Malaya Journal of Matematik 3(1)(2013) 55-63

Malaya Journal of Matematik

www.malayajournal.org

MJM an international journal of mathematical sciences with computer applications...

ISSN : 2319-3786

Total edge product cordial labeling of graphs

Samir K. Vaidya,[a,*] and Chirag M. Barasara[b]

[a] Department of Mathematics, Saurashtra University, Rajkot - 360005, Gujarat, India.

[b] Atmiya Institute of Technology and Science, Rajkot - 360005, Gujarat, India.

Abstract

The total product cordial labeling is a variant of cordial labeling. We introduce an edge analogue product cordial labeling as a variant of total product cordial labeling and name it as total edge product cordial labeling. Unlike to total product cordial labeling the roles of vertices and edges are interchanged in total edge product cordial labeling. We investigate several results on this newly defined concept.

Keywords: Cordial labeling, product cordial labeling, edge product cordial labeling, total edge product cordial labeling.

2010 MSC: 05C78.

1 Introduction

We begin with simple, finite, connected and undirected graph G = (V(G), E(G)) with order p and size q. The members of $V(G)$ and $E(G)$ are commonly termed as graph elements while $|V(G)|$ and $|E(G)|$ denotes number of vertices and edges in graph G respectively. For all standard terminology and notations we follow West [10]. We will give brief summary of definitions which are useful for the present investigations.

Definition 1.1. *A* graph labeling *is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of mapping is the set of vertices(edges) then the labeling is called* a vertex(an edge) labeling.

Most of the graph labeling techniques trace their origin to β - labelings introduced by Rosa [5] in 1967. This labeling was renamed as graceful labeling by Golomb [3] and it is now the popular term which is defined as follows.

Definition 1.2. *A graph $G = (V(G), E(G))$ of order p and size q is said to be* graceful *if there exists an injection $f : V(G) \to \{0, 1, 2, \ldots, q\}$ such that the induced function $f^* : E(G) \to \{1, 2, \ldots, q\}$ defined by $f^*(e = uv) = |f(u) - f(v)|$ for each edge $e = uv$ is a bijection and f is said to be* graceful labeling *of G.*

Vast amount of literature is available on different types of graph labeling. Labeling of graphs is a potential area of research and more than 1500 research papers have been published so far in past five decades. For an extensive survey on graph labeling and bibliographic references we refer to Gallian [2].

Graham and Sloane [4] have introduced harmonious labeling during their study on modular versions of additive bases problems stemming from error correcting codes.

Definition 1.3. *A graph $G = (V(G), E(G))$ is said to be* harmonious *if there exists an injection $f : V(G) \to Z_q$ such that the induced function $f^* : E(G) \to Z_q$ defined by $f^*(e = uv) = (f(u) + f(v))$ (mod q) is a bijection and f is said to be* harmonious labeling *of G.*

*Corresponding author.

E-mail addresses: samirkvaidya@yahoo.co.in (Samir K. Vaidya), chirag.barasara@gmail.com (Chirag M. Barasara)

In 1987, Cahit [1] have introduced cordial labeling as a weaker version of graceful labeling and harmonious labeling which is defined as follows.

Definition 1.4. *For a graph G, a vertex labeling function $f : V(G) \to \{0,1\}$ induces an edge labeling function $f^* : E(G) \to \{0,1\}$ defined as $f^*(uv) = |f(u) - f(v)|$. Let $v_f(i)$ be the number of vertices of G having label i under f and $e_f(i)$ be the number of edges of G having label i under f^* for $i = 0, 1$.The function f is called* cordial labeling *of G if $|e_f(0) - e_f(1)| \leq 1$ and $|v_f(0) - v_f(1)| \leq 1$. A graph is called* cordial *if it admits cordial labeling.*

In the same paper Cahit [1] proved many results on cordial labeling.

After this some labelings schemes like prime cordial labeling, A - cordial labeling, H-cordial labeling, product cordial labeling, etc. are also introduced as variants of cordial labeling.

The concept of E-cordial labeling was introduced by Yilmaz and Cahit [11] which is defined as follows.

