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Equitable edge domination in fuzzy graphs

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Abstract

In this paper, we introduced equitable edge dominating set, equitable edge domination number of fuzzy graphs and also obtain some bounds for equitable edge domination number. Also give some necessary and sufficient conditions for the equitable edge dominating set. Further introduction to connected equitable edge dominating set, connected equitable edge domination number of fuzzy graphs and their some relations are discussed.

Keywords

Strong neighbours, edge dominating set, edge domination number, equitable edge dominating set, equitable edge domination number.

AMS Subject Classification

03E72, 05C72.

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Contents

1. Introduction

Zadeh [9] introduced the concept of fuzzy sets in the year 1965. In 1975, fuzzy graph was introduced by Rosenfeld [6]. The edge domination number was studied in 1987 by Jayaram. S.R.,[4]. The notation of domination in fuzzy graphs was developed by A. Somasundaram and S. Somasundaram [7]. Nagoorgani and Chandrasekaran [5] discussed about domination in a fuzzy graph using strong arcs. The concept of degree equitable domination in graphs was introduced by Venkatasubramanian Swaminathan and Kuppusamy Markandan Dharmalingam [8].Anwar Alwardi and Soner.N.D.,[1] introduced equitable edge domination in graphs in the year 2013.The concept of equitable domination in fuzzy graphs was introduced by Dharmalingam and Rani [2]. We introduce the concept of paired equitable domination in fuzzy graph in 2018 [3].

2. Preliminaries

In this section, we recall some definitions and basic results of fuzzy graph which will be used throughout the paper.

Definition 2.1. *Let $G^* = (V,E)$ be a graph with vertex V and edge set $E \subseteq V \times V$. Let σ and μ be a fuzzy set of V and E respectively. Then $G = (\sigma, \mu)$ be a fuzzy graph if $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all $(u,v) \in E$ and is denoted by $G = (\sigma, \mu)$.*

Definition 2.2. *Let $G = (\sigma, \mu)$ be a fuzzy graph then the order and size are defined as $p = \sum_{u \in V} \sigma(u)$ and $q = \sum_{(u,v)\in E} \mu(u,v)$.*

Definition 2.3. *The neighbourhood degree of a vertex u is defined to be the sum of the weights of the vertices adjacent to u and is denoted by $d_{N(u)}$, the minimum neighbourhood degree of G is $\delta_{N(G)} = min\{d_{N(u)} : u \in V\}$ and the maximum neighbourhood degree of G is $\Delta_{N(G)} = max\{d_{N(u)} : u \in V\}$.*

Definition 2.4. *An arc (u,v) in a fuzzy graph $G = (\sigma, \mu)$ is said to be strong if $\mu^{\infty}(u,v) = \mu(u,v)$ then u,v are called strong neighbours.*

Definition 2.5. *The strong neighbourhood of the vertex u is defined as $N_{S(u)} = \{v \in V | (u,v) is a strong arc\}$.*

Definition 2.6. *Let $G = (\sigma, \mu)$ be a fuzzy graph for any edge $e \in \mu$ of the degree of $e = uv$ in G is defined by $d(e) = d(u) + d(v)\breve{}2\mu(u,v)$, since $(u,v) > 0$ for $uv \in E$, $\mu(u,v) = 0$ for $uv \notin E$.*

3. Equitable edge domination in fuzzy graphs

Definition 3.1. *A set $F \subseteq E$ of a strong edges in equitable edge dominating set of G if every strong edge f not in F is adjacent to atleast one edge $f^{'} \in F$ such that $|d(f)\breve{}d(f^{'})| \leq 1$.*

Definition 3.2. *The minimum cardinality of such equitable edge dominating set is denoted by $\gamma_e^{'}(G)$ and is called a equitable edge domination number G.*

Definition 3.3. *If F is minimal for any edge $f \in F, F\breve{}\{f\}$ is not an equitable edge dominating set of G.*

Definition 3.4. *The maximum equitable degree of edge in G are $\Delta_e^{'}(G) = max_{f \in E}|N_e(f)|$and the minimum equitable degree of edge in G are $\delta_e^{'}(G) = min_{f \in E}|N_e(f)|$.*

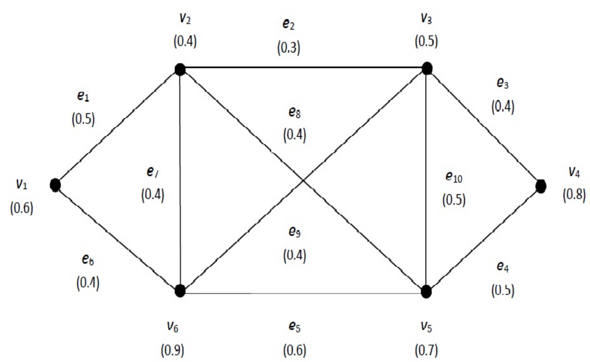

Figure 1

Example 3.5. *From the fuzzy graph G given in figure 1, Equitable edge domination set $F = \{e_4, e_7\}, \gamma_e^{'}(G) = 0.9$.*

