Malaya Journal of Matematik, Vol. S, No. 1, 41-45, 2018

https://doi.org/10.26637/MJM0S01/08

Performance analysis of a juice packaging plant using BFT

Nitin Kumar Sharma[1]*, Sachin Kumar[2] and Neelam Sharma[3]

Abstract

In this paper we are analyzing the performance of a juice packaging plant by finding various reliability parameters. The juice plant under consideration consists of a total of 11 components/ subsystems operating. According to working of them, a mathematical model has been developed and using Boolean function technique (BFT) expressions for various reliability parameters are obtained. At last a numerical example is included to study effect of various failures over the performance of the system.

Keywords

SVT, RPT, BFT, Series and Parallel configuration, Reliability and MTTF.

[1,2,3] *KIET Group of Institutions, Ghaziabad, Uttar Pradesh, India.*

***Corresponding author**: [1] nitmaths@yahoo.co.in;

Article History: Received **24** December **2017** ; Accepted **21** January **2018**

Contents

1. Introduction

With the development of technology and modernization of our society juice manufacturing and packaging business has attracted various industries and now a days so many companies are in this business of juice packaging. At present customers are having options of selecting best of available brands which is due to the reliability created by their product quality as well as availability and cost of the product. So each company wants to improve the performance of their plant.

There are so many techniques available to solve mathematical model such as supplementary variable technique (SVT), Regenerative point technique (RPT) and Boolean function technique (BFT) etc. [8] use supplementary variable to find out various reliability parameters of a three state repairable system, [9,11,12] used Boolean function technique in their papers to obtain expressions for various reliability parameters. [5] in his work discus the working of a juice plant and explain various stages involved in the juice packaging process. Keeping all these in view the authors has taken the problem of juice packaging plant and developed a mathematical model of same which consists of a total of 11 components/ subsystems operating. According to working of them, a mathematical model has been developed Boolean function technique (BFT) has been used to solve the model. Expression for Reliability and MTTF are obtained. At last a numerical example is included to study effect of components working over the performance of the system.

There are 11 components/ subsystems denoted by a_i, where $i = 1, 2, 3, \ldots, 11$ in the juice packaging plant under consideration. The raw fruits are used in extraction section which consists of two extractor units a_1, a_2 to extract juice. Juice is stored in two storage tanks a_3, a_5 which are in standby arrangement with perfect switching a_4. When a_3 is not in action, identical unit a_5 start to operate. Now juice is pumped forward in Processing unit a_6 for heat treatment and refining. Now the juice can be used to produce different variety as per its utility like ready to drink juice (RTD), concentrated juice. So in production section (a_7, a_9), the unit a_7 operate for blending and formulation of ready to drink juice (RTD) and unit a_9 operate for preparing juice concentrate by means of evaporation. At this section necessary ingredients like sugar, preservatives, etc. are mixed in product. Here a perfect switching a_8 is also used between unit $a_7 a_9$ to control the separation of processes. Now we pasteurize the products at unit a_{10} and forward to the packaging unit a_{11} where both the products are packed, as ready to drink juice (RTD) packed in bottle and concentrate packed in cans and made ready to supply in market.

State configuration of juice plant under consideration is given below:

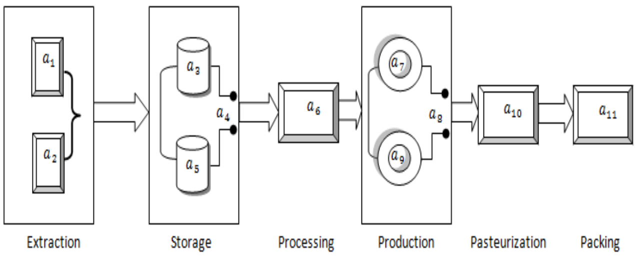

Figure 1

2. Assumptions or Notations

The determination of reliability of model described in this paper, using Boolean Function Technique, the following assumptions are (taken into account) made:

(1) All the components/subsystems are in good and operable condition, initially.

(2) The Failure times are arbitrary for all the components.

(3) The state of all components is statistically independent.

(4) In the system, each of the components is either in good or in failed state.