Definition 1.5. *For a graph G, an edge labeling function $f^* : E(G) \to \{0,1\}$ induces a vertex labeling function $f : V(G) \to \{0,1\}$ defined as $f(v) = \sum\{f^*(uv)/uv \in E(G)\}$ (mod 2). The function f^* is called* E-cordial labeling *of G if $|e_f(0) - e_f(1)| \leq 1$ and $|v_f(0) - v_f(1)| \leq 1$. A graph is called* E-cordial *if it admits E-cordial labeling.*

Definition 1.6. *For a graph G, an edge labeling function $f^* : E(G) \to \{0,1\}$ induces a vertex labeling function $f : V(G) \to \{0,1\}$ defined as $f(v) = \prod\{f^*(uv)/uv \in E(G)\}$. The function f^* is called* edge product cordial labeling *of G if $|e_f(0) - e_f(1)| \leq 1$ and $|v_f(0) - v_f(1)| \leq 1$. A graph is called* edge product cordial *if it admits edge product cordial labeling.*

The concept of edge product cordial labeling is introduced in recent past by Vaidya and Barasara [7] and they have investigated several results on this newly defined concept in [7–9].

Definition 1.7. *For a graph G, a vertex labeling function $f : V(G) \to \{0,1\}$ induces an edge labeling function $f^* : E(G) \to \{0,1\}$ defined as $f^*(uv) = f(u)f(v)$. The function f is called* total product cordial labeling *of G if $|\,(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))\,| \leq 1$. A graph is called* total product cordial *if it admits total product cordial labeling.*

In 2006, Sundaram *et al.* [6] have introduced total product cordial labeling and also proved some general results.

In this paper we introduce an edge analogue of total product cordial labeling which is defined as follows.

Definition 1.8. *For a graph G, an edge labeling function $f^* : E(G) \to \{0,1\}$ induces a vertex labeling function $f : V(G) \to \{0,1\}$ defined as $f(v) = \prod\{f^*(uv)/uv \in E(G)\}$. The function f^* is called a* total edge product cordial labeling *of G if $|\,(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))\,| \leq 1$. A graph is called* total edge product cordial *if it admits total edge product cordial labeling.*

This work also rules out any possibility of forbidden subgraph characterizations for total edge product cordial labeling as it is established that for $n > 2$, K_n is total edge product cordial graph.

Definition 1.9. *Let $C_n^{(t)}$ denote the one-point union of t cycles of length n.*

Definition 1.10. *The* wheel *W_n is defined to be the join $C_n + K_1$. The vertex corresponding to K_1 is known as apex vertex, the vertices corresponding to cycle are known as rim vertices.*

Definition 1.11. *Let $e = uv$ be an edge of graph G and w is not a vertex of G. The edge e is* subdivided *when it is replaced by the edges $e' = uw$ and $e'' = wv$.*

Definition 1.12. *The gear graph G_n is obtained from the wheel W_n by subdividing each of its rim edge.*

Definition 1.13. *The* fan *f_n is the graph obtained by taking $n - 2$ concurrent chords in cycle C_{n+1}. The vertex at which all the chords are concurrent is called the apex vertex. It is also given by $f_n = P_n + K_1$.*

Definition 1.14. *The* double fan *DF_n is defined as $P_n + 2K_1$.*

2 Main results

Theorem 2.1. *Every edge product cordial graph of either even order or even size admit total edge product cordial labeling.*

Proof. Let G be an edge product cordial graph with order p and size q. To prove our claim we consider following three cases.

Case 1: When p is even and q is even.

Since G is edge product cordial graph, $v_f(0) = v_f(1) = \frac{p}{2}$ and $e_f(0) = e_f(1) = \frac{q}{2}$. Therefore, $|\,(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))\,| = 0$.

Case 2: When p is even and q is odd.

Since G is edge product cordial graph, $v_f(0) = v_f(1) = \frac{p}{2}$ and $|e_f(0) - e_f(1)| = 1$. Therefore, $|\,(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))\,| = 1$.

