



Optimization of fuzzy inventory model for EOQ using Lagrangian method

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Abstract

This paper discusses the existence of defective items in a manufacturing process. A rework strategy is implemented to rectify the defective items. A rework cost function under fuzzy environment which includes both synchronous and asynchronous items. The paper develops fuzzy optimal total cost function and fuzzy production quantity using trapezoidal numbers and applying Lagrangian method. A numerical example follows to justify the solution procedure.

Keywords

Economic order quantity (EOQ), Fuzzy inventory, Graded mean integration and Lagrangian method.

AMS Subject Classification

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1. Introduction

In inventory management system, EOQ is the economic order quantity that minimizes the holding costs and ordering costs. Ford W. Harris[4] developed this model but it is R.H. Wilson [13] who had applied it extensively. Zadeh [14] in the

year 1965 had introduced the fuzzy set theory that dealt with uncertainty and also his application of fuzzy set theory in the field of production management plays a vital role. Chen et al. [1, 2] studied the back-order fuzzy inventory model under function principle and graded mean integration representations of generalized fuzzy number respectively. Presently, Muhammad Al-Salamah [7], analyzed the economic production quantity in an imperfect manufacturing process with synchronous and asynchronous flexible rework rates. For more details on this theory and on its applications, we suggest the reader to refer [3, 5, 6, 8–12, 15, 16].

Inspired by the above mentioned works, in this paper, we have considered trapezoidal fuzzy numbers and optimization method is carried out by Lagrangian method. For defuzzifying the total cost graded mean integration method has been applied. Finally, numerical examples highlight the contrast between the crisp and fuzzy scenarios.

2. Notations

κ : Production set up cost
 P : Production rate (items per time, $P > D$)
 T : Cycle length

B : Backorder time
 Q : lot size (items)
 $ATC(Q, B; PR)$: average total cost per unit of time

3. Preliminaries

Definition 3.1. A fuzzy set \tilde{A} defined on $R[\infty, -\infty]$, if the membership function of \tilde{A} is defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$

3.1 The Fuzzy Arithmetical Operations

Function principle is proposed to be as the fuzzy arithmetical operations by trapezoidal fuzzy numbers. Suppose $\tilde{A} = (x_1, x_2, x_3, x_4)$ and $\tilde{B} = (y_1, y_2, y_3, y_4)$ be two trapezoidal fuzzy numbers. Then

(1) The addition of \tilde{A} and \tilde{B} is

$$\tilde{A} \oplus \tilde{B} = (x_1 + y_1, x_2 + y_2, x_3 + y_3, x_4 + y_4),$$

where $x_1, x_2, x_3, x_4, y_1, y_2, y_3,$ and y_4 are any real numbers.

(2) The multiplication of \tilde{A} and \tilde{B} is

$$\tilde{A} \otimes \tilde{B} = (C_1, C_2, C_3, C_4),$$

where $Z_1 = \{x_1y_1, x_1y_4, x_4y_1, x_4y_4\}, Z_2 = \{x_2y_2, x_2y_3, x_3y_2, x_3y_3\},$
 $C_1 = \min Z_1, C_2 = \min Z_2, C_3 = \max Z_1, C_4 = \max Z_2.$

If $x_1, x_2, x_3, x_4, y_1, y_2, y_3,$ and y_4 are all zero positive real numbers then

$$\tilde{A} \otimes \tilde{B} = (x_1y_1, x_2y_2, x_3y_3, x_4y_4).$$

(3) The subtraction of \tilde{A} and \tilde{B} is

$$\tilde{A} \ominus \tilde{B} = (x_1 - y_4, x_2 - y_3, x_3 - y_2, x_4 - y_1),$$

where $-\tilde{B} = (-y_4, -y_3, -y_2, -y_1)$, also $x_1, x_2, x_3, x_4, y_1, y_2, y_3$ and y_4 are any real numbers.

