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Product fuzzy distance two labeling graph and its properties

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Abstract
Graph theoretical concepts are hugely exploited by the applications of computer science. Mainly in the field of research in computer science like communication networking (wired or wireless), image capturing, data base management etc. Fuzzy labeling graphs provide more accuracy, flexibility, and affinity to the system in comparison of standard and fuzzy graphs. These graphs are extremely applicable in the area of Computer Science, Physics, Chemistry and other branches of mathematics. In view of this article a new thought of product fuzzy distance two labeling graph has been established. This paper considers some complement properties of product fuzzy distance two labeling graph.

Keywords
Product fuzzy distance two labeling graph, product fuzzy graph, complement of product fuzzy distance two labeling graph.

AMS Subject Classification
45J05.

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Contents

1. Introduction

There are several explanations behind the speeding up of enthusiasm for graph theory. It has wound up popular to say that there are several uses of graph theory for taking care of combinatorial problems in various regions, for example, operation research, structural and electrical engineering, number theory, topology, algebra and computer science. Euler became the father of graph theory when in 1736 he ended a popular unsolved issue of his day called the Konigsberg Bridge Problem is considered to be the beginning of graph theory. The idea of fuzzy sets and fuzzy relations on a set was first explained by Zadeh in 1965 [9] is a mathematical implement for handling doubt like unclearness, imprecision and uncertainty in the linguistic variables. Fuzzy sets can be described mathematically by allocating a value to every conceivable individual in the universe of address (defining its grade of membership), which approaches to the degree, to which that individual is comparative or adaptable with the idea interpreted by the fuzzy set.

In 1973, Kaufmann introduced the principal meaning of a fuzzy graph, in view of Zadeh's definition of fuzzy relations in 1971. A more detailed definition is a result of Azriel Rosenfeld [1] who designed fuzzy relation theory on fuzzy sets and composed the hypothesis of fuzzy graphs structure in 1975 and got few graph theoretical ideas for bridges and trees. The various connectedness ideas in theory of fuzzy graphs and its applications introduced by Yeh and Bang [13], during the same time. Further Ramaswamy and Poornima [15] introduced product fuzzy graphs and demonstrated few outcomes which are closely related to fuzzy graphs. Nagoorgani et. al [4] discussed the properties of fuzzy labeling Graphs.

Mathematically in graph hypothesis, a graph labeling is the allocation of labels (commonly represented by an integer) to the graph vertices or graph edges or both. This article is a further contribution on fuzzy labeling graphs.

In fuzzy graph theory the assignment of fuzzy graph vertices and fuzzy graph edges has great importance in various interesting problems like traffic light problem, job allocation issues etc. In view of assignment of fuzzy graph vertices and fuzzy graph edges, a new concept of product fuzzy distance two labeling graph has been established in this research article. In this paper the characterization of product fuzzy distance two labeling graph is detailed as access to fuzzy labeling graph and some properties of complement of product fuzzy distance two labeling graph have been discussed.

2. Preliminaries

In this part, we review some basic definitions and fundamental results of fuzzy graphs that will be used in this research.

A fuzzy set F defined on a non-empty set X is characterized by a mapping $m : X \to [0,1]$, which is called the membership function such that $m(x) = 1$ if $x \in F$, $m(x) = 0$ if $x \notin F$ and and any intermediate value represents the degree in which x could belong to F. Fuzzy set is usually denoted by $F = (X,m)$. A crisp graph G a finite non-empty set of objects called vertices of G together with a set of unordered pair of distinct vertices of G called edges. The vertex set and the edge set of G are denoted by $V(G)$ or V_G and $E(G)$ or E_G respectively. A fuzzy graph $F(G) = (\sigma_V, \mu_E)$ is a pair of functions $\sigma_V : V_G \to [0,1]$ and $\mu_E : V_G \times V_G \to [0,1]$ such that for all $p,q \in V_G$ we have $\mu_E(p,q) \leq \sigma_V(p) \bigwedge \sigma_V(q)$ where $\bigwedge$ stands for minimum.Also, we denote the underlying crisp graph of $F(G)$ by $G^\star = (\sigma_V^\star, \mu_E^\star) = G^\star(V_G, E_G)$ where $\sigma_V^\star = \{p \in V_G : \sigma_V(p) > 0\}$ and $\mu_E^\star = \{(p,q) \in V_G \times V_G : \mu_E(p,q) > 0\}$.

