S \subseteq V(G)$ is called a dominating set if every vertex $v \in V(G)$ is either an element of S or is adjacent to an element of S. A dominating set S is a minimal dominating set if no proper subset $S^{'} \subset S$ is a dominating set. The domination number $\gamma(G)$ of a graph G is the minimum cardinality of a minimal dominating set in graph G.*

We denote the degree of a vertex v in a graph G by $d_G(v)$ while the maximum and minimum degree of the graph G are denoted by $\triangle(G)$ and $\delta(G)$ respectively.

Definition 1.2. *If uv is an edge of G then, u strongly dominates v if $d_G(u) \geq d_G(v)$. A set $S \subseteq V(G)$ is a strong dominating set $(sd-set)$ if every vertex $v \in V(G)-S$ is strongly dominated by some u in S. The minimum cardinality of a strong dominating set is called the strong domination number of G and it is denoted by $\gamma_{st}(G)$. Analogously, one can define a weak dominating set $(wd-set)$.*

The concept of strong (weak) domination was introduced by Sampathkumar and Pushpa Latha [13]. Some bounds on $\gamma_{st}(G)$ were investigated by Rautenbach [10, 11]. Also some bounds on strong and weak domination numbers were investigated by Sampathkumar and Pushpa Latha [13] while Bhat *et al.* [1] have improved these bounds and reported the graphs achieving such bounds. Rautenbach and Zverovich [12] have studied results on the NP-complete problems of strong dominating set and weak dominating set. Gani and Ahamed [4] have introduced the concept of strong and weak domination in fuzzy graphs and provided some examples to explain various notions. Vaidya and Karkar [16, 17] have investigated the strong domination number of some path related graphs and independent strong domination number of the graphs obtained by switching of a vertex in P_n. The strong domination in m-splitting graph of P_n, C_n and $K_{m,n}$ is discussed by Vaidya and Mehta [18] while the same authors in [19] have investigated the strong domination number for the generalized Petersen graph. The relation between the strong domination and weak domination number is given by Boutrig and Chellali [2] while Meena *et al.* [9] have compared strong efficient domination number with strong domination number. The relation between strong domination and maximum degree of the graph as well as weak domination and minimum degree of the graph were revealed by Swaminathan and Thangaraju [15]. To obtain strong domination number of larger graph (super graph) obtained from the given graph is challenging and interesting as well. We have studied such problem in the context of corona of two graphs, edge corona of two graphs and neighbourhood corona of two graphs.

2. Main Results

The concept of corona of two graphs was introduced by Frucht and Harary [3].

Definition 2.1. *Let G and H be two graphs on n and m vertices, respectively. The corona of the graphs G and H denoted by $G \circ H$ and is defined as the graph obtained by taking one copy of G and n copies of H, and then joining the i^{th} vertex of G to every vertex in the i^{th} copy of H.*

Proposition 2.2. [5] *Let G be a connected graph of order n and let H be any graph of order m. Then, $\gamma(G \circ H) = n$.*

We prove the following result.

Theorem 2.3. *Let G be a connected graph of order n and let H be any simple graph of order m. Then, $\gamma_{st}(G \circ H) = n$.*

Proof. Let $V(G) = \{v_1, v_2, \dots v_n\}$ and $V(\mathrm{H}) = \{u_1, u_2, \dots u_m\}$. In $G \circ H$, denote the i^{th} copy of H by H_i and the vertices of H_i by $u_1^i, u_2^i, \dots u_m^i$ for $1 \leq i \leq n$, that is, $V(H_i) = \{u_1^i, u_2^i, \dots u_m^i\}$. By the definition of the corona graph, $d_{G \circ H}(v_i) > d_{G \circ H}(u_j^i)$, $1 \leq i \leq n$ and $1 \leq j \leq m$. Therefore, every vertex v_i strongly dominates all the vertices of H_i as well as the vertices of G which are adjacent to v_i and having degree greater than or equal to that of vertex v_i. We also observe that any vertex v_i can not strongly dominate any vertex of H_j for $i \neq j$. Therefore, it is enough to include n vertices, namely $v_1, v_2, \dots v_n$ in any strong dominating set S. Hence, S becomes the strong dominating set of minimum cardinality implying that $\gamma_{st}(G \circ H) = n$.