(4) The division of \tilde{A} and \tilde{B} is

$$\tilde{A} \oslash \tilde{B} = \left(\frac{x_1}{y_4}, \frac{x_2}{y_3}, \frac{x_3}{y_2}, \frac{x_4}{y_1} \right),$$

where $\frac{1}{\tilde{B}} = \tilde{B}^{-1} = \left(\frac{1}{y_4}, \frac{1}{y_3}, \frac{1}{y_2}, \frac{1}{y_1} \right), y_1, y_2, y_3$ and y_4 are positive real numbers. Also $x_1, x_2, x_3, x_4, y_1, y_2, y_3$ and y_4 are nonzero positive numbers.

(5) For any $\alpha \in R,$

(a) If $\alpha \geq 0,$ then $\alpha \otimes \tilde{A} = (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4).$

(b) If $\alpha < 0,$ then $\alpha \otimes \tilde{A} = (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1).$

3.2 Extension of the Lagrangian method

Solving a nonlinear programming problem by obtaining the optimum solution was discussed by Taha with equality constraints, also by solving those inequality constraints using Lagrangian method.

Suppose if the problem is given as

$$\min y = f(x)$$

$$\text{Subject to } g_i(x) \geq 0, i = 1, 2, \dots, m.$$

The constraints are non-negative say $x \geq 0$ if included in the m constraints. Then the procedure of the extension of the Lagrangian method will involve the following steps.

Step 1: Solve the unconstrained problem

$$\min y = f(x)$$

If the resulting optimum satisfies all the constraints, then stop since all the constraints are inessential or else set $K = 1$ and move to step 2.

Step 2: Activate any K constraints (i.e., convert them into equalities) and optimize $f(x)$ subject to the K active constraints by the Lagrangian method. If the resulting solution is feasible with respect to the remaining constraints, we shall repeat the steps. If all sets of active constraints taken K at a time are considered without confront a feasible solution, go to step 3.

Step 3: If $K = m,$ stop; there's no feasible solution, otherwise set $K = K + 1$ and go to step 2.

3.3 Graded mean integration representation method

Graded mean integration representation method was introduced by Chen and Hsieh [2] based on the integral value of graded mean h -level of generalized fuzzy number for defuzzifying generalized fuzzy number. First we define a generalized fuzzy number as follows:

$$\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}.$$

By graded mean integration are the inverses of L and R are L^{-1} and R^{-1} respectively. The graded mean h -level value of the generalized fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$ is given by $\frac{h}{2}[L^{-1}(h) + R^{-1}(h)].$ Then the graded mean integration representation of $P(\tilde{A})$ with grade then

$$P(\tilde{A}) = \frac{\int_0^{\omega_A} \frac{h}{2} [L^{-1}(h) + R^{-1}(h)] dh}{\int_0^{\omega_A} h dh},$$

where $0 < h \leq \omega_A$ and $0 < \omega_A \leq 1.$

In this paper, we have used trapezoidal fuzzy numbers as fuzzy parameters for the production inventory model. Let $\tilde{B} = (b_1, b_2, b_3, b_4).$ Then the graded mean integration representation is given by the formula as

$$P(\tilde{B}) = \frac{\int_0^1 \frac{h}{2} [(b_1 + b_4) + h(b_2 - b_1 - b_4 + b_3)] dh}{\int_0^1 h dh} = \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}.$$

4. Fuzzy Integrated Inventory Model for Crisp production

$$ATC(Q; B; P) = \beta - \kappa B + \frac{\nu B^2}{Q} + \eta \frac{1}{Q} + \frac{\delta B}{Q} + \chi Q,$$



where $v > 0, \delta > 0$ and $\eta > 0$.

