



On weights of 2-repeated low-density bursts of length b (fixed)

Barkha Rohtagi

Abstract

The ever increasing demand for efficient and reliable data transmission and storage systems motivates the need of more reliable and efficient codes, so that large amount of data can be stored and transmitted efficiently. In a very busy communication channel due to large amount of noise, bursts repeat themselves. In such type of error patterns, it is quite obvious that not all digits in a burst are in error. Such repeated bursts are termed as low-density repeated bursts. In this paper some combinatorial results on 2-repeated low-density bursts of length b (fixed) for calculating their weights are derived.

Keywords

Repeated burst errors, low density repeated burst errors, weight consideration of repeated burst errors.

AMS Subject Classification

03D32.

Department of Applied Sciences, KIET, Ghaziabad, India.

Corresponding author: barkha.rohtagi@gmail.com

Article History: Received 11 March 2018; Accepted 12 June 2018

©2018 MJM.

Contents

1	Introduction	582
2	Results on weights of 2-repeated low-density bursts of length b (fixed)	583
3	Conclusion	584
	References	584

1. Introduction

In communication over a certain memory channel; the errors occur in the form of bursts. The work in this area was initiated by Abramson [1]. He developed codes for correcting single and double adjacent errors. Fire [6] generalized the concept of adjacent errors by introducing 'burst errors'.

A burst of length b is defined as follows [6]:

Definition 1.1. A burst of length b is a vector whose only non-zero components are among some b consecutive components, the first and the last of which are non-zero.

It is obvious that the behavior of the bursts depend upon the nature of the channel through which message is being communicated. Chien and Tang [3] noticed that in many channels errors occur in the form of a burst but not to the end digits of the burst. This type of errors generally occurs in the

channels due to Alexander, Gryb and Nast [2]. In literature such bursts are called CT bursts.

A CT burst [3] is defined as follows:

Definition 1.2. A burst of length b is a vector whose only non-zero components are among some b consecutive positions, the first of which is non-zero.

Dass [4] modified this definition, by imposing constraint on starting positions, as follows:

Definition 1.3. A burst of length b (fixed) is an n -tuple whose only non-zero components are confined to b consecutive positions, the first of which is non-zero and the number of its starting positions in an n -tuple is the first $(n - b + 1)$ components.

It is obvious that when density of noise is very high in a channel, errors repeat themselves. These repeated burst error patterns are termed as repeated burst errors. The concept of repeated burst errors was introduced by Dass, Garg and Zannetti [5]. They termed such a burst error as an ' m -repeated burst of length b (fixed)' and defined as follows [5]:

Definition 1.4. An m -repeated burst of length b (fixed) is a vector whose only non-zero components are among m distinct sets of b consecutive digits, the first component of each set is non-zero and the number of its starting positions is among the first $(n - mb + 1)$ components.

It is generally seen that in some channels, due to various disturbances or noise, during the process of data transmission over a given length of a burst some digits are received correctly while some are not. These type of errors are termed as low-density bursts [9]. In the view of this concept, Dass and Garg [6] obtained results for detection and detection/correction of 2-repeated low-density bursts of length b (fixed) with weight w or less ($w \leq b$) and for m -repeated low-density bursts of length b (fixed) with weight w or less ($w \leq b$).

Following is the definition of 2-repeated low-density bursts of length b (fixed) with weight w or less ($w \leq b$) as given by Dass and Garg [6]:

Definition 1.5. A 2-repeated low-density burst of length b (fixed) with weight w or less is a vector whose only non-zero components are among two distinct sets of b successive positions, the first component of each set is non-zero where each set can have at most w non-zero entries ($w \leq b$), and the number of its starting positions is among the first $(n - 2b + 1)$ components.

For example, (000001000011010) is a 2-repeated low-density burst of length at most 5 (fixed) with weight 3 or less.

Since weight consideration plays an important role in determining the error detection and correction capabilities of a code and also helpful in proposing decoding procedure for a code, several results have been obtained regarding weights of bursts. But the study of mathematical formulation of weights of open-loop bursts was initiated by Sharma and Dass [8].