Illustration 2.4. In Figure 1, $S = \{v_1, v_2, v_3, v_4, v_5\}$ is strong dominating set of the graph $C_5 \circ P_2$ and $\gamma_{st}(C_5 \circ P_2) = 5$. The strong dominating set is shown with solid vertices.

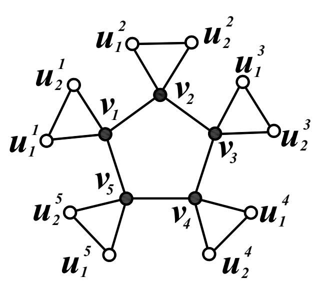

Figure 1. $C_5 \circ P_2$

Illustration 2.5. In Figure 2, $S = \{v_1, v_2, v_3, v_4\}$ is a strong dominating set of the graph $P_4 \circ K_1$ and $\gamma_{st}(P_4 \circ K_1) = 4$. The strong dominating set is shown with solid vertices.

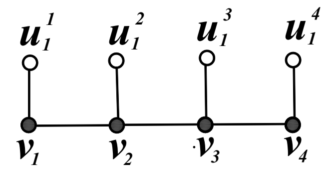

Figure 2. $P_4 \circ K_1$

Definition 2.6. *Duplication of a vertex v_k by a new edge $e = v_k' v_k''$ in a graph G produces a new graph G such that $N(v_k') = \{v_k, v_k''\}$ and $N(v_k'') = \{v_k, v_k'\}$.*

Corollary 2.7. *Let G be a graph with n vertices and G' be the graph obtained by duplication of every vertex of a connected graph G by an edge. Then, $\gamma_{st}(G') = n$.*

Proof. In $G \circ H$ let G be any connected graph and $H = P_2$ then $G' \cong G \circ H$. Therefore, by Theorem 2.3 $\gamma_{st}(G') = \gamma_{st}(G \circ H) = n$.

The concept of edge corona of two graphs was introduced by Hou and Shiu [8] and defined as follows.

Definition 2.8. *Let G and H be two graphs on n and m vertices, k and l edges, respectively. The edge corona $G \diamondsuit H$ of G and H is defined as the graph obtained by taking one copy of G and k copies of H, and then joining two end vertices of the i-th edge of G to every vertex in the i-th copy of H.*

Theorem 2.9. $\gamma_{st}(P_n \diamondsuit H) = \left\lfloor \frac{n}{2} \right\rfloor$, *where H is any simple graph.*

Proof. Let $V(P_n) = \{v_1, v_2, \dots v_n\}$, $E(P_n) = \{e_1, e_2, \dots e_{n-1}\}$ and $V(\mathrm{H}) = \{u_1, u_2, \dots u_m\}$. In $P_n \diamondsuit H$, denote the i^{th} copy of H by H_i and the vertices of H_i by $u_1^i, u_2^i, \dots u_m^i$ for $1 \leq i \leq n-1$, that is, $V(H_i) = \{u_1^i, u_2^i, \dots u_m^i\}$. By the definition of the edge corona graph, $d_{P_n \diamondsuit H}(v_i) \geq d_{P_n \diamondsuit H}(u_j^i)$, $1 \leq i \leq n-1$ and $1 \leq j \leq m$.

Therefore, the vertex v_i of P_n strongly dominates all vertices of H_{i-1} and H_i as well as the vertices of G which are adjacent to v_i and having degree greater than or equal to that of vertex v_i for $2 \leq i \leq n-1$. The vertex v_1 strongly dominates vertices of H_1 and v_2 only while the vertex v_n strongly dominates vertices of H_{n-1} and v_{n-1} only. If P_n is a path of odd order then the vertices $v_2, v_4, v_6, v_8 \dots v_{n-1}$ strongly dominate all the vertices of the graph $P_n \diamondsuit H$. In other words total $\left\lfloor \frac{n}{2} \right\rfloor$ vertices are necessary to strongly dominate all the vertices of $P_n \diamondsuit H$.

If P_n is a path of even order then the vertices $v_2, v_4, v_6, v_8 \dots v_{n-1}$ or $v_2, v_4, v_6, v_8 \dots v_n$ strongly dominate all the vertices of $P_n \diamondsuit H$. In other words total $\frac{n}{2}$ vertices are necessary to strongly dominate all the vertices of $P_n \diamondsuit H$. Therefore, $\left\lfloor \frac{n}{2} \right\rfloor$ vertices are enough to strongly dominate all the vertices of $P_n \diamondsuit H$ for any n. Hence, $\gamma_{st}(P_n \diamondsuit H) = \left\lfloor \frac{n}{2} \right\rfloor$.