Any sequence of distinct vertices $x_1, x_2, \ldots\ldots, x_n$ such that $\mu_E(x_r, x_{r+1}) > 0; 1 \leq r \leq n-1$ is called a path P in a fuzzy graph $F(G)$ and here $n > 1$ is defined as the length of the path P. The consecutive pairs (x_r, x_{r+1}) are called the arcs of the path P. If $x_1 = x_n$ and $n \geq 3$ then the path P is known as a cycle. Two nodes in a fuzzy graph $F(G)$ that are joined by a path are said to be connected. The strength of a path P defined to be the degree of membership of a weakest edge of the path P i.e the strength of a path P can be defined as $\bigwedge_{r=1}^{n-1} \mu_E(x_r, x_{r+1})$.

The most extreme value of the strength of all distinct paths between the nodes p and q in the fuzzy graph $F(G) = (\sigma_V, \mu_E)$ is characterized as the strength of connectedness between the nodes p and q and it can be written as $\mu_E^\infty(p,q)$. An edge in any fuzzy graph $F(G) = (\sigma_V, \mu_E)$ is said to be strong if its membership value is at least as maximum as the strength of connectedness of its end vertices when it is erased. If only strong edges appears in a path is defined as a strong path in fuzzy graph$F(G)$. An edge is known as a fuzzy bridge of the graph $F(G)$ if its expulsion diminishes the strength of connectedness between some combine nodes in graph $F(G)$. A node defines to be a fuzzy cut node of the fuzzy graph $F(G)$ if ejection of it decreases the strength of connectedness for some other pair of vertices. If a node in a graph $F(G)$ has only single strong neighbour is define a fuzzy end node.

Order of a fuzzy graph $F(G) = (\sigma_V, \mu_E)$ is defined as order $\sum_{s \in V_G} \sigma_V(s)$. The size of any fuzzy graph $F(G) = (\sigma_V, \mu_E)$ is defined as size $\sum_{t,s \in V_G} \mu_E(t,s)$. The degree of any vertex t in the fuzzy graph $F(G) = (\sigma_V, \mu_E)$ is defined by $d_{F(G)}t = \sum_{t \neq s; s \in V_G} \mu_E(t,s)$. The degree of an edge (p,q) in the fuzzy graph $F(G) = (\sigma_V, \mu_E)$ is denoted by $d_{F(G)}(p,q)$ and defined by $d_{F(G)}(p,q) = d_{F(G)}(p) + d_{F(G)}(q) - 2\mu_E(p,q)$.

3. Product Fuzzy Distance two Labeling Graph

In this portion, new theory related to product fuzzy distance two labeling graphs have been established. We, first begin by reviewing the following definitions.

Definition 3.1. *Suppose $G^\star = (V_G, E_G)$ is a underlying crisp graph, σ_V denote a fuzzy subset of V_G and μ_E denote a fuzzy subset of $V_G \times V_G$. We call graph $F(G) = (\sigma_V, \mu_E)$ be a product fuzzy graph if $\mu_E(p,q) \leq \sigma_V(p) \times \sigma_V(q)$ for all $p,q \in V_G$.*

Figure 1 illustrates a product fuzzy graph $F(G) = (\sigma_V, \mu_E)$ where $\sigma_V = \{a|0.2, b|0.3, c|0.4, d|0.5\}]$ and $\mu_E = \{(a,b)|0.015, (b,c)|0.05, (a,d)|0.04, (c,d)|0.11\}$.

