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Nano generalized pre c^*-closed sets in nano topological spaces

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Abstract
In this paper, we have introduced a new class of closed set namely nano generalized pre c^*-closed sets in nano topological space and study some of its basic results.

Keywords
Nano generalized pre c^*-closed set.

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Contents

1. Introduction

In 1970, Levine [4] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. In [5] Maki et al introduced the concepts of generalized pre closed sets and pre generalized closed sets. The notion of nano topology was introduced by Lellis Thivagar [3] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined nano closed sets, nano interior and nano closure. He studied about the weak forms of nano open sets such as nano α-open sets, nano semi open sets and nano pre open sets. In this paper, we have introduced a new class of sets on nano topological spaces called nano generalized pre c^*- closed sets and some of the properties are discussed.

2. Preliminaries

We recall the following definitions:

Definition 2.1. *[6] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$. Then,*
(i)The lower approximation of x with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is denoted by $L_R(X)$.
$L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by $x \in U$.
(ii)The upper approximation of x with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$.
$U_R(X) = \bigcup_{x\in U}\{R(x) : R(x)\cap X \neq \phi\}$
(iii)The boundary region of x with respect to R is the set of all objects which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$.
$B_R(X) = U_R(X)$ - $L_R(X)$.

Proposition 2.2. *[3] If (U,R) is an approximation space and $X,Y \subseteq U$, then*

1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\phi) = U_R(\phi) = \phi$
3. $L_R(U) = U_R(U) = U$
4. $U_R(X\cup Y) = U_R(X)\cup U_R(Y)$
5. $U_R(X\cap Y) \subseteq U_R(X) \cap U_R(Y)$
6. $L_R(X\cup Y) \supseteq L_R(X) \cup L_R(Y)$

7. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
8. $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$
9. $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$
10. $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
11. $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition 2.3. *[6] Let U be the universe, R be an equivalence relation on U and* $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ *where* $X \subseteq U$*. Then* $\tau_R(X)$ *satisfies the following axioms*
(i) U and $\phi \in \tau_R(X)$.
(ii) The union of the elements of any sub-collection of $\tau_R(X)$ *is in* $\tau_R(X)$.
(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ *is in* $\tau_R(X)$.
Then $\tau_R(X)$ *is a topology on U called the nano topology on U with respect to X. We call* $(U, \tau_R(X))$ *as nano topological space. The elements of* $\tau_R(X)$ *are called as nano-open sets. The complement of the nano-open sets are called nano-closed sets.*

Remark 2.4. *[6] If* $\tau_R(X)$ *is the nano topology on U with respect to X , then the set* $B = \{U, L_R(X), B_R(X)\}$ *is the basis for* $\tau_R(X)$.

Definition 2.5. *[3] If* $(U, \tau_R(X))$ *is a nano topological space with respect to X where* $X \subseteq U$ *and if* $A \subseteq U$*, then*
(i)The nano interior of A is defined as the union of all nano-open subsets of A is contained in A and is denoted by $Nint(A)$*. That is,* $Nint(A)$ *is the largest nano-open subset of A.*
(ii)The nano closure of A is defined as the intersection of all nano-closed sets containing A and is denoted by $Ncl(A)$)*. That is,* $Ncl(A)$) *is the smallest nano-closed set containing A.*

Definition 2.6. *[3]* $(U, \tau_R(X))$ *be a nano topological space and A then A is said to be*
i) nano pre open set if $A \subseteq Nint(Ncl(A))$
ii) nano semi open set if $A \subseteq Ncl(Nint(A))$
iii) nano α *-open set if* $A \subseteq Nint(Ncl(Nint(A)))$
iv) nano regular open set if $A = Nint(Ncl(A))$
The complements of the above mentioned sets are called their respective closed sets.

Definition 2.7. *[1] A subset A of a nano topological space* $(U, \tau_R(X))$ *is called,*
i) nano generalized closed set if $Ncl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is nano open.*
ii) Strongly nano g^**-closed set if* $Ncl(Nint(A)) \subseteq U$ *whenever* $A \subseteq U$ *and U is nano g-open.*
iii) nano c^**-set if* $S = G \cap F$ *where G is nano g-open and F is nano* α^**-set.*
iv) nano α^**-set if* $Nint(A) = Nint(Ncl(Nint(A)))$.

Definition 2.8. *[7] A subset* $M_x \subset U$ *is called a nano pre-neighbourhood (NP-nhd) of a point* $x \in U$ *iff there exists a* $A \in NPO(U,X)$ *such that* $x \in A \subset M_x$ *and a point x is called NP-nhd point of the set A.*

3. Nano generalized pre c^*- closed sets

In this section, we introduced and study the properties of nano generalized pre c^*- closed sets in nano Topological Spaces.