Case 3: When p is odd and q is even.

Since G is edge product cordial graph, $e_f(0) = e_f(1) = \frac{q}{2}$ and $|v_f(0) - v_f(1)| = 1$. Therefore, $|\,(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))\,| = 1$.

Thus in either case G satisfies the condition for total edge product cordial. i.e. G admits total edge product cordial labeling. □

Theorem 2.2. *The graph with degree sequences* $(1, 1), (2, 2, 2, 2)$ *or* $(3, 2, 2, 1)$ *are not total edge product cordial graphs.*

Proof. For the graph with degree sequence $(1, 1)$ has one edge and two vertices. If we label the edge with 1 or 0 then both the vertices will receive the same label. Consequently $|\,(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))\,| = 3$.

For the graph with degree sequence $(2, 2, 2, 2)$ or $(3, 2, 2, 1)$ has four edges and four vertices. If we assign label 0 to any edge then two end vertices will receive label 0 then $v_f(0) + e_f(0) = 3$. If we assign label 0 to two incident edges then three vertices will receive label 0(including a common vertex and two remaining vertices) then $v_f(0) + e_f(0) = 5$. If we assign label 0 to two non-incident edges then four end vertices will receive label 0 consequently $v_f(0) + e_f(0) = 6$. Hence in all situations $|\,(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))\,| > 2$.

Hence, the graph with degree sequences $(1, 1), (2, 2, 2, 2)$ or $(3, 2, 2, 1)$ are not total edge product cordial graphs. □

Theorem 2.3. *The cycle C_n is a total edge product cordial graph except for $n \neq 4$.*

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of cycle C_n. We will consider following two cases.

Case 1: When n is odd.

$$\begin{aligned}
&f(v_i v_{i+1}) = 0; && 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\
&f(v_i v_{i+1}) = 1; && \left\lceil \frac{n}{2} \right\rceil \le i \le n-1 \\
&f(v_1 v_n) = 1. &&
\end{aligned}$$

Case 2: When n is even and $n \neq 4$.

$$\begin{aligned}
&f(v_i v_{i+1}) = 0; && 1 \le i \le \frac{n-4}{2} \\
&f(v_i v_{i+1}) = 1; && i = \frac{n-2}{2} \\
&f(v_i v_{i+1}) = 0; && i = \frac{n}{2} \\
&f(v_i v_{i+1}) = 1; && \frac{n}{2} + 1 \le i \le n-1 \\
&f(v_1 v_n) = 1. &&
\end{aligned}$$

In both the cases we have $v_f(0) + e_f(0) = n$ and $v_f(1) + e_f(1) = n$. So, $|\,(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))\,| \le 1$.

Hence, the cycle C_n is a total edge product cordial graph except for $n \neq 4$. □

Example 2.1. *The cycle C_5 and its total edge product cordial labeling is shown in figure 1.*

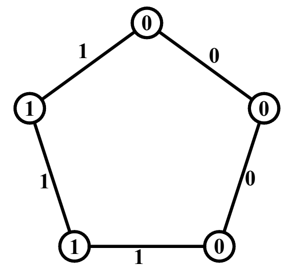

figure 1

Theorem 2.4. *The graph $C_n^{(t)}$ is a total edge product cordial graph.*

Proof. Let $v_{k,1}, v_{k,2}, \ldots, v_{k,n-1}$ be the vertices of k^{th} copy of cycle C_n and v be an common vertex of $C_n^{(t)}$. The vertices $v_{k,1}$ and $v_{k,n-1}$ of k^{th} copy of cycle C_n are adjacent to v. $|V(C_n^{(t)})| = t(n-1)+1$ and $|E(C_n^{(t)})| = tn$. We will consider following three cases.

Case 1: When t is even.

Here $C_n^{(t)}$ is of even size and it is edge product cordial graph as proved by Vaidya and Barasara [9]. Then by Theorem 2.1 result holds.