Differentiating partially with respect to Q and equating it to zero we arrive as follows

$$\frac{\partial ATC(Q, B; P)}{\partial Q} = \frac{-vB^2}{Q^2} - \eta \frac{1}{Q^2} - \frac{\delta B}{Q^2} + \chi$$

$$-\frac{vB^2}{Q^2} - \eta \frac{1}{Q^2} - \frac{\delta B}{Q^2} + \chi = 0$$

implies the EOQ is

$$Q = \sqrt{\frac{vB^2 + \eta + \delta B}{\chi}}$$

5. Fuzzy integrated inventory model for fuzzy EOQ

The annual integrated total inventory cost is

$$ATC(Q, B; P) = \beta - \kappa B + \frac{vB^2}{Q} + \eta \frac{1}{Q} + \frac{\delta B}{Q} + \chi Q.$$

Suppose

$$\tilde{Q} = (Q_1, Q_2, Q_3, Q_4)$$

$$\tilde{v} = (v_1, v_2, v_3, v_4)$$

$$\tilde{\kappa} = (\kappa_1, \kappa_2, \kappa_3, \kappa_4)$$

$$\tilde{\beta} = (\beta_1, \beta_2, \beta_3, \beta_4)$$

$$\tilde{B} = (B_1, B_2, B_3, B_4)$$

$$\tilde{\eta} = (\eta_1, \eta_2, \eta_3, \eta_4)$$

$$\tilde{\delta} = (\delta_1, \delta_2, \delta_3, \delta_4)$$

Therefore,

$$Q = \sqrt{\frac{(v_1^2 B_1^2 + 2v_2 B_2^2 + 2v_3 B_3^2 + v_4 B_4^2) + (\eta_1 + 2\eta_2 + 2\eta_3 + \eta_4) + (\delta_1 B_1 + 2\delta_2 B_2 + 2\delta_3 B_3 + \delta_4 B_4)}{(\chi_1 + 2\chi_2 + 2\chi_3 + \chi_4)}}$$

$$ATC(Q, B; P) = \frac{1}{6} \left\{ \left[\beta_1 - \kappa_4 B_4 + \frac{v_1 B_1^2}{Q_4} + \frac{\eta_1}{Q_4} + \frac{\delta_1 B_1}{Q_4} + \chi_1 Q_1 \right] + 2 \left[\beta_2 - \kappa_3 B_3 + \frac{v_2 B_2^2}{Q_3} + \frac{\eta_2}{Q_4} + \frac{\delta_2 B_2}{Q_4} + \chi_2 Q_2 \right] \right.$$

$$\left. + 2 \left[\beta_3 - \kappa_2 B_2 + \frac{v_3 B_3^2}{Q_2} + \frac{\eta_3}{Q_2} + \frac{\delta_3 B_3}{Q_2} + \chi_3 Q_3 \right] + \left[\beta_4 - \kappa_1 B_1 + \frac{v_4 B_4^2}{Q_1} + \frac{\eta_4}{Q_1} + \frac{\delta_4 B_4}{Q_1} + \chi_4 Q_4 \right] \right\}.$$

Solving the non-constraint problem, now we partially differentiating w.r.t Q_1, Q_2, Q_3, Q_4 respectively,

$$\frac{\partial P}{\partial Q_1} = \frac{1}{6} \left[\chi_1 - \frac{v_4 B_4^2}{Q_1^2} - \frac{\eta_4}{Q_1^2} - \frac{\delta_4 B_4}{Q_1^2} \right].$$

Letting $\frac{\partial P}{\partial Q_1} = 0$.

$$Q_1 = \sqrt{\frac{v_4 B_4^2 + \eta_4 + \delta_4 B_4}{\chi_1}}$$

$$\frac{\partial P}{\partial Q_2} = \frac{1}{6} \left[2\chi_2 - \frac{2v_3 B_3^2}{Q_2^2} - \frac{2\eta_3}{Q_2^2} - \frac{2\delta_3 B_3}{Q_2^2} \right]$$

