

# Further properties of nano pre- $T_0$ , nano pre- $T_1$ and nano pre- $T_2$ spaces

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#### **Abstract**

The basic objective of this paper is to define nano- $T_0$  space, nano semi- $T_0$  space, nano pre- $T_0$  space, nano pre- $T_0$  space, nano pre- $T_0$  space, nano semi- $T_0$  space and nano pre- $T_0$  space and investigate their properties. Also we have obtain some of their basic results and given an appropriate examples to understand the abstract concepts clearly.

#### **Keywords**

nano- $T_0$  space, nano semi- $T_0$  space, nano pre- $T_0$  space, nano- $T_1$  space, nano semi- $T_1$  space, nano pre- $T_1$  space, nano- $T_2$  space, nano semi- $T_2$  space and nano pre- $T_2$  space.

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#### 1. Introduction

In mathematics, topology is concerned with the properties of spaces that are preserved under continuous deformations, such as stretching, crumpling and bending, but not tearing or gluing. This can be studied by considering a collection of subsets, called open sets, that satisfy certain properties, turning the given set into what is known as a topological space. Important topological properties iNclude connectedness and compactness.

Recently Lellis Thivagar and Richard [1] introduced a nano topological space with respect to a subset of an universe which is defined in terms of lower and upper approximations of X. The elements of a nano topological space are called the nano-open sets. He also introduced weak form of nano-open

sets namely nano  $\alpha$ -open sets, nano semi-open sets and nano pre-open sets.

Sathishmohan et.al[3] defined nano pre-neighbourhoods, nano pre-interior, nano pre-limit point, nano pre-derived set, nano pre-frontier and nano pre-regular in nano topological spaces and obtained some of its properties.

Sathishmohan et.al[4] introduced and investigated the properties of nano semi pre-neighbourhoods, nano semi pre-interior, nano semi pre-frontier, nano semi pre-exterior, nanodense and nano-submaximal.

The main purpose of this paper is to bring up the idea about nano- $T_0$  space, nano semi- $T_0$  space, nano pre- $T_0$  space, nano- $T_1$  space, nano semi- $T_1$  space, nano pre- $T_1$  space, nano- $T_2$  space, nano semi- $T_2$  space, nano pre- $T_2$  space and obtain some of its basic results.

The structure of this manuscript as follows:

In section 2, some basic definitions and results in nano topological spaces are recalled, which are useful to prove the main results.

In section 3, we define and study the notions of nano- $T_0$  spaces, nano semi- $T_0$ , nano pre- $T_0$  spaces in nano topological spaces and obtained some of its basic results.

In section 4, we study the notions of nano- $T_1$ , nano semi- $T_1$ , nano pre- $T_1$  spaces in nano topological spaces and some of the properties has been investigated.

In section 5, we deal with nano- $T_2$  spaces, nano semi- $T_2$ ,

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nano pre- $T_2$  spaces in nano topological spaces and some of their properties are analyzed.

### 2. Preliminaries

In this section, some basic definitions and results in nano topological spaces are given, which are useful to prove the main results.

**Definition 2.1.** [2] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let  $X \subseteq U$ . Then,

(i)The lower approximation of x with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is denoted by  $L_R(X)$ .  $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$  where R(x) denotes the equivalence class determined by  $x \in U$ .

(ii) The upper approximation of x with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by  $U_R(X)$ .  $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$ 

(iii) The boundary region of x with respect to R is the set of all objects which can be classified neither as X nor as not-X with respect to R and it is denoted by  $B_R(X)$ .  $B_R(X) = U_R(X)$  -  $L_R(X)$ .

**Definition 2.2.** [2] Let U be the universe, R be an equivalence relation on U and  $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$  where  $X \subseteq U$ . Then  $\tau_R(X)$  satisfies the following axioms

(i) U and  $\phi \in \tau_R(X)$ .

(ii) The union of the elements of any sub-collection of  $\tau_R(X)$  is in  $\tau_R(X)$ .

(iii) The intersection of the elements of any finite sub collection of  $\tau_R(X)$  is in  $\tau_R(X)$ .

Then  $\tau_R(X)$  is a topology on U called the nano topology on U with respect to X. We call  $(U, \tau_R(X))$  as nano topological space.