Case 2: When t and n both are odd.

$$
\begin{array}{lll}
f(v_{i,j}v_{i,j+1}) = 0; & 1 \leq i \leq \dfrac{t-1}{2} & \text{and} \quad 1 \leq j \leq n-2 \\
f(vv_{i,1}) = 0; & 1 \leq i \leq \dfrac{t-1}{2} & \\
f(vv_{i,n-1}) = 0; & 1 \leq i \leq \dfrac{t-1}{2} & \\
f(v_{i,j}v_{i,j+1}) = 1; & \dfrac{t+1}{2} \leq i \leq t-1 & \text{and} \quad 1 \leq j \leq n-2 \\
f(vv_{i,1}) = 1; & \dfrac{t+1}{2} \leq i \leq t-1 & \\
f(vv_{i,n-1}) = 1; & \dfrac{t+1}{2} \leq i \leq t-1 & \\
f(v_{t,i}v_{t,i+1}) = 0; & 1 \leq i \leq \dfrac{n-3}{2} & \\
f(vv_{t,1}) = 0; & & \\
f(v_{t,i}v_{t,i+1}) = 1; & \dfrac{n-1}{2} \leq i \leq n-2 & \\
f(vv_{t,n-1}) = 1. & &
\end{array}
$$

Case 3: When t is odd and n is even.

$$
\begin{array}{lll}
f(v_{i,j}v_{i,j+1}) = 0; & 1 \leq i \leq \dfrac{t-3}{2} & \text{and} \quad 1 \leq j \leq n-2 \\
f(vv_{i,1}) = 0; & 1 \leq i \leq \dfrac{t-3}{2} & \\
f(vv_{i,n-1}) = 0; & 1 \leq i \leq \dfrac{t-3}{2} & \\
f(v_{i,j}v_{i,j+1}) = 0; & i = \dfrac{t-1}{2} & \text{and} \quad 1 \leq j \leq n-2 \\
f(vv_{i,1}) = 0; & i = \dfrac{t-1}{2} & \\
f(vv_{i,n-1}) = 1; & i = \dfrac{t-1}{2} & \\
f(v_{i,j}v_{i,j+1}) = 0; & i = \dfrac{t+1}{2} & \text{and} \quad 1 \leq j \leq \dfrac{n-2}{2} \\
f(vv_{i,1}) = 0; & i = \dfrac{t+1}{2} & \\
f(v_{i,j}v_{i,j+1}) = 1; & i = \dfrac{t+1}{2} & \text{and} \quad \dfrac{n}{2} \leq j \leq n-2 \\
f(vv_{i,n-1}) = 1; & i = \dfrac{t+1}{2} & \\
f(vv_{i,1}) = 1; & \dfrac{t+3}{2} \leq i \leq t & \\
f(v_{i,j}v_{i,j+1}) = 1; & \dfrac{t+3}{2} \leq i \leq t & \text{and} \quad 1 \leq j \leq n-2 \\
f(vv_{i,n-1}) = 1; & \dfrac{t+3}{2} \leq i \leq t &
\end{array}
$$

In case 2 and case 3 we have $v_f(0) + e_f(0) = \dfrac{2nt - t + 1}{2}$ and $v_f(1) + e_f(1) = \dfrac{2nt - t + 1}{2}$. Therefore $|\,(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))\,| \leq 1$. Hence, the graph $C_n^{(t)}$ is a total edge product cordial graph. $\square$

Example 2.2. *The graph $C_4^{(3)}$ and its total edge product cordial labeling is shown in figure 2.*

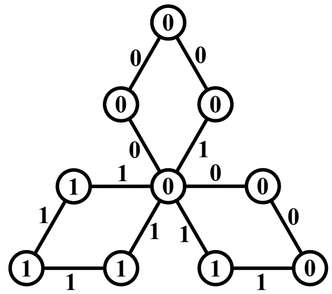

figure 2

Theorem 2.5. *The wheel W_n is a total edge product cordial graph.*

Proof. Let $v_1, v_2, \ldots, v_n$ be the rim vertices and v be an apex vertex of wheel W_n. To define $f : E(W_n) \to \{0, 1\}$ we will consider following two cases.