(5) Reliability of every component or stage is known in advance.

(6) The repair facility for any failed component is not available.

(7) The whole system will fail if there is failure in any one of its units.

3. Mathematical Notations

a_1, a_2: States of extraction unit

a_3, a_5: States of storage tanks

a_4,a_8: States of Perfect switching devices

a_6: State of Processing unit

a_7,a_9: States of Production section

a_{10}: State of Pasteurizing unit

a_{11}: State of Packaging unit

a_i: State of components which is equal to 1/0 in good/failed state $\forall\ i = 1, 2, 3, \ldots, 11$.

a_i': Negation of a_i

$\wedge$: Conjunction

$||$: Logical Matrix

R_i: Reliability of ith component of the system.

Q_i: $1 - R_i$, $\forall\ i = 1, 2, 3, \ldots, 11$

R_s: Reliability of the whole system.

R_{SE}/R_{SW}: Reliability of ith component of the system.

4. Formulation of Mathematical Model

Incorporating Boolean Function Technique, The possible successful operating conditions of juice plant in terms of logical matrix are:

$$F(a_1, a_2, \ldots, a_{11}) = \left|\begin{array}{cccccccc} a_1 & a_3 & a_6 & a_7 & a_{10} & a_{11} & & \\ a_1 & a_3 & a_6 & a_8 & a_9 & a_{10} & a_{11} & \\ a_1 & a_4 & a_5 & a_6 & a_7 & a_{10} & a_{11} & \\ a_1 & a_4 & a_5 & a_6 & a_8 & a_9 & a_{10} & a_{11} \\ a_2 & a_3 & a_6 & a_7 & a_{10} & a_{11} & & \\ a_2 & a_3 & a_6 & a_8 & a_9 & a_{10} & a_{11} & \\ a_2 & a_4 & a_5 & a_6 & a_7 & a_{10} & a_{11} & \\ a_2 & a_4 & a_5 & a_6 & a_8 & a_9 & a_{10} & a_{11} \end{array}\right| \tag{4.1}$$

5. Solution of Model

By means of algebra of logics, Equation (4.1) can be expressed as:

$$F(a_1, a_2, \ldots, a_{11}) = [a_{11} \wedge a_{10} \wedge f(a_1, a_2, a_3, \ldots, a_{11})] \tag{5.1}$$

where $f(a_1, a_2, a_3, \ldots, a_{11})$

$$= \left|\begin{array}{cccccc} a_1 & a_3 & a_6 & a_7 & & \\ a_1 & a_3 & a_6 & a_8 & a_9 & \\ a_1 & a_4 & a_5 & a_6 & a_7 & \\ a_1 & a_4 & a_5 & a_6 & a_8 & a_9 \\ a_2 & a_3 & a_6 & a_7 & & \\ a_2 & a_3 & a_6 & a_8 & a_9 & \\ a_2 & a_4 & a_5 & a_6 & a_7 & \\ a_2 & a_4 & a_5 & a_6 & a_8 & a_9 \end{array}\right| = \left|\begin{array}{c} L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \\ L_6 \\ L_7 \\ L_8 \end{array}\right| \tag{5.2}$$

where $L_1 = \left| \begin{array}{cccc} a_1 & a_3 & a_6 & a_7 \end{array} \right|$

$L_2 = \left| \begin{array}{ccccc} a_1 & a_3 & a_6 & a_8 & a_9 \end{array} \right|$

$L_3 = \left| \begin{array}{ccccc} a_1 & a_4 & a_5 & a_6 & a_7 \end{array} \right|$

$L_4 = \left| \begin{array}{cccccc} a_1 & a_4 & a_5 & a_6 & a_8 & a_9 \end{array} \right|$

$L_5 = \left| \begin{array}{cccc} a_2 & a_3 & a_6 & a_7 \end{array} \right|$

$L_6 = \left| \begin{array}{ccccc} a_2 & a_3 & a_6 & a_8 & a_9 \end{array} \right|$

$L_7 = \left| \begin{array}{ccccc} a_2 & a_4 & a_5 & a_6 & a_7 \end{array} \right|$