Illustration 2.10. In Figure 3, $S = \{v_2, v_3\}$ is a strong dominating set of $P_4 \diamondsuit P_2$ and $\gamma_{st}(P_4 \diamondsuit P_2) = 2$. The strong dominating set is shown with solid vertices.

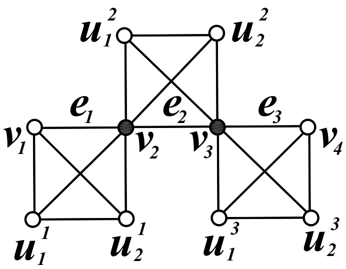

Figure 3. $P_4 \diamondsuit P_2$

Definition 2.11. *Duplication of an edge $e = uv$ by a new vertex v' in a graph G produces a new graph G' by adding a vertex v' such that $N(v') = \{u, v\}$.*

Corollary 2.12. *Let G be a graph obtained by duplication of each edge of P_n by a vertex then $\gamma_{st}(G) = \left\lfloor \frac{n}{2} \right\rfloor$.*

Proof. Taking $H = K_1$ in $P_n \diamondsuit H$, $G \cong P_n \diamondsuit H$. Therefore, by Theorem 2.9 $\gamma_{st}(G) = \left\lfloor \frac{n}{2} \right\rfloor$.

Theorem 2.13. $\gamma_{st}(C_n \diamondsuit H) = \left\lceil \frac{n}{2} \right\rceil$, *where H is any simple graph.*

Proof. Let $V(C_n) = \{v_1, v_2, \dots v_n\}$, $E(C_n) = \{e_1, e_2, \dots e_n\}$ and $V(\text{H}) = \{u_1, u_2, \dots u_m\}$. In $C_n \diamondsuit H$, denote the i^{th} copy of the graph H by H_i and the vertices of H_i by $u_1^i, u_2^i, \dots u_m^i$ for $1 \le i \le n$, that is, $V(H_i) = \{u_1^i, u_2^i, \dots u_m^i\}$. By the definition of the edge corona graph, $d_{C_n \diamondsuit H}(v_i) \ge d_{C_n \diamondsuit H}(u_j^i)$, $1 \le i \le n$ and $1 \le j \le m$. Therefore, the vertex v_i from C_n strongly dominates all vertices of H_{i-1} and H_i for as well as the vertices of G which are adjacent to v_i and having degree greater than or equal to that of vertex v_i for $2 \le i \le n$ while the vertex v_1 strongly dominates all vertices of H_n, H_1, v_2 and v_4. If C_n is a cycle of odd order then, $v_1, v_3, v_5, v_7 \dots v_n$ strongly dominate all the vertices of $C_n \diamondsuit H$. In other words atleast $\left\lceil \frac{n}{2} \right\rceil$ vertices are necessary to strongly dominate all the vertices of $C_n \diamondsuit H$. If C_n is a cycle of even order then the vertices $v_2, v_4, v_6, v_8 \dots v_n$ or $v_1, v_3, v_5, v_7 \dots v_{n-1}$ strongly dominate all the vertices of $C_n \diamondsuit H$. In other words atleast $\frac{n}{2}$ number of vertices are necessary to strongly dominate all the vertices of the graph $C_n \diamondsuit H$. Therefore, $\left\lceil \frac{n}{2} \right\rceil$ number of vertices are enough to strongly dominate all the vertices of $C_n \diamondsuit H$. Hence, $\gamma_{st}(C_n \diamondsuit H) = \left\lceil \frac{n}{2} \right\rceil$.

Illustration 2.14. In Figure 4, $S = \{v_2, v_4\}$ is a strong dominating set of graph $C_4 \diamondsuit P_2$ and $\gamma_{st}(C_4 \diamondsuit P_2) = 2$. The strong dominating set is shown with solid vertices.

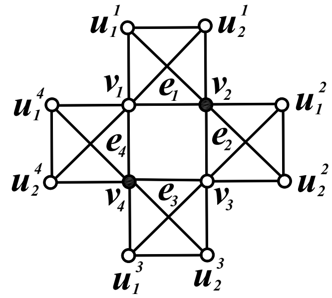

Figure 4. $C_4 \diamondsuit P_2$

Definition 2.15. *The middle graph $M(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent whenever either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it.*

Corollary 2.16. $\gamma_{st}(M(C_n)) = \left\lceil \frac{n}{2} \right\rceil$.