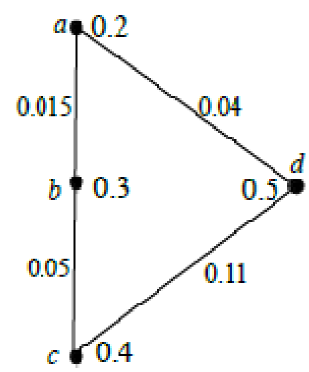

Figure.1: Product Fuzzy Graph

Definition 3.2. *Any product fuzzy graph $F(G) = (\sigma_V, \mu_E)$ with underlying crisp graph $G^\star = (V_G, E_G)$ is said to be complete product fuzzy graph if $\mu_E(p,q) = \sigma_V(p) \times \sigma_V(q)$ for all $p,q \in V_G$.*

Definition 3.3. *A graph $F(G)^P = (\sigma_V^P, \mu_E^P)$ with underlying crisp graph $G^\star = (V_G, E_G)$ represents a product fuzzy distance two labeling graph, if $\sigma_V^P : V_G \to [0,1]$ and $\mu_E^P : E_G \to [0,1]$ $(E_G = V_G \times V_G)$ are membership functions such that edges and vertices of the graph has distinct assignment and satisfying the following condition*

(i)- $\mu_E^P(p,q) < \sigma_V^P(p) \times \sigma_V^P(q)$ for each $p,q \in V_G$.

(ii)- $|\sigma_V^P(p) - \sigma_V^P(q)| \geq \mu_E^P(p,q)$ if $d(p,q) = 1$

and $|\mu_E^P(p,z) - \mu_E^P(z,q)| \leq \sigma_V^P(z)$ *if* $d(p,q) = 2$ *where* z *is a vertex lies on the path joined by* p *and* q.

Figure 2 illustrate a product fuzzy distance two labeling graph $F(G)^P = (\sigma_V^P, \mu_E^P)$ *where function* $\sigma_V^P = \{s|0.6, l|0.4, m|0.3, n|0.5, r|0.2, t|0.7, x|0.9\}$ *and* $\mu_E^P = \{(l,m)|0.1, (m,n)|0.12, (n,r)|0.09, (x,t)|0.17, (l,x)|0.35, (s,l)|0.19\}$.

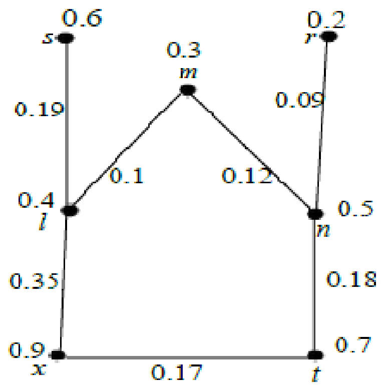

Figure.2: Product Fuzzy Distance two Labeling Graph

Remark 3.4. *Every product fuzzy distance two labeling graph is also a product fuzzy graph but a product fuzzy graph may or may not be a product fuzzy distance two labeling graph.*

Remark 3.5. *A complete product fuzzy graph cannot be a product fuzzy distance two labeling graph and converse also.*

4. Complement of Product Fuzzy Distance two Labeling Graph

In this part, we explain some important theorems analogous to the complement of the product fuzzy distance two labeling graphs. First start by introduce the following definitions.

Definition 4.1. *Let* $F(G) = (\sigma_V, \mu_E)$ *define a product fuzzy graph with crisp graph* $G^\star = (V_G, E_G)$. *Then the complement of the product fuzzy graph* $F(G) = (\sigma_V, \mu_E)$ *is a product fuzzy graph* $\overline{F(G)} = (\overline{\sigma}_V, \overline{\mu}_E)$ *where* $\overline{\sigma}_V = \sigma_V$ *and* $\overline{\mu}_E(p,q) = \overline{\sigma}_V(p) \times \overline{\sigma}_V(q) - \mu_E(p,q)$ *for each* $p,q \in V_G$.

Definition 4.2. *Let* $F(G) = (\sigma_V, \mu_E)$ *be a product fuzzy graph with underlying crisp graph* $G^\star = (V_G, E_G)$. *The* μ*-complement of the product fuzzy graph* $F(G) = (\sigma_V, \mu_E)$ *is a product fuzzy graph* $F(G)^\mu = (\sigma_V^\mu, \mu_E^\mu)$ *for which* $\sigma_V^\mu = \sigma_V$ *and* $\mu_E^\mu(p,q) = \sigma_V^\mu(p) \times \sigma_V^\mu(q) - \mu_E(p,q)$ *if* $\mu_E(p,q) > 0$ $\mu_E^\mu(p,q) = 0$ *if* $\mu_E(p,q) = 0$.