Case 1: When n is odd.

$$\begin{aligned}
&f(v_{2i-1}v_{2i}) = 0; && 1 \leq i \leq \frac{n-1}{2}\\
&f(v_1v_n) = 0; &&\\
&f(v_{2i}v_{2i+1}) = 1; && 1 \leq i \leq \frac{n-1}{2}\\
&f(vv_i) = 1; && 1 \leq i \leq n.
\end{aligned}$$

In view of the above defined labeling pattern we have $v_f(0) + e_f(0) = \dfrac{3n+1}{2}$ and $v_f(1) + e_f(1) = \dfrac{3n+1}{2}$.

Case 2: When n is even.

$$\begin{aligned}
&f(v_{2i-1}v_{2i}) = 0; && 1 \leq i \leq \frac{n}{2}\\
&f(v_1v_n) = 1; &&\\
&f(v_{2i}v_{2i+1}) = 1; && 1 \leq i \leq \frac{n-2}{2}\\
&f(vv_i) = 1; && 1 \leq i \leq n.
\end{aligned}$$

In view of the above defined labeling pattern we have $v_f(0) + e_f(0) = \dfrac{3n}{2}$ and $v_f(1) + e_f(1) = \dfrac{3n}{2} + 1$. Thus in both the cases we have $|\,(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))\,| \leq 1$. Hence, the wheel W_n is a total edge product cordial graph. $\square$

Example 2.3. *The wheel W_5 and its total edge product cordial labeling is shown in figure 3.*

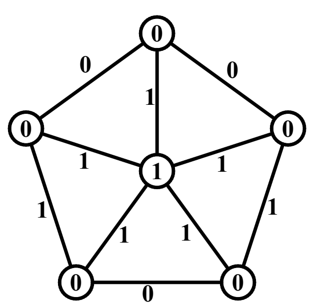

figure 3

Theorem 2.6. *The gear graph G_n is a total edge product cordial graph.*

Proof. Let $v_1, v_2, \ldots, v_{2n}$ be the rim vertices and v is apex vertex of gear graph G_n. To define $f : E(G_n) \to \{0, 1\}$ we will consider following two cases.

Case 1: When n is odd.

$$\begin{array}{ll} f(v_iv_{i+1}) = 0; & 1 \leq i \leq n-1 \\ f(v_iv_{i+1}) = 1; & n \leq i \leq 2n-1 \\ f(v_1v_{2n}) = 1; & \\ f(vv_{2i-1}) = 0; & 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\ f(vv_{2i-1}) = 1; & \left\lceil \frac{n}{2} \right\rceil + 1 \leq i \leq n. \end{array}$$

In view of the above defined labeling patten we have $v_f(0) + e_f(0) = \frac{5n+1}{2}$ and $v_f(1) + e_f(1) = \frac{5n+1}{2}$.

Case 2: When n is even.

Subcase 1: When $n \equiv 0 \pmod 4$.

$$\begin{array}{ll} f(vv_{2i-1}) = 0; & 1 \leq i \leq n \\ f(v_{2i-1}v_{2i}) = 0; & 1 \leq i \leq \frac{n}{4} \\ f(v_1v_{2n}) = 1; & \\ f(v_{2i}v_{2i+1}) = 1; & 1 \leq i \leq \frac{n}{4} \\ f(v_iv_{i+1}) = 1; & \frac{n}{4} + 1 \leq i \leq 2n-1. \end{array}$$

Subcase 2: When $n \equiv 2 \pmod 4$.

$$\begin{array}{ll} f(vv_{2i-1}) = 0; & 1 \leq i \leq n \\ f(v_{2i-1}v_{2i}) = 0; & 1 \leq i \leq \frac{n-2}{4} \\ f(v_2v_3) = 0; & \\ f(v_{2i}v_{2i+1}) = 1; & 2 \leq i \leq \frac{n-2}{4} \\ f(v_iv_{i+1}) = 1; & \frac{n+2}{4} \leq i \leq 2n-1 \\ f(v_1v_{2n}) = 1. & \end{array}$$