**Definition 2.3.** [1] A subset A of a nano topological space  $(U, \tau_R(X))$  is nano pre-open in U, if  $A \subseteq Nint(Ncl(A))$ . The complement of nano pre-open is nano pre-closed, and it is denoted by NPF(U).

**Definition 2.4.** [1] A subset A of a nano topological space  $(U, \tau_R(X))$  is nano regular-open in U, if Nint(Ncl(A)) = A. The complement of nano-regular open is nano regular-closed, and it is denoted by NRF(U).

**Remark 2.5.** [2] If  $\tau_R(X)$  is the nano topology on U with respect to X, then the set  $B = \{U, L_R(X), B_R(X)\}$  is the basis for  $\tau_R(X)$ .

**Definition 2.6.** [1] If  $(U, \tau_R(X))$  is a nano topological space with respect to X where  $X \subseteq U$  and if  $A \subseteq U$ , then

(i) The nano interior of A is defined as the union of all nanoopen subsets of A is contained in A and is denoted by Nint(A). That is, Nint(A) is the largest nano-open subset of A.

(ii) The nano closure of A is defined as the intersection of all nano-closed sets containing A and is denoted by Ncl(A)). That is, Ncl(A) is the smallest nano-closed set containing A.

**Definition 2.7.** [3] The union of all nano pre-open sets which are contained in A is called the nano pre-interior of A and is denoted by Npint(A) or  $NA_*$  or  $\{x\}_*$ .

**Definition 2.8.** [3] The intersection of nano pre-closed sets containing a set A is called the nano pre-closure of A and is denoted by Npcl(A) or  $NA^*$  or  $\{x\}^*$ .

# 3. Nano pre- $T_0$ spaces

In this section, we define and study the notions of nano- $T_0$  spaces, nano semi- $T_0$  and nano pre- $T_0$  spaces in nano topological spaces and obtained some of their basic results.

**Definition 3.1.** A space U is called nano- $T_0$  space for  $x, y \in U$  and  $x \neq y$ , there exists a nano-open set G such that  $x \in G$  and  $y \notin G$ .

**Definition 3.2.** A space U is called nano semi- $T_0$  space for  $x, y \in U$  and  $x \neq y$ , there exists a nano semi-open set G such that  $x \in G$  and  $y \notin G$ .

**Definition 3.3.** A space U is called nano pre- $T_0$  space for  $x, y \in U$  and  $x \neq y$ , there exists a nano pre-open set G such that  $x \in G$  and  $y \notin G$ .

We prove some properties of nano pre- $T_0$  spaces in the following.

**Theorem 3.4.** Let  $(U, \tau_R(X))$  be an nano topological space, then for every N-T<sub>0</sub> space is nano pre-T<sub>0</sub> (resp. nano semi-T<sub>0</sub>) spaces.

*Proof.* Let U be nano- $T_0$ -space, x and y be two distinct points of U, as U is nano- $T_0$  there exists nano-open set G such that  $x \in G$  and  $y \notin G$ , since every nano-open set is nano pre(resp. nano semi)-open and hence G is nano pre (resp. nano semi)open set such that  $x \in G$  and  $y \notin G \Rightarrow U$  is nano pre- $T_0$  (resp. nano semi- $T_0$ ) space.

But the converse of the theorem need not be true in general.  $\Box$ 

**Example 3.5.** Let  $U = \{a,b,c,d\}$ ,  $U/R = \{\{a\},\{b,d\},\{c\}\}\}$ ,  $X = \{a,b\}$  and  $\tau_R(X) = \{U,\phi,\{a\},\{a,b,d\},\{b,d\}\}$  be a nano topology on U. We have

 $NSO(U,X) = \{U, \phi, \{a\}, \{a,c\}, \{b,d\}, \{a,b,d\}, \{b,c,d\}\}, \\ NPO(U,X) = \{U, \phi, \{a\}, \{b\}, \{d\}, \{a,b\}, \{a,d\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}.$ 

1. Let  $x = \{a, c\}$  and  $y = \{c\}$  then it is nanosemi- $T_0$  space but not N- $T_0$  space.



2. Let  $x = \{b\}$  and  $y = \{c\}$  then it is nanopre- $T_0$  space but not N- $T_0$  space.

**Theorem 3.6.** Every nanosemi- $T_0$  space is nanopre- $T_0$  space.

*Proof.* Let U be  $nanosemi-T_0$  space and x and y be two distinct points of U, as U is  $nanosemi-T_0$  there exists nano semiopen set G such that  $x \in G$  and  $y \notin G$ , since every nano semiopen set is nanopre-open and hence G is nanopre-open set such that  $x \in G$  and  $y \notin G \Rightarrow U$  is  $nanopre T_0$  space.