Proof. In graph, $C_n \diamondsuit H$ if we consider $H = K_1$ then $C_n \diamondsuit H \cong M(C_n)$. Therefore, by Theorem 2.13, $\gamma_{st}(M(C_n)) = \left\lceil \frac{n}{2} \right\rceil$.

Theorem 2.17. $\gamma_{st}(K_{1,n} \diamondsuit H) = 1$, *where H is any simple graph.*

Proof. Let $V(K_{1,n}) = \{v_0, v_1, v_2, \dots v_n\}$, $E(K_{1,n}) = \{e_1, e_2, \dots e_n\}$ and $V(\text{H}) = \{u_1, u_2, \dots u_m\}$ where v_0 is the vertex of degree n in $K_{1,n}$. In $K_{1,n} \diamondsuit H$, denote the i^{th} copy of the graph H by H_i and the vertices of H_i by $u_1^i, u_2^i, \dots u_m^i$ for $1 \le i \le n$, that is, $V(H_i) = \{u_1^i, u_2^i, \dots u_m^i\}$. By definition of the edge corona graph $d_{K_{1,n} \diamondsuit H}(v_i) \ge d_{K_{1,n} \diamondsuit H}(u_j^i)$, $1 \le i \le n$ and $1 \le j \le m$. The vertex v_0 strongly dominates all vertices of $K_{1,n} \diamondsuit H$. Therefore, $S = \{v_0\}$ is the strong dominating set of minimum cardinality for $K_{1,n} \diamondsuit H$. Hence, $\gamma_{st}(K_{1,n} \diamondsuit H) = 1$.

Illustration 2.18. In Figure 5, $S = \{v_0\}$ is a strong dominating set of the graph $K_{1,n} \diamondsuit K_1$ and $\gamma_{st}(K_{1,n} \diamondsuit K_1) = 1$. The strong dominating set is shown with solid vertices.

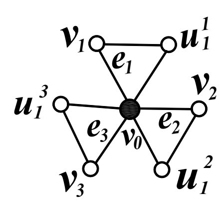

Figure 5. $K_{1,n} \diamondsuit K_1$

Definition 2.19. *A friendship graph F_n is a one point union of n copies of cycle C_3.*

Corollary 2.20. $\gamma_{st}(F_n) = 1$.

Proof. Taking $H = K_1$ in $K_{1,n} \diamondsuit H$, $F_n \cong K_{1,n} \diamondsuit K_1$. Therefore, by Theorem 2.17 $\gamma_{st}(K_{1,n} \diamondsuit H) = \gamma_{st}(F_n) = 1$.

The following concept was introduced by Gopalapillai [6] recently.

Definition 2.21. *Let G and H be two graphs on n and m vertices respectively. Then the neighborhood corona, $G\bigstar H$ is the graph obtained by taking one copy of G and n copies of H, and then joining each neighbor of i^{th} vertex of G to every vertex in the i^{th} copy of H.*

In neighborhood corona $G\bigstar H$ if we consider $H = K_1$ then $G\bigstar H$ becomes a splitting graph. The splitting graph is introduced by Sampathkumar and Walikar [14].

Theorem 2.22. *For $n \geq 4$,*

$$\gamma_{st}(P_n\bigstar G) = \begin{cases} \dfrac{n}{2} & \text{if } n \equiv 0 \ (mod4), \\ \dfrac{n+1}{2} & \text{if } n \equiv 1 \ or\ 3 \ (mod4), \\ \dfrac{n}{2}+1 & \text{if } n \equiv 2 \ (mod4). \end{cases}$$

Proof. Let $V(P_n) = \{v_1, v_2, \dots v_n\}$ and $V(\mathrm{G}) = \{u_1, u_2, \dots u_m\}$. In $P_n\bigstar G$, denote the vertices of i^{th} copy of the graph G by $u^i_1, u^i_2, \dots u^i_m$ for $1 \leq i \leq n$, and $H_i = \{u^i_1, u^i_2, \dots u^i_m\}$, $1 \leq i \leq n$. It is very clear that $d_{P_n\bigstar G}(v_i) = (m+1)d_{P_n}(v_i)$, for $1 \leq i \leq n$ and $d_{P_n\bigstar G}(u^i_j) = d_{P_n}(v_i) + d_G(u_j)$, for $1 \leq i \leq n$ and $1 \leq j \leq m$. Hence, $d_{P_n\bigstar G}(v_i) \geq d_{P_n\bigstar G}(u^i_j)$.