$L_8 = \left| \begin{array}{cccccc} a_2 & a_4 & a_5 & a_6 & a_8 & a_9 \end{array} \right|$

By orthogonalization algorithm, we may write (5.2) as

$$f(a_1,a_2,a_3,\ldots,a_{11}) = \left|\begin{array}{llllllll} L_1 \\ L_1' & L_2 \\ L_1' & L_2' & L_3 \\ L_1' & L_2' & L_3' & L_4 \\ L_1' & L_2' & L_3' & L_4' & L_5 \\ L_1' & L_2' & L_3' & L_4' & L_5' & L_6 \\ L_1' & L_2' & L_3' & L_4' & L_5' & L_6' & L_7 \\ L_1' & L_2' & L_3' & L_4' & L_5' & L_6' & L_7' & L_8 \end{array}\right| \tag{5.3}$$

Now $L_1 = \left|a_1 \quad a_3 \quad a_6 \quad a_7\right|$

$$L_1'L_2 = \left|\begin{array}{llll} a_1' \\ a_1 & a_3' \\ a_1 & a_3 & a_6' \\ a_1 & a_3 & a_6 & a_7 \end{array}\right| \wedge \left|a_1 \quad a_3 \quad a_6 \quad a_8 \quad a_9\right|$$

$L_1'L_2 = \left|a_1 \quad a_3 \quad a_6 \quad a_7' \quad a_8 \quad a_9\right|$

$L_1'L_2'L_3 = \left|a_1 \quad a_3' \quad a_4 \quad a_5 \quad a_6 \quad a_7\right|$

$L_1'L_2'L_3'L_4 = \left|a_1 \quad a_3' \quad a_4 \quad a_5 \quad a_6 \quad a_7' \quad a_8 \quad a_9\right|$

$L_1'L_2'L_3'L_4'L_5 = \left|a_1 \quad a_2 \quad a_3 \quad a_6 \quad a_7\right|$

$L_1'L_2'L_3'L_4'L_5'L_6 = \left|a_1' \quad a_2 \quad a_3 \quad a_6 \quad a_7' \quad a_8 \quad a_9\right|$

$L_1'L_2'L_3'L_4'L_5'L_6'L_7 = \left|a_1' \quad a_2 \quad a_3' \quad a_4 \quad a_5 \quad a_6 \quad a_7\right|$

$L_1'L_2'L_3'L_4'L_5'L_6'L_7'L_8 = \left|a_1' \quad a_2' \quad a_3 \quad a_4 \quad a_5 \quad a_6 \quad a_8 \quad a_9\right|$

Putting all in (5.3), we get

$$f(a_1,a_2,a_3,\ldots,a_{11}) = \left|\begin{array}{llllllll} a_1 & a_3 & a_6 & a_7 \\ a_1 & a_3 & a_6 & a_7' & a_8 & a_9 \\ a_1 & a_3' & a_4 & a_5 & a_6 & a_7 \\ a_1 & a_3' & a_4 & a_5 & a_6 & a_7' & a_8 & a_9 \\ a_1 & a_2 & a_3 & a_6 & a_7 \\ a_1' & a_2 & a_3 & a_6 & a_7' & a_8 & a_9 \\ a_1' & a_2 & a_3' & a_4 & a_5 & a_6 & a_7 \\ a_1' & a_2' & a_3 & a_4 & a_5 & a_6 & a_8 & a_9 \end{array}\right| \tag{5.4}$$

Therefore (5.1) becomes

$$f(a_1,a_2,a_3,\ldots,a_{11}) \tag{5.5}$$

$$= \left|\begin{array}{llllllllll} a_1 & a_3 & a_6 & a_7 & a_{10} & a_{11} \\ a_1 & a_3 & a_6 & a_7' & a_8 & a_9 & a_{10} & a_{11} \\ a_1 & a_3' & a_4 & a_5 & a_6 & a_7 & a_{10} & a_{11} \\ a_1 & a_3' & a_4 & a_5 & a_6 & a_7' & a_8 & a_9 & a_{10} & a_{11} \\ a_1 & a_2 & a_3 & a_6 & a_7 & a_{10} & a_{11} \\ a_1' & a_2 & a_3 & a_6 & a_7' & a_8 & a_9 & a_{10} & a_{11} \\ a_1' & a_2 & a_3' & a_4 & a_5 & a_6 & a_7 & a_{10} & a_{11} \\ a_1' & a_2' & a_3 & a_4 & a_5 & a_6 & a_8 & a_9 & a_{10} & a_{11} \end{array}\right| \tag{5.6}$$