In subcase 1 and subcase 2 we have $v_f(0) + e_f(0) = \frac{5n}{2} + 1$ and $v_f(1) + e_f(1) = \frac{5n}{2}$. Thus in both the cases we have $|\,(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))\,| \leq 1$. Hence, the gear graph G_n is a total edge product cordial graph. □

Example 2.4. *The gear graph G_5 and its total edge product cordial labeling is shown in figure 4.*

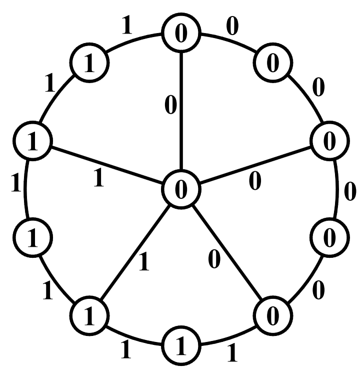

figure 4

Theorem 2.7. *The complete graph K_n admits total edge product cordial labeling for $n > 2$.*

Proof. For complete graph K_n, $|V(K_n)| = n$ and $|E(K_n)| = \frac{n(n-1)}{2}$.Hence total number of elements in K_n is $\frac{n(n+1)}{2}$. For $m < n$, K_m is a subgraph of K_n. Now we search for the smallest integer m for which $\left\lceil \frac{n(n+1)}{4} \right\rceil \leq \frac{m(m+1)}{2}$. Denote $\frac{m(m+1)}{2} - \left\lceil \frac{n(n+1)}{4} \right\rceil$ by l and assign label 0 to $\frac{m(m-1)}{2} - l$ edges of subgraph K_m and assign label 1 to all the remaining edges of supergraph K_n. Then $v_f(0) = m$, $e_f(0) = \frac{m(m-1)}{2} - l$, $v_f(1) = n - m$ and $e_f(1) = \frac{n(n-1)}{2} - \frac{m(m-1)}{2} + l$. Thus

$|(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))|$

$$= \left|\left(m + \frac{m(m-1)}{2} - l\right) - \left(n - m + \frac{n(n-1)}{2} - \frac{m(m-1)}{2} + l\right)\right|$$
$$= \left|\left(\frac{m(m+1)}{2} - l\right) - \left(\frac{n(n+1)}{2} - \frac{m(m+1)}{2} + l\right)\right|$$
$$= \left|\left\lceil\frac{n(n+1)}{4}\right\rceil - \left(\frac{n(n+1)}{2} - \left\lceil\frac{n(n+1)}{4}\right\rceil\right)\right|$$
$$= \left|\left\lceil\frac{n(n+1)}{4}\right\rceil - \left\lfloor\frac{n(n+1)}{4}\right\rfloor\right|$$
$$\leq 1.$$

Hence, K_n admits total edge product cordial labeling for $n > 2$. □

Example 2.5. *The complete graph K_5 and its total edge product cordial labeling is shown in figure 5. Here $m = 4$ and $l = 2$.*

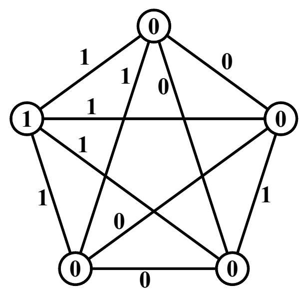

figure 5

Remark 2.1. *There is no possibility for any forbidden subgraph characterization for total edge product cordial labeling as K_n admits total edge product cordial labeling.*

Theorem 2.8. *The complete bipartite graph $K_{m,n}$ is a total edge product cordial graph except $K_{1,1}$ and $K_{2,2}$.*

Proof. For complete bipartite graph $K_{m,n}$, $|V(K_{m,n})| = m+n$ and $|E(K_{m,n})| = mn$. Therefore total number of elements in $K_{m,n}$ is $m+n+mn$. Without loss of generality assume that $m \leq n$. Let $v_1, v_2, \dots, v_m$ be the vertices of one partite set and $u_1, u_2, \dots, u_n$ be the vertices of other partite set. We will consider following two cases.

Case 1: When $m = 1$ and $n > 1$.