Case: I. $n \equiv 0 \ (mod4)$
Let $V(P_n\bigstar G) = \{v_1, v_2, v_3, v_4\} \cup \{v_5, v_6, v_7, v_8\} \cup \dots \cup \{v_{n-3}, v_{n-2}, v_{n-1}, v_n\} \cup H_1 \cup H_2 \cup \dots H_n$ be the partition of $V(P_n\bigstar G)$. As per the discussion about the degree of the vertices of graph $P_n\bigstar G$ in beginning of proof, the vertices v_1 and v_n strongly dominate only $m+1$ vertices while the vertices $v_2, v_3, \dots v_{n-1}$ strongly dominate $2m+3$ vertices from $P_n\bigstar G$. Therefore, it is very clear that if we consider the vertex v_2 in a strong dominating set S then the vertices v_1, v_2, v_3 and all the vertices of H_1 and H_3 are strongly dominated by it. Now, to strongly dominate the vertices of H_2 the vertex v_1 or the vertex v_3 must belong to S. But the vertex v_3 strongly dominates more vertices than the vertex v_1, Therefore, to form the strong dominating set S of minimum cardinality the vertex v_3 must belong to S. Thus, the vertices v_2 and v_3 strongly dominate all the vertices of the sets $\{v_1, v_2, v_3, v_4\}$, H_1, H_2, H_3 and H_4. We can also observe that the vertices v_6 and v_7 strongly dominate all the vertices of the sets $\{v_5, v_6, v_7, v_8\}$, H_5, H_6, H_7 and H_8. Similarly, the vertices v_{n-2} and v_{n-1} strongly dominate all the vertices of the sets $\{v_{n-3}, v_{n-2}, v_{n-1}, v_n\}$, H_{n-3}, H_{n-2}, H_{n-1} and H_n. Therefore, $v_2, v_3, v_6, v_7, \dots v_{n-2}, v_{n-1}$ vertices strongly dominate all the vertices of $P_n\bigstar G$. So, it is enough to consider $\dfrac{n}{2}$ vertices from $V(P_n)$ to strongly dominate all the vertices of the graph $P_n\bigstar G$. Hence, $\gamma_{st}(P_n\bigstar G) = \dfrac{n}{2}$.

Case: II. $n \equiv 1 \ (mod4)$
Let $V(P_n\bigstar G) = \{v_1, v_2, v_3, v_4\} \cup \{v_5, v_6, v_7, v_8\} \cup \dots \cup \{v_{n-4}, v_{n-3}, v_{n-2}, v_{n-1}\} \cup \{v_n\} \cup H_1 \cup H_2 \cup \dots H_n$ be the partition of $V(P_n\bigstar G)$. As per the discussion in beginning of the case(I), $\dfrac{n-1}{2}$ number of vertices $(v_2, v_3, v_6, v_7, \dots v_{n-3}, v_{n-2})$ from $V(P_n)$ can strongly dominate the vertices $v_1, v_2, v_3, \dots v_{n-1}$ and all the vertices of sets $H_1, H_2, \dots H_{n-1}$. To strongly dominate the remaining vertices (v_n and all the vertices of the set H_n) the vertex v_{n-1} must be in a strong dominating set S. Therefore, $\dfrac{n-1}{2} + 1 = \dfrac{n+1}{2}$ number of vertices are enough to strongly dominate all the vertices of the graph $P_n\bigstar G$. Hence, $\gamma_{st}(P_n\bigstar G) = \dfrac{n+1}{2}$.

