Malaya Journal of Matematik, Vol. 6, No. 3, 483-484, 2018

https://doi.org/10.26637/MJM0603/0004

# $A$-perfect lattice

Seema Bagora[1*]

**Abstract**

In the Paper [3] authors define the concept of $A$-perfect Group. Inspired by [3], we give a new concept of $A$-perfect lattice. If $g \in L$ and $\alpha \in A$, then the element $[g;\alpha] = g^{-1}\alpha(g)$ is an auto commutator of $g$ and $\alpha$, if is taken to be an inner automorphism, then the autocommutator sublattice is the derived sublattice $L'$ of $L$. A lattice $L$ is said to be perfect if $L = L'$. Here, the perception of A-perfect lattices would be introduced. A lattice L would be known as $A$-perfect, if $L = K(L)$.

**Keywords**

Perfect lattice, $A$-perfect group, finite abelian group.

**AMS Subject Classification**

03G10, 11E57.

[1] *Department of Applied Mathematics, Shri Vaishnav Vidyapeeth Vishwavidyalaya, Gram Baroli, Sanwer Road, Indore (M.P.) 453331 India.*

*Corresponding author: [1] bagoraseema@gmail.com

**Article History**: Received **21** March **2018**; Accepted **19** April **2018**



## Contents



## 1. Introduction

In 1877, Korkine and Zolotareff [3] introduced the concept of a Perfect Lattice. According to them, a perfect lattice (or perfect form) is a lattice in an Euclidean vector space, that is completely determined by the set $S$ of its minimal vectors in the sense that there is only one positive definite quadratic form taking value 1 at all points of $S$. Inspired by the results of Korkine and Zolotareff [3], in this paper we give the concept of $A$-perfect lattice.

Let $L$ be a lattice and $A =$Aut$(L)$ represent the lattice of automorphisms of $L$. If $g \in L$ and $\alpha \in A$, then the element $[g;\alpha] = g^{-1}\alpha(g)$ is an auto commutator of $g$ and $\alpha$. Now, we define the auto commutator sublattice of $L$ as $K(L) = [L;A] = g^{-1}\alpha(g), g \in L, \alpha \in A$ which is a characteristic sublattice of $L$. Particularly, if $\alpha$ is taken to be an inner automorphism, then the autocommutator sublattice is the derived sublattice $L'$ of $L$. A lattice $L$ is said to be perfect if $L = L'$. Here, the perception of $A$-perfect lattices will be introduced.

**Definition 1.1.** *A lattice L would be known as A-perfect, if* $L = K(L)$.

## 2. Some Important Results

**Theorem 2.1.** *Let H and T be two lattices. Suppose, the following conditions are satisfied:*

*(i)* $K(H) \times K(T) \subseteq K(H \times T)$;

*(ii) H and T are such that* $(|H|;|T|) = 1$.

*Then,* $K(H) \times K(T) = K(H \times T)$.

*Proof.* (i) For $\alpha \in$Aut$(H)$ and $\beta \in$Aut$(T)$ we define the automorphism of lattice $H \times T$, given by

$$(\alpha \times \beta(h,t)) = \alpha(t)\beta(t) \quad \forall\, h \in H,\ t \in T.$$

It is easy to check that $[h;\alpha];[t;\beta] = [(h;t);\alpha \times \beta]$. This implies the result.

(ii) It is sufficient to prove $K(H \times T) \subseteq K(H) \times K(T)$. It is easy to check that $\lambda/H \in$Aut$(H)$ and $\lambda/T \in$Aut$(T)$, for all $\lambda \in$Aut$(H \times T)$. Now

$$[(h;\lambda t);\lambda] = ([h;\lambda H];[t;\lambda/T]), \forall h \in H, t \in T, \text{Aut}(H \times T).$$

This implies the result. □

**Theorem 2.2.** *For all nonnegative integers* $n > m_1 \geq m_2 \geq \cdots \geq m_k$.

**Corollary 2.3.** *If G is a finite abelian group of odd order, then G is A-perfect.*

*Proof.* $L$ is a direct product of finitely many $Z_{p^t}$, where $p$ is an odd prime number and $t \geq 1$. Hence, the result is true due to previous theorem. □