$K_{1,n}$ is a tree of either even order or even size. But Vaidya and Barasara [7] have proved that all trees of order greater than 2 are edge product cordial graph. Hence the result holds from Theorem 2.1.

Case 2: When $m > 2$.

For $l < n$, $K_{m,l}$ is a subgraph of $K_{m,n}$. Now we search for the largest integer l for which $m + l + ml \leq \left\lfloor\frac{m+n+mn}{2}\right\rfloor$. Let $r = \left\lfloor\frac{m+n+mn}{2}\right\rfloor - (m + l + ml)$. We define $f : E(K_{m,n}) \to \{0,1\}$ as follows.

$$\begin{array}{lll} f(v_iu_j) = 0; & 1 \leq i \leq m & \text{and} \quad 1 \leq j \leq l \\ f(v_iu_{l+1}) = 0; & 1 \leq i \leq r-1 & \\ f(v_iu_{l+1}) = 1; & r \leq i \leq m & \\ f(v_iu_j) = 1; & 1 \leq i \leq m & \text{and} \quad l+2 \leq j \leq n. \end{array}$$

In view of the above defined labeling pattern we have $v_f(0) + e_f(0) = \left\lfloor\frac{m+n+mn}{2}\right\rfloor$ and $v_f(1) + e_f(1) = \left\lceil\frac{m+n+mn}{2}\right\rceil$. Therefore, $|\,(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))\,| \leq 1$.

Hence, the complete bipartite graph $K_{m,n}$ is a total edge product cordial graph except $K_{1,1}$ and $K_{2,2}$. □

Example 2.6. *The complete bipartite graph $K_{3,4}$ and its total edge product cordial labeling is shown in figure 6. Here $m = 3$, $n = 4$. Hence $l = 1$ and $r = 2$. For which $v_f(0) = 5$, $e_f(0) = 4$, $v_f(1) = 2$ and $e_f(1) = 8$. Therefore, $|\,(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))\,| = 1$*

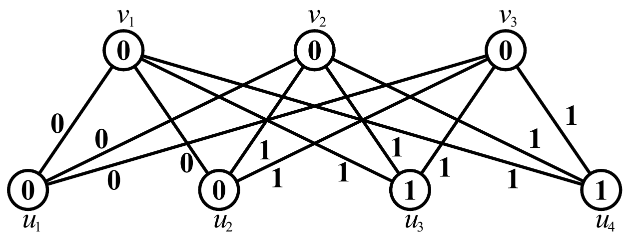

figure 6

Theorem 2.9. *The fan f_n is a total edge product cordial graph.*

Proof. Let v be an apex vertex and $v_1, v_2, \ldots, v_n$ be the other vertices of the fan f_n. To define $f : E(f_n) \to \{0, 1\}$ we will consider following two cases.

Case 1: When n is odd.

$$\begin{aligned}
&f(v_i v_{i+1}) = 0; \quad && 1 \leq i \leq \frac{n-1}{2} \\
&f(v_i v_{i+1}) = 1; && \frac{n+1}{2} \leq i \leq n-1 \\
&f(vv_i) = 0; && 1 \leq i \leq \frac{n-1}{2} \\
&f(vv_i) = 1; && \frac{n+1}{2} \leq i \leq n.
\end{aligned}$$

In view of the above defined labeling pattern we have $v_f(0) + e_f(0) = \dfrac{3n+1}{2}$ and $v_f(1) + e_f(1) = \dfrac{3n-1}{2}$.

Case 1: When n is even.

$$\begin{aligned}
&f(v_i v_{i+1}) = 0; \quad && 1 \leq i \leq \frac{n-2}{2} \\
&f(v_i v_{i+1}) = 1; && \frac{n}{2} \leq i \leq n-1 \\
&f(vv_i) = 0; && 1 \leq i \leq \frac{n}{2} \\
&f(vv_i) = 1; && \frac{n+2}{2} \leq i \leq n.
\end{aligned}$$

In view of the above defined labeling pattern we have $v_f(0) + e_f(0) = \dfrac{3n}{2}$ and $v_f(1) + e_f(1) = \dfrac{3n}{2}$.