But the converse of the theorem need not be true in general.  $\Box$ 

**Example 3.7.** From the Example 3.5, Let  $x = \{b\}$  and  $y = \{c\}$  then it is nanopre- $T_0$  space but not nanosemi- $T_0$  space.

**Theorem 3.8.** A space U is nano pre- $T_0$  iff  $\{x\}^* \neq \{y\}^*$  for every pair of distinct points  $x, y \in U$ .

*Proof.* Let x and y be any two distinct points of nano pre- $T_0$  space U. We show that  $\{x\}^* \neq \{y\}^*$ . By hypothesis, suppose  $X \in NPO(U)$  such that  $x \in X$  and  $y \notin X$ . Hence  $y \in U - X$  and U - X is nano pre-closed. Therefore  $\{y\}^* \subset U - X$ . Hence  $y \in \{y\}^*$  but  $x \notin \{y\}^*$  as  $x \in U - X$ . Hence  $\{x\}^* \notin \{y\}^*$ . Conversely, suppose for any  $x, y \in U$  with  $x \neq y$ ,  $\{x\}^* \neq \{y\}^*$ . Now without loss of generality, let  $z \in U$  such that  $z \in \{x\}^*$  but  $z \notin \{y\}^*$ . Now, we claim that  $x \notin \{y\}^*$ . For if  $x \in \{y\}^*$  then  $\{x\} \subset \{y\}^*$  which implies that  $\{\{x\}^* \subset \{y\}^*\}$ . Thus,  $z \in \{x\}^*$  and  $z \in \{y\}^*$ . This is a contradiction.

Therefore,  $x \in \{y\}^*$ . Hence  $U - \{y\}^*$  is a nano pre-open set containing x but not y. It gives that U is nano pre- $T_0$  space.

**Theorem 3.9.** If  $P \in NPO(X)$  and  $Q \in NPO(U)$  then  $Q \in NPO(X)$ .

**Theorem 3.10.** A space U is nano pre- $T_0$  iff for each  $x \in U$ , there exists a nano pre-open set X of U containing x such that the subspace X is nano pre- $T_0$ .

- *Proof.* 1. If U is nano pre- $T_0$ , take U as X. Then, X is a nano pre-open set containing x such that the subspace X is nano pre- $T_0$  for every  $x \in U$ .
  - 2. Next suppose that  $x_1$ ,  $x_2$  be any two distinct points of U. By hypothesis, there exist  $X_j \in NPO(U)$  such that  $x_j \in X_j$  and the subspace  $X_j$  is nano pre- $T_0$ , for j = 1, 2. If  $x_2 \notin X_1$  then the proof is completed. If  $x_2 \in X_1$  then as  $X_1$  is nano pre- $T_0$ , there exists  $W_1 \in NPO(X_1)$  such that  $x_1 \in w_1$  and  $x_2 \notin w_1$  or there exists  $W_2 \in NPO(X_1)$  such that  $x_2 \in w_2$  and  $x_1 \notin w_2$ . Since  $X_1 \in NPO(U)$ , it follows that from Theorem 3.9,  $w_j \in NPO(U)$  for j = 1, 2. This means that the space U is nano pre- $T_0$ .

Next, we define the following.

**Definition 3.11.** A function  $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$  is said to be point nano pre-closure 1-1 iff for  $x,y \in U$  such that  $\{x\}^* \neq \{y\}^*$ , then  $\{f(x)\}^* \neq \{f(y)\}^*$ .

**Example 3.12.** From the Example 3.5, Let  $x = \{a\}$ ,  $y = \{c\}$  then  $\{x\}^* = \{a\}$ ,  $\{y\}^* = \phi \Rightarrow \{x\}^* \neq \{y\}^*$ . Then  $\{f(x)\}^* \neq \{f(y)\}^*$ .