Definition 4.3. *A simple undirected crisp graph* $G^\star = (V_G, E_G)$ *in which each pair of different nodes is combined with a unique line is known as complete graph.*

Definition 4.4. *Let us consider two product fuzzy graphs* $F(G)_1 = (\sigma_1, \mu_1)$ *and* $F(G)_2 = (\sigma_2, \mu_2)$ *with fundamental graphs* $G_1^\star = (V_{G1}, E_{G1})$ *and* $G_2^\star = (V_{G2}, E_{G2})$ *respectively. A bijective mapping* $\xi : V_{G1} \to V_{G2}$ *is called an isomorphism between the fuzzy graphs* $F(G)_1 = (\sigma_1, \mu_1)$ *and* $F(G)_2 = (\sigma_2, \mu_2)$ *if it satisfies the following condition* $\sigma_1(r) = \sigma_2 \xi(r)$ $\forall r \in V_{G1}$ *and* $\mu_1(r,s) = \mu_2(\xi(r), \xi(s))$ $\forall r,s \in V_{G1}$. *If* $\xi : V_{G1} \to V_{G2}$ *is isomorphic then product fuzzy graphs* $F(G)_1 = (\sigma_1, \mu_1)$ *and* $F(G)_2 = (\sigma_2, \mu_2)$ *are called isomorphic to each other and it is denoted* $F(G)_1 \cong F(G)_2$. *If* $F(G)_1 = F(G)_2$ *then*ξ *is said to be an automorphism and fuzzy graphs* $F(G)_1 = (\sigma_1, \mu_1)$ *and* $F(G)_2 = (\sigma_2, \mu_2)$ *are called automorphic product fuzzy graphs to each other.*

In general, the complement and μ-complement of any product fuzzy distance two labeling graph need not be a product fuzzy distance two labeling graph. But for some special types of underlying crisp graph and under some specific membership functions, the complement and μ-complement of product fuzzy distance two labeling graph is also a product fuzzy distance two labeling graph. In view of this, we state and prove some following theorems in this section.

Theorem 4.5. *Suppose* $F(G)^P = (\sigma_V^P, \mu_E^P)$ *be a product fuzzy distance two labeling graph with crisp underlying graph* $G^\star = (V_G, E_G)$ *as a complete graph satisfying the condition* $\mu_E^P(r,s) = \frac{\sigma_V^P(r) \times \sigma_V^P(s)}{2}$ *for all* $(r,s) \in E_G$. *Then the complement fuzzy graph* $\overline{F(G)^P} = (\overline{\sigma_V^P}, \overline{\mu_E^P})$ *is also a product fuzzy distance two labeling graph.*

Proof. Consider an identity map $\xi : V_G \to V_G$ such that $\sigma_V^P(r) = \sigma_V^P \xi(r)$ for each vertex $r \in V_G$. Then by the definition of complement of product fuzzy graph, we have $\overline{\sigma_V^P} = \sigma_V^P$ and
$\overline{\mu_E^P}(r,s) = \overline{\sigma_V^P}(r) \times \overline{\sigma_V^P}(s) - \mu_E^P(r,s)$ for each vertex $r,s \in V_G$
$= \sigma_V^P(r) \times \sigma_V^P(s) - \mu_E^P(r,s)$ for each vertex $r,s \in V_G$
$= \sigma_V^P(r) \times \sigma_V^P(s) - \frac{\sigma_V^P(r) \times \sigma_V^P(s)}{2}$ for each vertex $r,s \in V_G$
$= \frac{\sigma_V^P(r) \times \sigma_V^P(s)}{2}$ for each vertex $r,s \in V_G$
$\Rightarrow \overline{\mu_E^P}(r,s) = \mu_E^P(r,s)$ for each vertex $r,s \in V_G$.
Thus the identity function $\xi : V_G \to V_G$ be an isomorphism then product fuzzy graphs $F(G)^P = (\sigma_V^P, \mu_E^P)$ and $\overline{F(G)^P} = (\overline{\sigma_V^P}, \overline{\mu_E^P})$ are isomorphic to each other. This implies that $F(G)^P \cong \overline{F(G)^P}$
Hence product fuzzy distance two labeling graph $F(G)^P = (\sigma_V^P, \mu_E^P)$ is self complementary. This means that the complement fuzzy graph $\overline{F(G)^P} = (\overline{\sigma_V^P}, \overline{\mu_E^P})$ is also a product fuzzy distance two labeling graph. □