Case: III. $n \equiv 2 \ (mod4)$
Let $V(P_n\bigstar G) = \{v_1, v_2, v_3, v_4\} \cup \{v_5, v_6, v_7, v_8\} \cup \dots \cup \{v_{n-5}, v_{n-4}, v_{n-3}, v_{n-2}\} \cup \{v_{n-1}, v_n\} \cup H_1 \cup H_2 \cup \dots H_n$ be the partition of $V(P_n\bigstar G)$. As per the discussion in beginning of the case(I), $\dfrac{n-2}{2}$ number of vertices $v_2, v_3, v_6, v_7, \dots v_{n-4}, v_{n-3}$ from $V(P_n)$ can strongly dominate the vertices $v_1, v_2, v_3, \dots v_{n-2}$ and all the vertices of sets $H_1, H_2, \dots H_{n-2}$. To strongly dominate the remaining vertices (v_{n-1}, v_n and all the vertices of the sets H_{n-1} and the set H_n) the vertex v_{n-1} and either v_{n-2} or v_n must be in a strong dominating set S. Therefore, $\dfrac{n-2}{2} + 2 = \dfrac{n}{2} + 1$ vertices are enough to strongly dominate all the vertices of the graph $P_n\bigstar G$. Hence, $\gamma_{st}(P_n\bigstar G) = \dfrac{n}{2} + 1$.

Case: IV. $n \equiv 3 \ (mod4)$
Let $V(P_n\bigstar G) = \{v_1, v_2, v_3, v_4\} \cup \{v_5, v_6, v_7, v_8\} \cup \dots \cup \{v_{n-6}, v_{n-5}, v_{n-4}, v_{n-3}\} \cup \{v_{n-2}, v_{n-1}, v_n\} \cup H_1 \cup H_2 \cup \dots H_n$ be the partition of $V(P_n\bigstar G)$. As per the discussion in beginning of the case(I), $\dfrac{n-3}{2}$ vertices namely $v_2, v_3, v_6, v_7, \dots v_{n-5}, v_{n-4}$ from $V(P_n)$ can strongly dominate the vertices $v_1, v_2, v_3, \dots v_{n-3}$ and all the vertices of sets $H_1, H_2, \dots H_{n-3}$. To strongly dominate the remaining vertices (v_{n-2}, v_{n-1}, v_n and all the vertices of the sets H_{n-2}, H_{n-1} and H_n) the vertex v_{n-1} and either v_{n-2} or v_n must be in a strong dominating set S. Therefore, $\dfrac{n-3}{2} + 2 = \dfrac{n+1}{2}$ vertices are enough to strongly dominate all the vertices of the graph $P_n\bigstar G$. Hence, $\gamma_{st}(P_n\bigstar G) = \dfrac{n+1}{2}$.

Remark 2.23. $\gamma_{st}(P_2\bigstar G) = \gamma_{st}(P_3\bigstar G) = 2$.

Illustration 2.24. In Figure 6, $S = \{v_2, v_3, v_4\}$ is a strong dominating set of the graph $P_5\bigstar P_2$ and $\gamma_{st}(P_5\bigstar P_2) = 3$. The strong dominating set is shown with solid vertices.

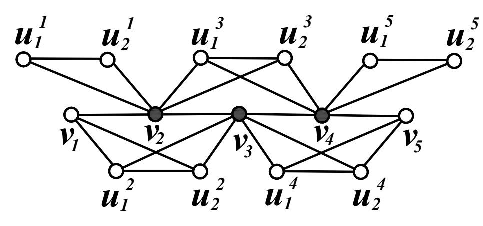

Figure 6. $P_5\bigstar P_2$

Definition 2.25. *For a graph G the splitting graph $S'(G)$ of a graph G is obtained by adding a new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$.*

Corollary 2.26. *For $n \geq 4$,*

$$\gamma_{st}(S'(P_n)) = \begin{cases} \dfrac{n}{2} & if\, n \equiv 0\ (mod 4), \\ \dfrac{n+1}{2} & if\, n \equiv 1\ or\ 3\ (mod 4), \\ \dfrac{n}{2}+1 & if\, n \equiv 2\ (mod 4). \end{cases}$$

Proof. In the graph $P_n \bigstar G$ if we consider $G = K_1$ then $P_n \bigstar K_1 \cong S'(P_n)$ becomes splitting graph of path P_n. Thus, $\gamma_{st}(S'(P_n)) = \gamma_{st}(P_n \bigstar K_1)$. Therefore, by the Theorem 2.22, the result holds.

Conclusion

The concept of strong domination is a variant of usual domination in graphs. The strong domination number of some standard graphs are already available in the literature while we have investigated the strong domination number for corona product and corona like products of two graphs. To derive similar results for other graph families as well as in the context of various domination models are potential areas of research.

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