Proposition 3.6. *An equitable edge dominating set F is minimal if and only if for each edge $f \in F$ one of the following conditions holds :*

(i) $N_e(f) \cap F = \emptyset$

(ii) *there exists a strong edge $g \in E - F$ such that $N_e(g) \cap F = \{f\}$.*

Proof. Suppose that F is a minimal equitable edge dominating set. Assume that (i) and (ii) do not hold. Then for some $f \in F$ there exists a strong edge $g \in N_e(f) \cap F$ and for every edge be a strong edges $e \in E\breve{}F, N_e(e) \cap F \neq \{f\}$. Therefore $F - \{f\}$ is an equitable edge dominating set but not minimal. Which is a contradiction to the minimality of F, this implies that (i) and (ii) holds. Conversely, suppose for every edges in a fuzzy graph G be strong edges $f \in F$ one of the conditions holds. Suppose F is not minimal, then there exist $f \in F$ such that $F\breve{}\{f\}$ is an equitable edge dominating set. Therefore there exist and strong edge $g \in F - \{f\}$ such that $g \in N_e(f)$. Hence f does not satisfy (i). Then f must satisfy (ii), so that there exists an strong edge $g \in E\breve{}F$ such that $N_e(g) \cap F = \{f\}$. Since $F\breve{}\{f\}$ is an equitable edge dominating set there exist an strong edge $f^{'} \in F\breve{}\{f\}$ such that $f^{'}$ is equitable adjacent to g. Therefore $f^{'} \in N_e(g) \cap F$ and $f^{'} \neq f$, a contradiction to $N_e(g) \cap F = \{f\}$. Hence F is minimal equitable edge dominating set. □

Proposition 3.7. *For any fuzzy graph G, $\gamma^{'}(G) \leq \gamma_e^{'}(G)$*

Proof. It is obvious. □

Proposition 3.8. *For any fuzzy graph G without any equitable isolated edges, if F is minimal equitable edge dominating set then $E\breve{}F$ is equitable edge dominating set.*

Proof. Let F be minimal equitable edge dominating set of G. Suppose $E\breve{}F$ is not an equitable edge dominating set, then there exist an edge f such that $f \in F$ is not equitable adjacent to any strong edge in $E\breve{}F$, since G has no equitable isolated edges then f is equitable dominated by at least one edge in $F\breve{}\{f\}$. Thus $F\breve{}\{f\}$ is equitable edge dominating set a contradiction to the minimality of F. Therefore $E\breve{}F$ is equitable edge dominating set. □

Proposition 3.9. *If G is a regular or bi-regular fuzzy graph then $\gamma^{'}(G) = \gamma_e^{'}(G)$*

Proof. Suppose G is a regular fuzzy graph, it has every edges in G are strong edges. Which has the same degree say a. Let F be an edge dominating set of G. Then $\gamma_e^{'}(G) = |F|$, let us consider $e \in E - F$, then F is an edge dominating set, there exists an strong edge $e^{'} \in F$ and $ee^{'}$ are adjacent. Also $d(e) = d(e^{'}) = a$. Therefore $d(e) - d(e^{'}) = 0 < 1$. Which implies F is an equitable edge dominating set of a fuzzy graph G, so that $\gamma_e^{'}(G) \leq |F| = \gamma^{'}(G)$. But $\gamma^{'}(G) \leq \gamma_e^{'}(G)$. Therefore $\gamma^{'}(G) = \gamma_e^{'}(G)$. □

Proposition 3.10. *For any fuzzy graph G, $\gamma_e^{'}(G) \leq q - \Delta_e^{'}(G)$*

Proof. Let f be an strong edge in a fuzzy graph G of equitable degree $\Delta_e^{'}(G)$. Clearly $E(G)\breve{}N_e(f)$ is an equitable edge dominating set. Hence $\gamma_e^{'}(G) \leq q - \Delta_e^{'}(G)$. □

Corollary 3.11. *For any fuzzy graph G, $\gamma_e^{'}(G) \leq q - \delta_e^{'}(G)$*

Proposition 3.12. *For a fuzzy star G, $\gamma_e^{'}(G) = min\{\mu(e_i); e_i \in E\}$*