Remark 4.6. *For any product fuzzy distance two labeling graph* $F(G)^P = (\sigma_V^P, \mu_E^P)$ *with underlying crisp underlying graph* $G^\star = (V_G, E_G)$ *and* $\mu_E^P(r,s) = \frac{\sigma_V^P(r) \times \sigma_V^P(s)}{2}$ *for all* $(r,s) \in E_G$, *if fundamental graph* $G^\star = (V_G, E_G)$ *is a complete graph then the complement fuzzy graph* $\overline{F(G)^P} = (\overline{\sigma_V^P}, \overline{\mu_E^P})$ *will be a product fuzzy distance two labeling graph. Figure 3 illustrates* *this statement.*

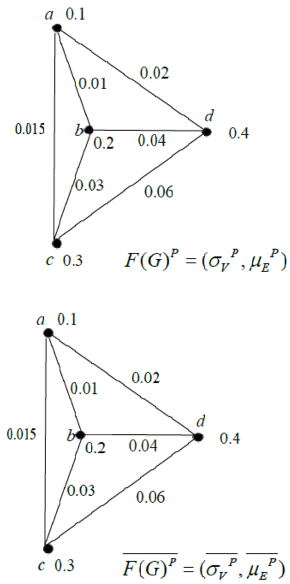

Figure.3

But if fundamental graph $G^\star = (V_G, E_G)$ is not a complete graph, then the complement fuzzy graph $\overline{F(G)^P} = (\overline{\sigma_V^P}, \overline{\mu_E^P})$ cannot be a product fuzzy distance two labeling graph. Figure 4 illustrates this statement.

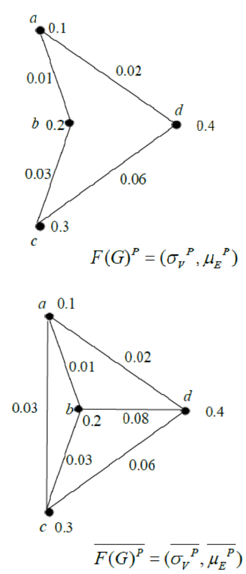

Figure.4

Theorem 4.7. *Consider $F(G)^P = (\sigma_V^P, \mu_E^P)$ be a product fuzzy distance two labeling graph with crisp fundamental graph $G^\star = (V_G, E_G)$ following the condition $\mu_E^P(r,s) \leq \frac{\sigma_V^P(r) \times \sigma_V^P(s)}{2}$ for every arc $(r,s) \in E_G$. Then the μ-complement fuzzy graph $(F(G)^P)^\mu = ((\sigma_V^P)^\mu, (\mu_E^P)^\mu)$ is also a product fuzzy distance two labeling graph.*

Proof. Consider an identity map $\xi : V_G \to V_G$ with the condition $\sigma_V^P(r) = \sigma_V^P(\xi(r))$ for each vertex r in V_G. Then with the help of the concept of μ-complement of a product fuzzy graph, following result can be find as $(\sigma_V^P)^\mu = \sigma_V^P$ and $(\mu_E^P)^\mu(r,s) = (\sigma_V^P)^\mu(r) \times (\sigma_V^P)^\mu(s) - \mu_E^P(r,s)$ for $\mu_E^P(r,s) > 0$ where vertex $r,s \in V_G$
$= \sigma_V^P(r) \times \sigma_V^P(s) - \mu_E^P(r,s)$ for $\mu_E^P(r,s) > 0$ where vertex $r,s \in V_G$
$\geq \sigma_V^P(r) \times \sigma_V^P(s) - \frac{\sigma_V^P(r) \times \sigma_V^P(s)}{2}$ for $\mu_E^P(r,s) > 0$ where vertex $r,s \in V_G$
$\geq \frac{\sigma_V^P(r) \times \sigma_V^P(s)}{2}$ for $\mu_E^P(r,s) > 0$ where vertex $r,s \in V_G$
$\Rightarrow (\mu_E^P)^\mu(r,s) \geq \mu_E^P(r,s)$ for $\mu_E^P(r,s) > 0$ where vertex $r,s \in V_G$.
Also $(\mu_E^P)^\mu(r,s) = 0$ if $\mu_E^P(r,s) = 0$.