**Theorem 2.4.** *For all nonnegative integers $n > m_1 \geq m_2 \geq \cdots \geq m_k$*

$$K(Z_{2^n} \times Z_{2^{m_1}} \times \cdots \times Z_{2^{m_k}}) = Z_{2^n} \times Z_{2^{m_1}} \times \cdots \times Z_{2^{m_k}}.$$

**Theorem 2.5.** *For all nonnegative integers $n > m_1 \geq m_2 \geq \cdots \geq m_k$*

$$K(Z_{2^n} \times Z_{2^n} \times Z_{2^{m_1}} \times \cdots \times Z_{2^{m_k}}) = Z_{2^n} \times Z_{2^n} \times Z_{2^{m_1}} \times \cdots \times Z_{2^{m_k}}.$$

*Proof.* We define the automorphisms $\alpha, \alpha', \beta_1, \ldots, \beta_k$ of the lattice $L$ given by:

$$\begin{aligned}
\alpha(a,b,c_1,\ldots,c_k) &= (a+b,c_1,\ldots,c_k) \\
\alpha'(a,b,c_1,\ldots,c_k) &= (a,a+b,c_1,\ldots,c_k) \\
\beta_1(a,b,c_1,\ldots,c_k) &= (a,b,a+c_1,c_2,\ldots,c_k) \\
&\vdots \\
\beta_k(a,b,c_1,\ldots,c_k) &= (a,b,c_1,c_2,\ldots,a+c_k).
\end{aligned}$$

for all $a,b \in \{0,1,2,\ldots,2^n-1\}$ and $c_i \in \{0,1,2,\ldots,2^{m_i}-1\}$, $1 \leq i \leq k$. Clearly,

$$\begin{aligned}
(a,0,\ldots,0) &= [(0,a,0,\ldots,0),\alpha],(0,b,0,\ldots,0) \\
&= [(b,0,\ldots,0),\alpha'] \\
(0,0,c_1,0,\ldots,0) &= [(c_1,0,\ldots,0),\beta_1] \\
(0,0,0,c_2,0,\ldots,0) &= [(c_2,0,\ldots,0),\beta_2] \\
&\vdots \\
(0,0,\ldots,0,c_2) &= [(c_k,0,\ldots,0),\beta_k].
\end{aligned}$$

These imply that

$$\begin{aligned}
&K(Z_{2^n} \times Z_{2^n} \times Z_{2^{m_1}} \times \cdots \times Z_{2^{m_k}}) \\
\supseteq\ & Z_{2^n} \times Z_{2^n} \times Z_{2^{m_1}} \times \cdots \times Z_{2^{m_k}}.
\end{aligned}$$

□

**Theorem 2.6.** *A finite abelian lattice $L$ is A-perfect if and only if*

$$L \approx Z_{2^n} \times Z_{2^{m_1}} \times \cdots \times Z_{2^{m_k}} \times M.$$

*for some nonnegative integers $n > m_1 \geq m_2 \geq \cdots \geq m_k$, where $M$ is a finite lattice of odd order.*

*Proof.* The necessary condition follows from Theorem 2.5. Now, for the reverse conclusion, we assume that $L$ is not a product of $Z_{2^n} \times Z_{2^{m_1}} \times \cdots \times Z_{2^{m_k}} \times M$, so it is $Z_{2^t} \times Z_{2^{s_1}} \times \cdots \times Z_{2^{s_k}} \times N$ where $N$ is a finite abelian lattice of odd order. Theorem 2.1 implies that

$$\begin{aligned}
&K(Z_{2^t} \times Z_{2^{s_1}} \times \cdots \times Z_{2^{s_k}} \times N) \\
&= K(Z_{2^t} \times Z_{2^{s_1}} \times \cdots \times Z_{2^{s_k}}) \times K(N).
\end{aligned}$$

Now, the lattice $L$ is not $A$-perfect due to previous. It completes the proof. □

## Acknowledgment

Author is thankful to Dr. Satish Shukla for his kind support and guidance for research work.

## References

[1] C. Chis, M. Chis, and G. Silberberg, Abelian groups as autocommutator group, Arch. Math. (Basel), **90** no. 6 (2008) 490–492.

[2] P. Hegarty, The absolute centre of a group, J. Algebra, **169** no. 3 (1994) 929–935.

[3] M.M. Nasrabadi and A Gholamian, on finite A-perfect abelian Groups, International Journal of group theory, **1**, no.3 (2012) 11–14.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