This equation is the disjoint of disjoint conjunctions; therefore we can calculate the reliability of whole system which is given by

$$R_S = P_r\{f(a_1,a_2,a_3,\ldots,a_{11})\} = 1 \tag{5.7}$$

or,

$$\begin{aligned} R_S = R_{10}R_{11}\{&R_1R_3R_6R_7 + R_1R_3R_6Q_7R_8R_9 \\ &+ R_1Q_3R_4R_5R_6R_7 + R_1Q_3R_4R_5R_6Q_7R_8R_9 \\ &+ R_1R_2R_3R_6R_7 + Q_1R_2R_3R_6Q_7R_8R_9 \\ &+ Q_1R_2Q_3R_4R_5R_6R_7 + Q_1Q_2R_3R_4R_5R_6R_8R_9 \end{aligned} \tag{5.8}$$

where R_i is the reliability of ith state of the system and $Q_i = 1 - R_i, \forall\, i = 1,2,3,\ldots,11$.

$$\begin{aligned} R_S = {} & R_1R_3R_6R_7R_{10}R_{11} + R_1R_3R_6R_8R_9R_{10}R_{11} \\ & - R_1R_3R_6R_7R_8R_9R_{10}R_{11} + R_1R_4R_5R_6R_7R_{10}R_{11} \\ & - R_1R_3R_4R_5R_6R_7R_{10}R_{11} + R_1R_4R_5R_6R_8R_9R_{10}R_{11} \\ & - R_1R_3R_4R_5R_6R_8R_9R_{10}R_{11} - R_1R_4R_5R_6R_7R_8R_9R_{10}R_{11} \\ & + R_1R_3R_4R_5R_6R_7R_8R_9R_{10}R_{11} + R_1R_2R_3R_6R_7R_{10}R_{11} \\ & + R_2R_3R_6R_8R_9R_{10}R_{11} - R_1R_2R_3R_6R_8R_9R_{10}R_{11} \\ & - R_2R_3R_6R_7R_8R_9R_{10}R_{11} + R_1R_2R_3R_6R_7R_8R_9R_{10}R_{11} \\ & + R_2R_4R_5R_6R_7R_{10}R_{11} - R_1R_2R_4R_5R_6R_7R_{10}R_{11} \\ & - R_2R_3R_4R_5R_6R_7R_{10}R_{11} + R_1R_2R_3R_4R_5R_6R_7R_{10}R_{11} \\ & + R_3R_4R_5R_6R_8R_9R_{10}R_{11} - R_1R_3R_4R_5R_6R_8R_9R_{10}R_{11} \\ & - R_2R_3R_4R_5R_6R_8R_9R_{10}R_{11} + R_1R_2R_3R_4R_5R_6R_8R_9R_{10}R_{11} \end{aligned}$$

$$\begin{aligned} R_S = {} & x_1 + x_2 - y_1 + x_3 - y_2 + x_4 - y_3 - y_4 + x_5 \\ & + x_6 + x_7 - y_5 - y_6 + x_8 + x_9 - y_7 - y_8 + x_{10} \\ & + x_{11} - y_9 - y_{10} + x_{12} \end{aligned} \tag{5.9}$$

Case – (1) when reliability of each component is R, Then (5.9) gives:

$$R_S = 2R^6 + 3R^7 - 4R^8 - 2R^9 + 2R^{10} \tag{5.10}$$

Case – (5.1) when failure rates follow Weibull time distribution:

Let m_i is the rate of failure for ith state of system where $i, j = 1,2,3,\ldots,11$, then reliability of system at any time "t", is calculated by

$$R_{SW}(t) = \sum_{i=1}^{12} \exp\{-x_it^p\} - \sum_{j=1}^{10} \exp\{-y_jt^p\} \tag{5.11}$$

where p is a parameter $(p > 0)$ and x_i, y_j, are given above

Case – (3) when failure rates follow Exponential time distribution:

Taking $p = 1$ in case (5.1), it will become Exponential time distribution, then we have

$$R_{SE}(t) = \sum_{i=1}^{12} \exp\{-x_it\} - \sum_{j=1}^{10} \exp\{-y_jt\},\ldots,. \tag{5.11}$$

Where x_i, y_j taken as in case (5.1).