Case I: If $(\mu_E^P)^\mu(r,s) = \mu_E^P(r,s)$ then the identity function $\xi : V_G \to V_G$ is isomorphic. In this case product fuzzy graphs $F(G)^P = (\sigma_V^P, \mu_E^P)$ and $(F(G)^P)^\mu = ((\sigma_V^P)^\mu, (\mu_E^P)^\mu)$ are called isomorphic to each other. Hence product fuzzy distance two labeling graph $F(G)^P = (\sigma_V^P, \mu_E^P)$ will be self complementary. This implies that the μ-complement fuzzy graph $(F(G)^P)^\mu = ((\sigma_V^P)^\mu, (\mu_E^P)^\mu)$ is also a product fuzzy distance two labeling graph. Consider the Figure 3 for this case.
Case II: If $(\mu_E^P)^\mu(r,s) > \mu_E^P(r,s)$ then $(\mu_E^P)^\mu(r,s) < (\sigma_V^P)^\mu(r) \times (\sigma_V^P)^\mu(s)$ for all $r,s \in V_G$. Also $(\mu_E^P)^\mu(r,s) = 0$ if $\mu_E^P(r,s) = 0$, therefore μ-complement product fuzzy graph $(F(G)^P)^\mu = ((\sigma_V^P)^\mu, (\mu_E^P)^\mu)$ will not contain any extra edge in comparison to product fuzzy graph $F(G)^P = (\sigma_V^P, \mu_E^P)$. Hence both graphs $F(G)^P$ and $(F(G)^P)^\mu$ will be isomorphic to each other. This implies that the μ-complement fuzzy graph $(F(G)^P)^\mu = ((\sigma_V^P)^\mu, (\mu_E^P)^\mu)$ is also a product fuzzy distance two labeling graph. Consider the Figure 5 for this case.

□

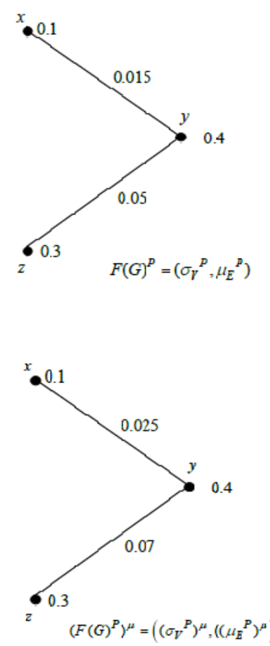

Figure.5

Theorem 4.8. *Suppose $\overline{F(G)^P} = (\overline{\sigma_V^P}, \overline{\mu_E^P})$ is a complement fuzzy graph of the product fuzzy distance two labeling graph $F(G)^P = (\sigma_V^P, \mu_E^P)$ with crisp fundamental graph $G^\star = (V_G, E_G)$ as a complete graph. If $F(G)^P \cong \overline{F(G)^P}$ then the complement fuzzy graph $\overline{F(G)^P} = (\overline{\sigma_V^P}, \overline{\mu_E^P})$ is also a product fuzzy distance two labeling graph.*

Proof. Suppose the graphs $F(G)^P$ and $\overline{F(G)^P}$ are isomorphic to each other. Then by the definition of isomorphism of two product fuzzy graphs, there exist a bijective function $\xi : V_G \to V_G$ with the following condition $\overline{\sigma_V^P}(\xi t) = \sigma_V^P \xi(t)$ for each vertex $t \in V_G$ and $\overline{\mu_E^P}(\xi(r), \xi(s)) = \mu_E^P(r,s) \forall r,s \in V_G$. Now the concept of complement fuzzy graph $\overline{F(G)^P} = (\overline{\sigma_V^P}, \overline{\mu_E^P})$ gives