and

$$\tilde{\chi} = (\chi_1, \chi_2, \chi_3, \chi_4)$$

$$= \frac{1}{6} \left\{ \left[\beta_1 - \kappa_4 B_4 + \frac{v_1 B_1^2}{Q} + \frac{\eta_1}{Q} + \frac{\delta_1 B_1}{Q} + \chi_1 Q \right] \right.$$

$$+ 2 \left[\beta_2 - \kappa_3 B_3 + \frac{v_2 B_2^2}{Q} + \frac{\eta_2}{Q} + \frac{\delta_2 B_2}{Q} + \chi_2 Q \right]$$

$$+ 2 \left[\beta_3 - \kappa_2 B_2 + \frac{v_3 B_3^2}{Q} + \frac{\eta_3}{Q} + \frac{\delta_3 B_3}{Q} + \chi_3 Q \right]$$

$$\left. + \left[\beta_4 - \kappa_1 B_1 + \frac{v_4 B_4^2}{Q} + \frac{\eta_4}{Q} + \frac{\delta_4 B_4}{Q} + \chi_4 Q \right] \right\}.$$

Partially differentiating w.r.t Q and equating to zero,

$$= \frac{1}{6} \left\{ \left[-\frac{v_1 B_1^2}{Q^2} - \frac{\eta_1}{Q^2} - \frac{\delta_1 B_1}{Q^2} + \chi_1 \right] \right.$$

$$+ 2 \left[-\frac{v_2 B_2^2}{Q^2} - \frac{\eta_2}{Q^2} - \frac{\delta_2 B_2}{Q^2} + \chi_2 \right]$$

$$+ 2 \left[-\frac{v_3 B_3^2}{Q^2} - \frac{\eta_3}{Q^2} - \frac{\delta_3 B_3}{Q^2} + \chi_3 \right]$$

$$\left. + \left[-\frac{v_4 B_4^2}{Q^2} - \frac{\eta_4}{Q^2} - \frac{\delta_4 B_4}{Q^2} + \chi_4 \right] \right\} = 0.$$

Letting $\frac{\partial P}{\partial Q_2} = 0$.

$$Q_2 = \sqrt{\frac{v_3 B_3^2 + \eta_3 + \delta_3 B_3}{\chi_2}}$$

$$\frac{\partial P}{\partial Q_3} = \frac{1}{6} \left[2\chi_3 - \frac{2v_2 B_2^2}{Q_3^2} - \frac{2\eta_2}{Q_3^2} - \frac{2\delta_2 B_2}{Q_3^2} \right].$$



Letting $\frac{\partial P}{\partial Q_3} = 0$.

$$Q_3 = \sqrt{\frac{v_2 B_2^2 + \eta_2 + \delta_2 B_2}{\chi_3}}$$

$$\frac{\partial P}{\partial Q_4} = \frac{1}{6} \left[\chi_4 - \frac{v_1 B_1^2}{Q_4^2} - \frac{\eta_1}{Q_4^2} - \frac{2\delta_1 B_1}{Q_4^2} \right].$$

Letting $\frac{\partial P}{\partial Q_4} = 0$.

$$Q_4 = \sqrt{\frac{v_1 B_1^2 + \eta_1 + \delta_1 B_1}{\chi_4}}$$

The above results shows that $Q_1 > Q_2 > Q_3 > Q_4$, but contrastingly we have $0 < Q_1 \leq Q_2 \leq Q_3 \leq Q_4$. Hence w and set $k = 1$ and we convert the inequality constraint by optimizing the total cost subject to Lagrangian method subject to $Q_2 - Q_1 = 0$.