In this case, the expression for another reliability parameter i.e. M.T.T.F. given by

$$M.T.T.F = \int_0^\infty R_{SE}(t)dt = \sum_{i=1}^{12}\left[\frac{1}{x_i}\right] - \sum_{j=1}^{10}\left[\frac{1}{y_j}\right] \tag{5.12}$$

6. Numerical Calculations

For Numerical computation, let us setting the values as:

(1) $m_i = 0.001$, $t = 0,1,2,\ldots$, $p = 2$ in equation (5.11) where $i = 1,2,3,\ldots,11$.

(2) $m_i = 0.001$, $t = 0,1,2,\ldots$ in equation (5.11) where $i = 1,2,3,\ldots,11$.

(3) $m_i = 0.0, 0.1, 0.2, \ldots, 1.0$ in equation (5.12) where $i = 1,2,3,\ldots,11$.

The computed results are given in Table 1 and Table 2 and graphically shown in Figure 2 and Figure 3 respectively.

Table 1

Sr. No.	t	R_{SW}	R_{SE}
1	0	1.0000000000	1.0000000000
2	1	0.9970005075	0.9970005075
3	2	0.9880084829	0.9940020602
4	3	0.9730460385	0.9910047034
5	4	0.9521593699	0.9880084829
6	5	0.9254331315	0.9850134446
7	6	0.8930106164	0.9820196345
8	7	0.8551185369	0.9790270991
9	8	0.8120937128	0.9760358849
10	9	0.7644075017	0.9730460385
11	10	0.7126828900	0.9700576069
12	11	0.6576992298	0.9670706371
13	12	0.6003808648	0.9640851762

Table 2

m	MTTF
0.01	40.6349206349
0.02	20.3174603175
0.03	13.5449735450
0.04	10.1587301587
0.05	8.1269841270
0.06	6.7724867725
0.07	5.8049886621
0.08	5.0793650794
0.09	4.5149911817
0.1	4.0634920635

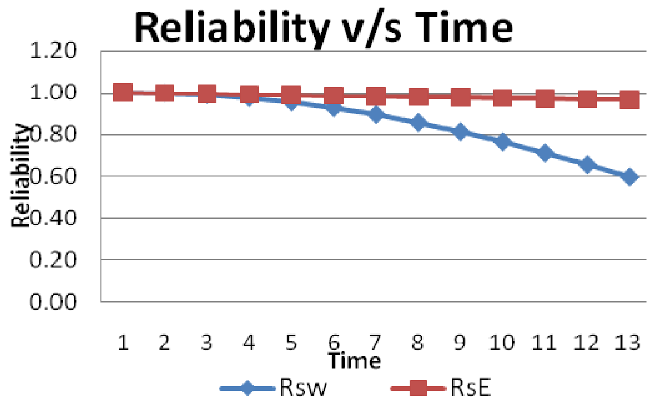

Figure 2

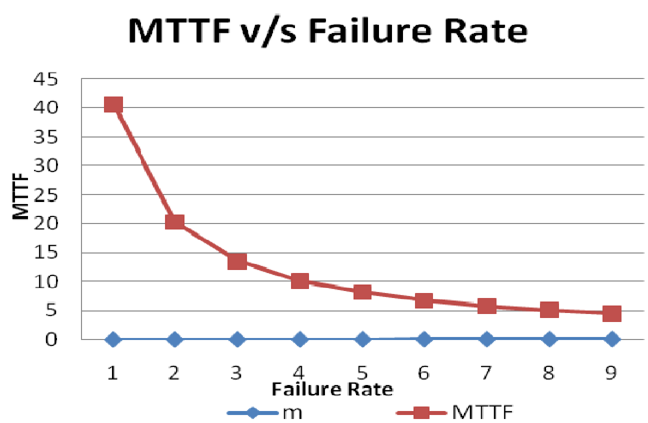

Figure 3

7. Conclusion

The author has analyzed a **Juice Packaging Plant** for determining various reliability parameters by applying the Boolean Function Technique & algebra of logics.