**Theorem 3.13.** Let  $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$  is a point nano-pre closure 1-1 and U is nano pre- $T_0$ , then f is 1-1.

*Proof.* Let  $x, y \in U$  with  $x \neq y$ , since U is nano pre- $T_0$  space then  $\{x\}^* \neq \{y\}^*$  by Theorem 3.8. But f is a point nano preclosure 1-1 implies that  $\{f(x)\}^* \neq \{f(y)\}^*$ . Hence  $f(x) \neq f(y)$ . Thus, f is 1-1.

**Theorem 3.14.** Let  $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$  be a mapping from nano pre- $T_0$  space U into nano pre- $T_0$  space V. Then f is point nano pre-closure 1-1 iff f is 1-1.

*Proof.* Proof follows from Theorem 3.13.

# 4. Nano pre- $T_1$ spaces

In this section, we study the notions of nano- $T_1$ , nano semi- $T_1$ , nano pre- $T_1$  spaces in nano topological spaces and some of the properties has been investigated.

**Definition 4.1.** A space U is called nano- $T_1$  space for  $x, y \in U$  and  $x \neq y$ , there exists a nano-open sets G and H such that  $x \in G$ ,  $y \notin G$  and  $y \in H$ ,  $x \notin H$ .

**Definition 4.2.** A space U is called nano semi- $T_1$  space for  $x, y \in U$  and  $x \neq y$ , there exists a nano semi-open sets G and H such that  $x \in G$ ,  $y \notin G$  and  $y \in H$ ,  $x \notin H$ .

**Definition 4.3.** A space U is called nano pre- $T_1$  space for  $x, y \in U$  and  $x \neq y$ , there exists a nano pre-open sets G and H such that  $x \in G$ ,  $y \notin G$  and  $y \in H$ ,  $x \notin H$ .

**Theorem 4.4.** Every nano- $T_1$  space is nano pre- $T_1$  (resp. nano semi- $T_1$ ) space.

*Proof.* Let U be nano- $T_1$  space and let  $x \neq y$  in U. Then there exists distinct nano-open sets G and H such that  $x \in G$  and  $y \in H$ . As every nano-open set is nano pre(resp. nano semi)-open G and H are distinct nano pre(resp. nano semi)-open sets such that  $x \in G$  and  $y \in H$ .

Converse of above theorem is need not be true in general.  $\Box$ 

**Example 4.5.** Let  $U = \{a, b, c, d\}$ ,  $U/R_1 = \{\{a\}, \{b, d\}, \{c\}\}$ ,  $X_G = \{a, b\}$  and  $\tau_R(X_G) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$  be a nano topology on U, we have

 $NSO(U, X_G) = \{U, \phi, \{a\}, \{a, c\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}\},\$   $NPO(U, X_G) = \{U, \phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}\}$  $U/R_2 = \{\{a, b\}, \{c\}, \{d\}\}, X_H = \{b, c\} \text{ and } \{a, c\}, \{a,$ 

 $\tau_R(X_H) = \{U, \phi, \{c\}, \{a, c, d\}, \{a, d\}\}\}$  we have  $NSO(U, X_H) = \{U, \phi, \{c\}, \{a, d\}, \{b, c\}, \{a, b, d\}, \{a, c, d\}\},$ 



 $NPO(U, X_H) = \{U, \phi, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, c, d\}\},\$ 

- 1. Let  $x = \{a,c\}$  and  $y = \{a,b\}$  then it is nano semi- $T_1$  space but not nano- $T_1$  space.
- 2. Let  $x = \{b\}$  and  $y = \{c\}$  then it is nano pre- $T_1$  space but not nano- $T_1$  space.

**Theorem 4.6.** Every nano semi- $T_1$  space is nano pre- $T_1$  space.

*Proof.* Let U be nano semi- $T_1$  space and let  $x \neq y$  in U. Then there exists distinct nano-open sets G and H such that  $x \in G$  and  $y \in H$ . As every nano-open set is nano pre-open G and H are distinct nano pre-open sets such that  $x \in G$  and  $y \in H$ . Converse of above theorem is need not be true in general.  $\square$ 

**Example 4.7.** From the Example 4.5, Let  $x = \{a\}$  and  $y = \{c\}$  then it is nano pre- $T_1$  space but not nano semi- $T_1$  space.