Proof. Let G be fuzzy star and let each edges will be a strong edges and all edges will incident to a vertex v, which is every edge will dominate remaining edges of a fuzzy star G. Hence $\gamma_e^{'}(G) = min\{\mu(e_i); e_i \in E\}$. □

4. Equitable independent edge domination in fuzzy graphs

Definition 4.1. *An edge dominating set F is called an equitable independent edge dominating set if no two strong edges in F are equitable adjacent.*

Definition 4.2. *The equitable independent edge domination number $\gamma'_{ei}(G)$ is the minimum cardinality taken over all equitable independent edge dominating set of G.*

Definition 4.3. *The edge independence number $\beta'_e(G)$ is defined to be the number of edges in a maximum equitable independent set of edges of G.*

Example 4.4. *From the fuzzy graph G given in figure 1, Equitable independent edge domination set $F = \{e_7, e_{10}\}, \gamma'_{ei}(G) =$ 0.9*

Proposition 4.5. *An equitable independent set F is maximal equitable independent set if and only if it is equitable edge independent and equitable edge dominating set of a fuzzy graph G.*

Proof. Suppose an equitable independent set F is maximal. Then for every edge $f \in E-F$, the set $F \cup \{f\}$ is not equitable independent, that is for every edge $f \in E-F$, there is an strong edge $g \in F$ such that f is equitable adjacent to g. Thus F is equitable edge dominating set. Hence F is both equitable edge independent and equitable edge dominating set of a fuzzy graph G. Conversely, suppose F is both equitable edge independent and equitable edge dominating set of G. Suppose F is not maximal equitable edge independent set. Then there exist an edge $f \in E-F$ such that $F \cup \{f\}$ is equitable independent, and then there is no strong edge in F equitable adjacent to f. Hence F is not equitable edge dominating set which is a contradiction. Hence F is maximal equitable independent set. □

Proposition 4.6. *For any γ'_e- set F of a graph $G = (\sigma, \mu), |E-F| \leq \Sigma_(f \in F)d_e(f)$ and the equality holds if and only if*

(i) *F is equitable independent*

(ii) *for every edge $f \in E\check{}F$ there exists only one strong edge $g \in F$ such that $N_e(f) \cap F = \{g\}$*

Proof. Since each edge in $E\check{}F$ is equitable adjacent to at least one edge of F. Therefore each edge in $E\check{}F$ contributes at least one to the sum of the equitable degrees of the edges of F. Hence $|E-F| \leq \Sigma(f \in F)d_e(f)$. Let $|E-F| \leq \Sigma(f \in F)d_e(f)$ and suppose that F is not equitable independent. Clearly each edge in $E\check{}F$ is counted in the sum $\Sigma_(f \in F)d_e(f)$. Hence if f_1 and f_2 are equitable adjacent, then f_1 is counted in $d_e(f_1)$ and vice versa. Then the sum exceeds $|E-F|$ be at least two contrary to the hypothesis. Hence F must be equitable independent. Now suppose (ii) is not true. Then $N_e(f) \cap F \geq 2$ for some edge $f \in E-F$. Let f_1 and f_2 belong to $N_e(f) \cap F$, hence $\Sigma_(f \in F)d_e(f)$ exceeds $E-F$ by at least one since f_1 counted twice once in $d_e(f_1)$ and once in $d_e(f_2)$. Hence if the equality holds then the condition (i) and (ii) must be true. The converse is obvious. □

Proposition 4.7. *If $E-F$ is equitable edge dominating set for every minimal edge dominating set F of a fuzzy graph G is without equitable isolated edges.*

Proof. Suppose $E-F$ is not equitable edge dominating set for F be minimal equitable edge dominating set of a fuzzy graph G, there exist an strong edge f such that f is not equitable adjacent to any edge in $E-F$. We have G has no equitable isolated edges, f is equitable adjacent to at least one strong edge in $F-\{f\}$ is equitable edge dominating set. Which contradicts to the minimal equitable edge dominating set of F for a fuzzy graph G. Hence $E-F$ is equitable edge dominating set of G. □

Proposition 4.8. *For any fuzzy graph $G = (\sigma, \mu)$ of order p and size q, $\frac{q}{\Delta'_e(G)+1} \leq \gamma'_e(G) \leq q - \beta'_e + q_0$ where q_0 is the total sum of equitable isolated edges.*