In Table 1, Reliability of system is computed with respect to time when failure rates follow Weibull time distribution and exponential time distribution. The graph of "Reliability v/s Time" in the figure-2 depicts that the reliability of the complex system decreases approximately at a uniform rate in case of Exponential Time Distributions, but it decreases very rapidly, in case when failure rates follow Weibull Time Distributions.

In Table 2, MTTF is calculated and the graph "MTTF v/s Failure Rate" in figure-3 depicts that MTTF of the system decreases catastrophically in beginning but later it decreases approximately at a uniform rate.

References

[1] E Balagurusamy, *Reliability Engineering*, Tata McGraw – Hill Education, 1984.

[2] Marvin Rausanda, *System Reliability Theory; Models, Statistical Methods, and Applications*, 2nd edition, John Wiley & Sons, Inc., Publication, 2004.

[3] M. Van Der Spiegel and P.A. Luning, W.J. De Boer, G.W. Ziggers and W.M.F. Jongen, *How to Improve Food Quality Management in the Bakery Sector*, NJAS 53-2, pp. 131-150, 2005.

[4] D. Sharma and N. Sharma, *Evaluation Of Some Reliability Parameters For Tele-Communication System By Boolean Function Technique*, International Journal of Research and Reviews in Applied Sciences, 1 (3) (December 2009).

[5] J.J.H. Fey, *Design of a Fruit Blending and Packaging Plant*, Ph.D. Thesis, University of Technology, Eindhoven, 2000.

[6] S. Fang-Ming and H. Pin-Han, *Reliability optimization of distributed access networks with constrained total cost*, IEEE Trans. on Reliability, 54 (3), 221-230, 2005.

[7] S.H. Sim, and J. Endrehyi, *A failure repair model with minimal and major maintenance. IEEE Trans. On Reliability*, 55 (1), 134-140, 2006.

[8] S. Kumar and A. Sachin, *Evaluation of some reliability parameters of a three state repairable system with environmental failure*, International Journal of Research and Reviews in Applied Sciences, 1 (2) (November 2009).

[9] S. Bansal, S.C. Agarwal and K. Sharma, *Evaluation of Reliability Factors Using Boolean Function Technique in Milk Powder Manufacturing Plant*, IJRRAS, 4 (4) (September 2010), 416–424.

[10] S. Gulati, P.K. Yadav and K. Bhatia, *A Reliability Model for the Task Schedule in Distributed System based on Fuzzy Theory*, CIIT Int. Journel of Networking and Commuication Engineering, 4 (11), 684-688 (August 2012).

[11] M. Kumar and R. Avtar, *Reliability Analysis of Rice Mill by Boolean Function Technique*, IJETT, 4 (5) (May 2013, 1947-1953.

[12] J. Singh and R.A. Jaswal, *Evaluation Of Reliability Parameter Of Thermal Power Plant By BFT,* Int. Journal of Adv. Engg. Tech/IV/III, 79-81, July-Sept 2013.

[13] N. Yadav and R.B. Yadav, *Reliability Estimation In Milk Powder Manufacturing Plant Mathematics Subject Classification*, IJERIA, 7 (II) (May 2014), 15-31.

[14] A.K. Aggarwal, V. Singh and S. Kumar, *Availability Analysis and Performance Optimization of a Butter Oil Production System: A Case Study*, Int. Journal of Sys. Assur, Engg. and Management, Springer, Nov. 2014.

[15] M. Ahmad, *A Case Study of Maintenance Management System for Bread Baking Plant*, Pak. Journal of Engg. Tech. & Science, 5 (1), 1-12, 2015.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319-3786
Malaya Journal of Matematik
ISSN(O):2321-5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