**Theorem 4.8.** Let  $(U, \tau_R(X))$  be a nano topological space, then for each nano pre- $T_1$  (resp. nano semi- $T_1$ ) space is nano pre- $T_0$  space.

*Proof.* Let U be nano pre- $T_1$  (resp. nano semi- $T_1$ ) space and x and y be two distinct points of U, as U is nano pre- $T_1$ (resp. nano semi- $T_1$ ) space there exists nano pre-(resp. nano semi) open set G such that  $x \in G$  and  $y \notin G$ , since every nano-open set is nano pre-(resp. nano semi) open and hence G is nano pre-(resp. nano semi) open set such that  $x \in G$  and  $y \notin G \Rightarrow U$  is nano pre- $T_0$ .

**Example 4.9.** From Example 4.5, Let  $x = \{a\}$  and  $y = \{c\}$ ,  $x, y \in U$  and  $x \neq y$ , then it is clear that  $x \in G$ ,  $y \notin G$  and  $y \in H$  and  $x \notin H$ . Then we can say that it is nanopre- $T_0$  space.

**Lemma 4.10.** Union of nano pre-open sets is nano pre-open.

**Theorem 4.11.** A space U is nano pre- $T_1$  space iff for any point  $x \in U$ , the singleton set  $\{x\}$  is nano pre-closed set.

*Proof.* Let every singleton set  $\{x\}$  of U be nano pre-closed. Therefore  $U - \{x\}$  is nano pre-open. Now we show that U is nano pre- $T_1$  space. Let  $x, y \in U$  with  $x \neq y$ . Then  $\{x\}$  and  $\{y\}$  are nano pre-closed sets. Therefore  $U - \{x\}$  is a nano pre-open set containing y but not x and  $U - \{y\}$  is a nano pre-open set containing x but not y. Thus U is nano pre- $T_1$  space.

Conversely, let U be a nano pre- $T_1$  space. Assume that  $x \in U$  be an arbitrary point. Now, we show that  $\{x\}$  is nano pre-closed or  $U - \{x\}$  is nano pre-open. Let  $z \in U - \{x\}$  then clearly  $z \neq x$ . Now, U is nano pre- $T_1$  and z is a point different from x so there exists a nano pre-open set  $G_z$  such that  $z \in G_z$  but  $x \notin G_z$ . Hence  $z \in G_z \subset U - \{x\}$ . Therefore  $U - \{x\} = \bigcup \{G_z | z \neq x\}$ . So  $U - \{x\}$  being the union of nano pre-open sets is nano pre-open. Hence  $\{x\}$  is nano pre-closed set.

**Theorem 4.12.** A space U is nano pre- $T_1$  iff for each point  $x \in U$ , there exists a nano pre-open set X of U containing x such that the subspace X is nano pre- $T_1$ .

*Proof.* Proof is similar to Theorem 3.10.  $\square$ 

## 5. Nano pre- $T_2$ spaces

In this section, we define and study the notions of nano- $T_2$  spaces, nano semi- $T_2$ , nano pre- $T_2$  spaces in nano topological spaces and some of their properties are analyzed.

**Definition 5.1.** A space U is called nano- $T_2$  (or N- $T_2$ ) space for  $x, y \in U$  and  $x \neq y$ , there exists disjoint N-open sets G and H such that  $x \in G$  and  $y \in H$ .

**Definition 5.2.** A space U is called nano semi- $T_2$  (or nanosemi- $T_2$ ) space for  $x, y \in U$  and  $x \neq y$ , there exists disjoint nanosemi-open sets G and H such that  $x \in G$  and  $y \in H$ .

**Definition 5.3.** A space U is called nano pre- $T_2$  (or nanopre- $T_2$ ) space for  $x, y \in U$  and  $x \neq y$ , there exists disjoint nanopre-open sets G and H such that  $x \in G$  and  $y \in H$ .

Next, we shall characterize nano pre- $T_2$  spaces in the following.

**Theorem 5.4.** Every nano- $T_2$  space is nanopre- $T_2$  (resp. nanosemi- $T_2$ ) space.

*Proof.* Let U be nano- $T_2$  space and let  $x \neq y$  in U. Then there exists disjoint nano-open sets G and H such that  $x \in G$  and  $y \in H$ . As every nano-open set is *nanopre* (resp. *nanosemi*)-open G and H are disjoint *nanopre*-(resp. *nanosemi*)-open sets such that  $x \in G$  and  $y \in H$ .