In this paper we obtained results regarding the weights and the number of low-density repeated burst errors of length b (fixed) with particular weights. These results will be useful in developing codes for correcting and detecting repeated low-density bursts of length b (fixed) by economizing the number of parity-checks required in comparison of the usual open-loop burst error correcting and/or detecting codes while considering such repeated bursts.

This paper is organized as follows:

1. In Section 1 basic definitions along with brief introduction related to our study are given.
2. In Section 2, results for weight calculations of 2-repeated low-density bursts of length b (fixed) are derived.
3. Section 3 presents conclusion of the chapter.

2. Results on weights of 2-repeated low-density bursts of length b (fixed)

In this section the results are obtained for 2-repeated low-density bursts of length b (fixed).

Following lemma is a result for counting the numbers of 2-repeated low-density bursts of length b (fixed).

Lemma 2.1. *The total number of vectors having 2-repeated low-density bursts of length b (fixed) with weight w ($1 \leq w \leq b$)*

in the space of all n -tuples is given by

$$\frac{(n - 2b + 1)(n - 2b + 2)}{2} [L_{w,q}^{b-1}]^2, \tag{2.1}$$

where

$$\left[\sum_{s=1}^w \binom{b-1}{s-1} (q-1)^s \right] = L_{w,q}^{b-1} \tag{2.2}$$

is the incomplete binomial expansion of $[1 + (q - 1)]^{b-1}$ up to the $(q - 1)^w$ in the ascending powers of $(q - 1)$, $w \leq b$.

Proof. Let us consider a vector having 2-repeated low-density bursts of length b (fixed) with weight at most w . Its only nonzero components are confined to two distinct sets of b consecutive components, the first component of each set is nonzero, where each set can have at most w non-zero components ($w \leq b$), and the number of its starting positions is among the first $n - 2b + 1$ components.

Now, each of these, the first component of each set may be any of the $q - 1$ nonzero field elements. As we are considering only 2-repeated low-density bursts of length b (fixed) with weight at most w , in a vector of length n , this will have non-zero positions as follows:

1. First position of first burst.
2. First position of second burst.
3. Some $r - 1$ amongst the $b - 1$ in-between positions of first burst ($1 \leq r \leq w$) and then some $s - 1$ in the in-between $b - 1$ positions of the second burst ($1 \leq s \leq w$).
4. Other positions have the value 0.

Thus analyzing in combinatorial ways, in the earlier counting factor $[(q - 1)q^{b-1}]^2$ replacing one factor q^{b-1} by $\binom{b-1}{r-1}(q - 1)^{r-1}$ and the other by $\binom{b-1}{s-1}(q - 1)^{s-1}$ each 2-repeated low-density burst will give its number by:

$$\begin{aligned} & (q - 1)(q - 1) \sum_{r=1}^w \binom{b-1}{r-1} (q - 1)^{r-1} \\ & \cdot \sum_{s=1}^w \binom{b-1}{s-1} (q - 1)^{s-1} \\ & = \sum_{r=1}^w \binom{b-1}{r-1} (q - 1)^r \left[\sum_{s=1}^w \binom{b-1}{s-1} (q - 1)^s \right] \end{aligned}$$

Then from equation (2.2) the number of each 2-repeated low-density burst of length b (fixed) with weight w is given by,

$$[L_{w,q}^{b-1}]^2.$$

Therefore, the total number of 2-repeated low-density bursts of length b (fixed) and weight w , with sum of their starting position $\frac{(n-2b+1)(n-2b+2)}{2}$ is

$$\frac{(n - 2b + 1)(n - 2b + 2)}{2} [L_{w,q}^{b-1}]^2.$$

This proves the lemma. □

Now, we find an expression for W_{2b} , the total weight of all vectors having 2-repeated low-density bursts of length b (fixed) in the space of all n -tuples.



Theorem 2.1. For $n \geq b$

$$W_2 = \frac{n(n-1)}{2}(q-1)^2 \tag{2.3}$$

and

$$W_{2b} = \frac{(n-2b+1)(n-2b+2)}{2}w^2[L_{w,q}^{b-1}]^2. \tag{2.4}$$

Proof. The value of W_2 follows simply by considering all vectors having any two non-zero entries out of n . Their number clearly is given by

$$\binom{n}{2}(q-1)^2 = \frac{n(n-1)}{2}(q-1)^2.$$

This gives the value of W_2 as stated above.