$$L(Q_1, Q_2, Q_3, Q_4, \lambda) = P(ATC(Q, B, P)) - \lambda(Q_2 - Q_1).$$

Now taking the partial derivatives w.r.t Q_1, Q_2, Q_3, Q_4 and λ and the minimize $L(Q_1, Q_2, Q_3, Q_4, \lambda)$, we have

$$\frac{\partial L}{\partial Q_1} = 0$$

$$\frac{1}{6} \left[\chi_1 - \frac{v_4 B_4^2}{Q_1^2} - \frac{\eta_4}{Q_1^2} - \frac{\delta_4 B_4}{Q_1^2} \right] + \lambda = 0$$

$$\frac{\partial L}{\partial Q_2} = 0$$

$$\frac{1}{6} \left[2\chi_2 - \frac{2v_3 B_3^2}{Q_2^2} - \frac{2\eta_3}{Q_2^2} - \frac{2\delta_3 B_3}{Q_2^2} \right] - \lambda = 0$$

$$\frac{\partial L}{\partial Q_3} = 0$$

$$\frac{1}{6} \left[2\chi_3 - \frac{2v_2 B_2^2}{Q_3^2} - \frac{2\eta_2}{Q_3^2} - \frac{2\delta_2 B_2}{Q_3^2} \right] = 0$$

$$\frac{\partial L}{\partial Q_4} = 0$$

$$\frac{1}{6} \left[\chi_4 - \frac{v_1 B_1^2}{Q_4^2} - \frac{\eta_1}{Q_4^2} - \frac{\delta_1 B_1}{Q_4^2} \right] = 0.$$

$$\frac{\partial L}{\partial \lambda} = -(Q_2 - Q_1)$$

$$Q_1 = Q_2 = \sqrt{\frac{(v_4 B_4^2 + v_3 B_3^2) + (\eta_4 + \eta_3) + (\delta_4 B_4 + \delta_3 B_3)}{\chi_1 + 2\chi_2}}$$

$$Q_3 = \sqrt{\frac{v_2 B_2^2 + \eta_2 + \delta_2 B_2}{\chi_3}}$$

$$Q_4 = \sqrt{\frac{v_1 B_1^2 + \eta_1 + \delta_1 B_1}{\chi_4}}$$

Now converting the inequality constraints $Q_2 - Q_1 \geq 0, Q_3 - Q_2 \geq 0$ into equality constraints $Q_2 - Q_1 = 0$ and $Q_3 - Q_2 = 0$.

Optimizing $L(Q_1, Q_2, Q_3, Q_4, \lambda_1, \lambda_2) = P(ATC(Q, B, P)) - \lambda_1(Q_2 - Q_1) - \lambda_2(Q_3 - Q_2)$.

$$\frac{\partial L}{\partial Q_1} = 0$$

$$\frac{1}{6} \left[\chi_1 - \frac{v_4 B_4^2}{Q_1^2} - \frac{\eta_4}{Q_1^2} - \frac{\delta_4 B_4}{Q_1^2} \right] + \lambda_1 = 0$$

$$\frac{\partial L}{\partial Q_2} = 0$$

$$\frac{1}{6} \left[2\chi_2 - \frac{2v_3 B_3^2}{Q_2^2} - \frac{2\eta_3}{Q_2^2} - \frac{2\delta_3 B_3}{Q_2^2} \right] - \lambda_1 + \lambda_2 = 0$$

$$\frac{\partial L}{\partial Q_3} = 0$$

$$\frac{1}{6} \left[2\chi_3 - \frac{2v_2 B_2^2}{Q_3^2} - \frac{2\eta_2}{Q_3^2} - \frac{2\delta_2 B_2}{Q_3^2} \right] - \lambda_2 = 0$$

$$\frac{\partial L}{\partial Q_4} = 0$$

$$\frac{1}{6} \left[\chi_4 - \frac{v_1 B_1^2}{Q_4^2} - \frac{\eta_1}{Q_4^2} - \frac{\delta_1 B_1}{Q_4^2} \right] = 0$$

$$\frac{\partial L}{\partial \lambda_1} = -(Q_2 - Q_1)$$

$$\frac{\partial L}{\partial \lambda_2} = -(Q_3 - Q_2)$$

$$Q_1 = Q_2 = Q_3 = \sqrt{\frac{(v_4 B_4^2 + v_3 B_3^2 + v_2 B_2^2) + (\eta_4 + \eta_3 + \eta_2) + (\delta_4 B_4 + \delta_3 B_3 + \delta_2 B_2)}{\chi_1 + 2\chi_2 + 2\chi_3}}$$