Converse of above theorem is need not be true in general.  $\Box$ 

**Example 5.5.** From the Example 4.5,

- 1. Let  $x = \{a,c\}$  and  $y = \{a,d\}$  then it is nanosemi- $T_2$  space but not N- $T_2$  space.
- 2. Let  $x = \{b\}$  and  $y = \{c\}$  then it is nanopre- $T_2$  space but not N- $T_2$  space.

**Theorem 5.6.** Every nanosemi- $T_2$  space is nanopre- $T_2$  space.

*Proof.* Let U be  $nanosemi-T_2$  space and let  $x \neq y$  in U. Then there exists disjoint nano-open sets G and H such that  $x \in G$  and  $y \in H$ . As every nano-open set is  $nanopre-T_2$  open G and H are disjoint  $nanopre-T_2$  open sets such that  $x \in G$  and  $y \in H$ 

Converse of above theorem is need not be true in general.  $\Box$ 

**Theorem 5.7.** Let  $(U, \tau_R(X))$  be an nano topological space, then for each nanopre- $T_2$  (resp. nanosemi- $T_2$ ) space is nanopre- $T_0$  space.



*Proof.* Let U be  $nanopre-T_2$  (resp.  $nanosemi-T_2$ ) space and x and y be two distinct points of U, as U is  $nanopre-T_2$  (resp.  $nanosemi-T_2$ ) space there exists nanopre- (resp. nanosemi) open set G such that  $x \in G$  and  $y \notin G$ , since every nano-open set is nanopre- (resp. nanosemi) open and hence G is nanopre- (resp. nanosemi) open set such that  $x \in G$  and  $y \notin G \Rightarrow U$  is  $nanopre-T_0$ .

**Example 5.8.** From Example 4.5, Let  $x = \{a\}$  and  $y = \{c\}$ ,  $x, y \in U$  and  $x \neq y$ , then it is clear that  $x \in G$ ,  $y \notin G$  and  $y \in H$  and  $x \notin H$ . Then we can say that it is nanopre- $T_0$  space but not nanopre- $T_0$  (resp. nanosemi- $T_0$ ) space.

**Lemma 5.9.**  $NPO(U, \tau) = NPO(U, \tau^{\alpha});$   $NPF(U, \tau) = NPF(U, \tau^{\alpha}).$ 

**Lemma 5.10.** A space  $(U, \tau_R(X))$  is nano pre- $T_2$  iff  $(U, \tau_R(X)^{\alpha})$  is nano pre- $T_2$ .

*Proof.* Let  $x, y \in U$  with  $x \neq y$ . Since U is nano pre- $T_2$  there exist disjoint nano pre-open sets X and Y in U such that  $x \in X$  and  $y \in Y$ . Also  $X, Y \in NPO(U, \tau_R(X)^\alpha)$  by Lemma 5.9. Thus  $(U, \tau_R(X)^\alpha)$  is nano pre- $T_2$ . Converse follows similarly, since  $NPO(U, \tau_R(X)) = NPO(U, \tau_R(X)^\alpha)$ .

**Lemma 5.11.** For any subset of a space U, the following conditions are equivalent.

- 1.  $A \in NSPO(U)$ .
- 2.  $A \subset Ncl(Nint(Ncl(A)))$ .
- 3.  $Ncl(A) \in NRF(U)$

**Lemma 5.12.** Every nano  $\alpha$ -open subspace of a nano pre- $T_2$  space is nano pre- $T_2$  space.

*Proof.* Let X be an nano  $\alpha$ -open subspace of a nano pre- $T_2$  space U. Let x and y be any two distinct points of X. Since U is nano pre- $T_2$  space and  $X \subset U$ , there exist two disjoint nano pre-open sets G and H in U such that  $x \in G$  and  $y \in H$ . Let  $A = G \cap X$  and  $B = H \cap X$ . Then A and B are nano pre-open sets in X containing x and y respectively by Lemma 5.11. Also,  $A \cap B = (G \cap X) \cap (H \cap X) = \phi$ . Hence X is nano pre- $T_2$  space.

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