Next, for $b > 1$, using the Lemma 1.1, the total weight of all vectors having 2-repeated low-density bursts of length b (fixed) each with weight of each burst at most w , is given by

$$\begin{aligned} \sum_{i=1}^w \sum_{j=1}^w \frac{(n-2b+1)(n-2b+2)}{2} i[L_{w,q}^{b-1}] \cdot j[L_{w,q}^{b-1}] \\ = \frac{(n-2b+1)(n-2b+2)}{2} w^2 [L_{w,q}^{b-1}]^2 \end{aligned}$$

This completes the proof of the theorem. □

Further, in coding theory, an important criterion is to look for minimum weight in a group of vectors. Our following theorem gives a bound on the largest minimum weight that can be attained by a 2-repeated low-density burst of length $b > 1$ (fixed).

Theorem 2.2. The minimum weight of a vector having 2-repeated low-density burst of length $b > 1$ (fixed) in the space of all n -tuples is at most

$$\left[\frac{wL_{w,q}^{b-1}}{(q-1)q^{b-1}} \right]^2 \tag{2.5}$$

Proof. From Lemma 2.1, it is clear that the number of 2-repeated low-density bursts of length b (fixed) in the space of all n -tuples with symbols taken from the field of q elements is

$$[q^{(b-1)}(q-1)]^2 \frac{(n-2b+1)(n-2b+2)}{2}.$$

Also from Theorem 2.1, their total weight is

$$\frac{(n-2b+1)(n-2b+2)}{2} w^2 [L_{w,q}^{b-1}]^2.$$

Since the minimum weight of an element can at most be equal to the average weight, an upper bound on minimum weight of a 2-repeated low-density burst of length b (fixed) is given by

$$\begin{aligned} \frac{(n-2b+1)(n-2b+2)}{2} w^2 [L_{w,q}^{b-1}]^2 \\ \cdot \frac{2}{(n-2b+1)(n-2b+2)(q-1)^2 q^{2(b-1)}} \\ = w^2 \text{ or } b^2 \quad (\text{because } w \leq b). \end{aligned}$$

This proves the result. □

Note. Similar bound was obtained by Sharma and Dass [8] for open-loop burst.

3. Conclusion

In this paper we developed mathematical formulation for weight calculation of low-density 2- repeated bursts of length b (fixed). Similar study can be done for such error patterns in their respective generalized manner. This paper also presents the generalization of the idea of study of random errors as well as burst errors and also bursts of bursts or more precisely multiple bursts with a different perspective of weight consideration.

References

- [1] N.M. Abramson, A class of systematic codes for non-independent errors, *IEEE Trans. Inf. Theory* 5 (1959), 150-157.
- [2] A.A. Alexander, R.M. Gryb and D.W. Nast, Capabilities of the telephone network for data transmission, *Bell System Tech. J.* 39 (1960), 431-476.
- [3] R.T. Chien and D.T. Tang, On definitions of a burst, *IBM Journal of Research and Development* 9 (4) (1965), 292-293.
- [4] B.K. Dass, On a burst-error correcting code, *Journal of information and Optimization Sciences* 1 (3) (1980), 291-295.
- [5] B.K. Dass, P.Garg and M. Zannetti, Some combinatorial aspects of m -repeated burst error detecting codes, *Journal of Statistical Theory and Practice* 2 (4) (2008), 707-711.
- [6] B.K. Dass and P. Garg, On repeated low-density burst error detecting linear codes, *Mathematical Communication* 16 (2011), 37-47.
- [7] P. Fire, A class of multiple error-correcting binary codes for non-independent errors, *Sylvania Report RSL-E-2, Sylvania Reconnaissance System Laboratory, Mount View, Calif.*, 1959.
- [8] B.D. Sharma and B.K. Dass, On weight of bursts, *Presented at 38th Annul. Conf. of IMS, Bhopal, India*, 1972.
- [9] A.D. Wyner, Low-density-burst-correcting-codes, *IEEE Trans. Inf. Theory* IT-95 (1963), 124.

 ISSN(P):2319 – 3786
 Malaya Journal of Matematik
 ISSN(O):2321 – 5666