Thus in both the cases we have $|\,(v_f(0) + e_f(0)) - (v_f(1) + e_f(1))\,| \leq 1$.

Hence, the fan f_n is a total edge product cordial graph. $\square$

Example 2.7. *The fan f_4 and its total edge product cordial labeling is shown in figure 7.*

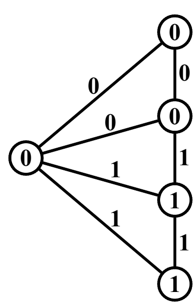

figure 7

Theorem 2.10. *The double fan Df_n is a total edge product cordial graph.*

Proof. Let v and u be vertices with degree $n-1$ and $v_1, v_2, \ldots, v_n$ be the other vertices of the double fan $Df(n)$. We define $f : E(Df_n) \to \{0, 1\}$ as follows.

$$\begin{aligned}
&f(vv_i) = 0; \quad && 1 \leq i \leq n \\
&f(v_i v_{i+1}) = 1; && 1 \leq i \leq n-1 \\
&f(uv_i) = 1; && 1 \leq i \leq n.
\end{aligned}$$

In view of the above defined labeling pattern we have $v_f(0)+e_f(0) = 2n+1$ and $v_f(1)+e_f(1) = 2n$. Therefore, $|\,(v_f(0)+e_f(0)) - (v_f(1)+e_f(1))\,| \leq 1$.

Hence, the double fan Df_n is a total edge product cordial graph. □

Example 2.8. *The double fan Df_4 and its total edge product cordial labeling is shown in figure 8.*

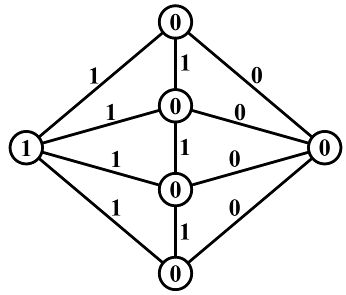

figure 8

3 Concluding remarks

Labeling of discrete structure is a potential area of research. We have introduced the concept of total edge product cordial labeling and derive several results on it. To investigate analogous results for various graphs as well as in the context of different graph labeling problems is an open area of research.

References

[1] I. Cahit, Cordial Graphs: A weaker version of graceful and harmonious graphs, *Ars Combin.*, 23(1987), 201-207.

[2] J. A. Gallian, *A Dynamic Survey of Graph Labeling*, The electronic journal of combinatorics, 19, #DS6, 2012.

[3] S. W. Golomb, *How to number a graph* in: R. C. Read(ed.), Graph theory and Computing, Academic Press, New York, 1972, 23-37.

[4] R. L. Graham and N. J. A.Sloane, On additive bases and harmonious graphs, *SIAM Journal on Algebraic and Discrete Methods*, 1(4)(1980), 382-404.

[5] A. Rosa, On certain valuations of the vertices of a graph, *Theory of graphs*, International Symposium, Rome, July (1966), Gordon and Breach, New York and Dunod Paris, 1967, 349-355.

[6] M. Sundaram, R. Ponraj, and S. Somasundaram, Total product cordial labeling of graphs, *Bulletin Pure and Applied Sciences (Mathematics & Statistics)*, 25E(2006), 199-203.

[7] S. K. Vaidya and C. M. Barasara, Edge Product Cordial Labeling of Graphs, *J. Math. Comput. Sci.*, 2(5)(2012), 1436-1450.

[8] S. K. Vaidya and C. M. Barasara, Some New Families of Edge Product Cordial Graphs, *Advanced Modeling and Optimization*, 15(1)(2013), 103-111.

[9] S. K. Vaidya and C. M. Barasara, *Some Edge Product Cordial Graphs*, Communicated.

[10] D. B. West, *Introduction to Graph theory*, Prentice - Hall of India, 2001.

[11] R. Yilmaz and I. Cahit, E-cordial graphs, *Ars Combinatoria*, 46(1997), 251-266.

Received: May 06, 2013; *Accepted*: June 03, 2013