Proof. From the previous theorem, $q - \gamma'_e(G) \leq \sum_{(f \in F)} d_e(f) \leq \gamma'_e(G)\Delta'_e$. Which implies $\frac{q}{\Delta'_e(G)+1} \leq \gamma'_e(G)$. Let $G' = \langle E(G)\check{}I_e(G)\rangle$ where $I_e(G)$ is the set of equitable isolated edges of G. Let S be the maximal equitable independent set of edges of G'. Hence S is also equitable edge dominating set of G'. Since G' does not have equitable isolated edges $E(G')\check{}S$ is also equitable edge dominating set of G'. Therefore $\gamma'_e(G') \leq |E(G')\check{}S| = q(G') - \beta'_e(G')$. But $\gamma'_e(G') = \gamma'_e(G) - q_0$ and $\beta'_e(G') = \beta'_e(G) - q_0$. Hence $\gamma'_e(G) - q_0 \leq q(G) - q_0 - (\beta'_e(G) - q_0)$ then $\gamma'_e(G) \leq q - \beta'_e + q_0$ from the above results we have $\frac{q}{\Delta'_e(G)+1} \leq \gamma'_e(G) \leq q - \beta'_e + q_0$. □

5. Connected equitable edge domination in fuzzy graphs

Definition 5.1. *An equitable edge dominating set F of a fuzzy graph G is connected equitable edge dominating set if the induced subgraph $\langle F \rangle$ is connected.*

Definition 5.2. *The connected equitable edge domination number γ'_{ce} of G is the minimum cardinality of a connected equitable edge dominating set.*

Example 5.3. *From the fuzzy graph G given in figure 2, Connected equitable edge domination set $F = \{e_2, e_3, e_4\}, \gamma'_{ei}(G) =$ 1.1*

Observation 5.4. *Every connected equitable edge dominating set is an equitable edge dominating set.*

Proposition 5.5. *For any fuzzy graph G, $\gamma'(G) \leq \gamma'_e(G) \leq \gamma'_{ce}(G)$*

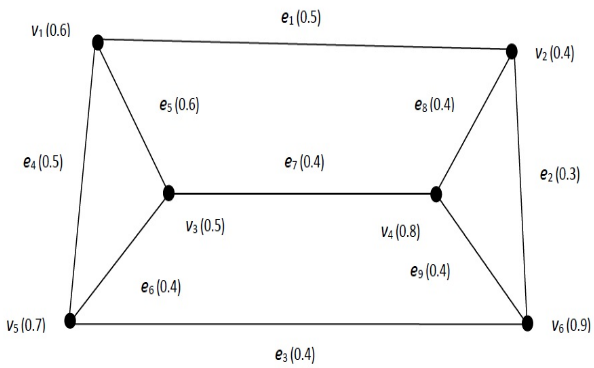

Figure 2

Proof. This is obviously true. □

Proposition 5.6. *For any connected fuzzy graph G of order p then* $\gamma'_{ce}(G) \leq p - \Delta_e(G)$.

Proof. Let T be a spanning tree of G. If u is an end vertex of T then its strong edges are incident with u form a connected equitable edge dominating set of G. Hence $\gamma'_{ce}(G) \leq p - \Delta_e(G)$. □

Corollary 5.7. *For any connected fuzzy graph G of order p then* $\gamma'_{ce}(G) \leq p - \delta_e(G)$.

Proposition 5.8. *An connected equitable edge dominating set F of a fuzzy graph G is minimal if and only if for each edge* $e \in F$ *one of the following conditions holds :*

(i) $N_e(e) \cap F = \emptyset$

(ii) *there exists a strong edge* $f \in E - F$ *such that* $N_e(f) \cap F = \{e\}$.

Proof. Let F be a minimal connected equitable edge dominating set and $e \in F$ then $F_e = F\check{~}\{e\}$ is not an connected equitable edge dominating set and hence there exists $f \in E - F_e$ such that f is not dominated by any strong edges of F, if $f = e$ then $N(e) \cap D = \emptyset$ and if $f \neq e$ then there exists an strong edge $f \in E - F$ such that $N(f) \cap F = \{e\}$ and f is an strong edge. The converse is always true. □

Proposition 5.9. *If G be a fuzzy graph without isolated edges and p be order and q be the size of G then* $\frac{q}{\Delta'(G)+1} \geq \gamma'_{ce}(G)$

Proof. Let F be a connected equitable edge dominating set of G. Since $|F|\Delta'(G) \leq \sum_(e \in F) d_e(e) = \sum_(e \in F)|N(e)| \leq |\cup_(e \in F)N(e)| \leq |E - F| \leq q - |F|, |F|\Delta'(G) + |F| \leq q$ hence $\frac{q}{\Delta'(G)+1} \geq \gamma'_{ce}(G)$. □

Proposition 5.10. *For any fuzzy graph G,* $\gamma'_{ce}(G) \leq q - \Delta'(G)$

6. Conclusion

In this paper, we discuss and establish equitable edge domination in fuzzy graph. Also derive some relation between equitable edge domination and other parameters. Also extend this result in future.

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