$\overline{\mu_E^P}(\xi(r), \xi(s)) = \overline{\sigma_V^P}(\xi(r)) \times \overline{\sigma_V^P}(\xi(s)) - \mu_E^P(\xi(r), \xi(s))$
$\forall r,s \in V_G$
or $\mu_E^P(r,s) = \sigma_V^P(r) \times \sigma_V^P(s) - \mu_E^P(\xi(r), \xi(s)) \forall r,s \in V_G$
or $\mu_E^P(r,s) + \mu_E^P(\xi(r), \xi(s)) = \sigma_V^P(r) \times \sigma_V^P(s) \forall r,s \in V_G$

Thus $\sum_{r \neq s; r,s \in V_G} \mu_E^P(r,s) + \sum_{r \neq s; r,s \in V_G} \mu_E^P(\xi(r), \xi(s))$
$= \sum_{r \neq s; r,s \in V_G} \sigma_V^P(r) \times \sigma_V^P(s)$

Furthermore we have
$\sum_{r \neq s; r,s \in V_G} \mu_E^P(r,s) = \sum_{r \neq s; r,s \in V_G} \mu_E^P(\xi(r), \xi(s))$.
Then above equation becomes
$2 \times \sum_{r \neq s; r,s \in V_G} \mu_E^P(r,s) = \sum_{r \neq s; r,s \in V_G} \sigma_V^P(r) \times \sigma_V^P(s)$
$\Rightarrow (Size(F(G)^P) = \sum_{r \neq s; r,s \in V_G} \mu_E^P(r,s)) =$
$\frac{1}{2} \times \sum_{r \neq s; r,s \in V_G} \sigma_V^P(r) \times \sigma_V^P(s)$
$F(G)^P \cong \overline{F(G)^P} \Rightarrow (Size(F(G)^P) =$
$\sum_{r \neq s; r,s \in V_G} \mu_E^P(r,s)) = \frac{1}{2} \times \sum_{r \neq s; r,s \in V_G} \sigma_V^P(r) \times \sigma_V^P(s)$

Also any product fuzzy distance two labeling graph $F(G)^P = (\sigma_V^P, \mu_E^P)$ will satisfies these conditions only when $\mu_E^P(r,s) = \frac{\sigma_V^P(r) \times \sigma_V^P(s)}{2}$ for every vertex $r,s \in V_G$ and hence in such manner graph $F(G)^P = (\sigma_V^P, \mu_E^P)$ is self complementary. Thus the complement fuzzy graph $\overline{F(G)^P} = (\overline{\sigma_V^P}, \overline{\mu_E^P})$ will also be a product fuzzy distance two labeling graph. □

5. conclusions

Fuzzy graphs play an important role in many areas, including decision-making, computer networking and management sciences. A graph which has product fuzzy distance two labeling is to investigate the role of product fuzzy distance two labeling graph in Computer Science field. Product fuzzy distance two labeling graphs is a powerful tool that makes things ease in various field of computer science as said above. The product fuzzy distance two labeling graphs give more exactness, adaptability and similarity to the framework when more than one agreement are to be managed. The fuzzy distance two labeling graph is useful for handling various interesting problems like traffic light problem, networking problems and job allocation problem etc.

It is realized that graphs are actually the natural illustration of relations. Graph is an standard process to depict data which including connection between some objects. In general, Objects are expressed by means of nodes (vertices) and connections by arcs (edges). At the point when there is there is a dubiousness or uncertainty in the illustration of objects or in object connections or both, we require to construct a fuzzy graph model. The real world problems often involve distance restrictions, unmatched assignment and multi-objects. Then product fuzzy distance two labeling graphs give more accuracy and precision as compared to fundamental fuzzy graphs. In this research article, a new concept of product fuzzy distance two labeling graphs and its complement have been introduced. We are extending our research work to operations on product fuzzy distance two labeling graphs.

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