Definition 3.1. *A subset A of a space* $(U, \tau_R(X))$ *is called nano generalized* c^**-closed set if* $Ncl(Nint(A)) \subseteq U$*, whenever* $A \subseteq U$ *and U is nano* c^**-set in* $(U, \tau_R(X))$.

Remark 3.2. *Every nano closed set is nano g-closed set.*

Theorem 3.3. *Every nano open set is nano* c^**-set.*

Proof. Every nano open set is nano g-open set [2]. But every nano c^*-set is the intersection of nano g-open set and nano α^*-set. So, by definition of nano c^*-set. The proof is immediate. The converse of the above theorem need not be true as seen from the following example. □

Example 3.4. *Let* $U = \{a, b, c, d\}$ *with* $U/R = \{\{a\}, \{b, c\}, \{d\}\}$ *and* $X = \{b, d\}$*. Then the nano topology* $\tau_R(X) = \{\phi, U, \{d\}, \{b, c\}, \{b, c, d\}\}$*. Here the set* $\{a, b, c\}$ *is nano* c^**-set but not nano open set.*

Theorem 3.5. *Every nano g-closed set is nano generalized pre* c^**-closed set.*

Proof. Let A be a nano g-closed set of U and $A \subseteq V$, V is nano open in U. Every nano open set is nano c^*-set, V is nano c^*-set and $Ncl(Nint(A)) \subseteq Ncl(A) \subseteq V$. Hence A is nano generalized pre c^*-closed set.
The converse of the above theorem need not be true as seen from the following example. □

Example 3.6. *Let* $U = \{a, b, c, d\}$ *with* $U/R = \{\{a\}, \{b, d\}, \{c\}\}$ *and* $X = \{b, d\}$*. Then the nano topology* $\tau_R(X) = \{\phi, U, \{a\}, \{b, d\}, \{a, b, d\}\}$*. Here the set* $\{b\}$ *is nano generalized pre* c^**-closed set but not nano generalized closed set.*

Remark 3.7.
- *Every nano* α*-closed set is nano generalized pre* c^**-closed set.*
- *Every nano regular closed set is nano generalized pre* c^**-closed set.*

Remark 3.8. *The notion of nano generalized pre* c^**-closed set is independent with nano semi closed.*

Example 3.9. *Let* $U = \{a, b, c, d\}$ *with* $U/R = \{\{a\}, \{b, d\}, \{c\}\}$ *and* $X = \{a, b\}$*. Then the nano topology* $\tau_R(X) = \{\phi, U, \{a\}, \{b, d\}, \{a, b, d\}\}$*. Here the set* $\{b\}$ *is nano semi closed set but not a nano generalized pre* c^**-closed set. The set* $\{c, d\}$ *is nano generalized pre* c^**-closed set but not a nano semi closed set.*

Theorem 3.10. *The union the two nano generalized pre* c^**-closed sets in* $(U, \tau_R(X))$ *is also a nano generalized pre* c^**-closed set in* $(U, \tau_R(X))$.

Proof. Let A and B be two nano generalized pre c^*-closed sets in $(U, \tau_R(X))$. Let V be a nano c^*-set in U such that $A \subseteq V$ and $B \subseteq V$. Then we have $A \cup B \subseteq V$, A and B are nano generalized pre c^*-closed sets in $(U, \tau_R(X))$, $Ncl(Nint(A)) \subseteq V$ and $Ncl(Nint(B)) \subseteq V$. Now $Ncl(Nint(A \cup B)) = Ncl(Nint(A))$ $\cup Ncl(Nint(B)) \subseteq V$. Thus we have $Ncl(Nint(A \cup B)) \subseteq V$ whenever $A \cup B \subseteq V$, V is nano c^*-set in $(U, \tau_R(X))$. This implies $A \cup B$ is a nano generalized pre c^*-closed set in $(U, \tau_R(X))$. □

Remark 3.11. *The intersection of two nano generalized pre c^*-closed sets in $(U, \tau_R(X))$ is also a nano generalized pre c^*-closed set in $(U, \tau_R(X))$ as seen from the following example.*

Example 3.12. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{b,d\},\{c\}\}$ and $X = \{a,b\}$. Then the nano topology $\tau_R(X) = \{\phi, U, \{a\}, \{b,d\}, \{a,b,d\}\}$. The nano generalized pre c^*-closed sets are $= \{\phi, U, \{b\}, \{c\}, \{d\}, \{a,c\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}\}$. Here $\{a,c\} \cap \{b,c\} = \{c\}$ which is again a nano generalized pre c^*-closed set.*

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