$$Q_4 = \sqrt{\frac{v_1 B_1^2 + \eta_1 + \delta_1 B_1}{\chi_4}}$$

Again converting the inequality constraints $Q_2 - Q_1 \geq 0, Q_3 - Q_2 \geq 0$ and $Q_4 - Q_3 \geq 0$ into equality constraints by $Q_2 - Q_1 = 0, Q_3 - Q_2 = 0$ and $Q_4 - Q_3 = 0$ using Lagrangian method. The Lagrangian function is given by

$$L(Q_1, Q_2, Q_3, Q_4, \lambda_1, \lambda_2, \lambda_3) = P(ATC(Q, B, P)) - \lambda_1(Q_2 - Q_1) - \lambda_2(Q_3 - Q_2) - \lambda_3(Q_4 - Q_3).$$

$$\frac{\partial L}{\partial Q_1} = 0$$

$$\frac{1}{6} \left[\chi_1 - \frac{v_4 B_4^2}{Q_1^2} - \frac{\eta_4}{Q_1^2} - \frac{\delta_4 B_4}{Q_1^2} \right] + \lambda_1 = 0$$

$$\frac{\partial L}{\partial Q_2} = 0$$

$$\frac{1}{6} \left[2\chi_2 - \frac{2v_3 B_3^2}{Q_2^2} - \frac{2\eta_3}{Q_2^2} - \frac{2\delta_3 B_3}{Q_2^2} \right] - \lambda_1 + \lambda_2 = 0$$

$$\frac{\partial L}{\partial Q_3} = 0$$

$$\frac{1}{6} \left[2\chi_3 - \frac{2v_2 B_2^2}{Q_3^2} - \frac{2\eta_2}{Q_3^2} - \frac{2\delta_2 B_2}{Q_3^2} \right] - \lambda_2 + \lambda_3 = 0$$



$$\frac{\partial L}{\partial Q_4} = 0$$

$$\frac{1}{6} \left[\chi_4 - \frac{v_1 B_1^2}{Q_4^2} - \frac{\eta_1}{Q_4^2} - \frac{\delta_1 B_1}{Q_4^2} \right] - \lambda_3 = 0.$$

$$\frac{\partial L}{\partial \lambda_1} = -(Q_2 - Q_1)$$

$$\frac{\partial L}{\partial \lambda_2} = -(Q_3 - Q_2)$$

$$\frac{\partial L}{\partial \lambda_3} = -(Q_4 - Q_3)$$

$$Q_1 = Q_2 = Q_3 = Q_4$$

$$= \sqrt{\frac{(v_4 B_4^2 + v_3 B_3^2 + v_2 B_2^2 + v_1 B_1^2) + (\eta_4 + \eta_3 + \eta_2 + \eta_1) + (\delta_4 B_4 + \delta_3 B_3 + \delta_2 B_2 + \delta_1 B_1)}{\chi_1 + 2\chi_2 + 2\chi_3 + \chi_4}}$$

Therefore $\tilde{Q} = (Q_1, Q_2, Q_3, Q_4)$ satisfies all the inequality constraints and we obtain the optimum solution for the problems. Let $Q_1 = Q_2 = Q_3 = Q_4 = \tilde{Q}^*$. Then optimal fuzzy EOQ is given by

$$\tilde{Q}^* = \sqrt{\frac{(v_4 B_4^2 + v_3 B_3^2 + v_2 B_2^2 + v_1 B_1^2) + (\eta_4 + \eta_3 + \eta_2 + \eta_1) + (\delta_4 B_4 + \delta_3 B_3 + \delta_2 B_2 + \delta_1 B_1)}{\chi_1 + 2\chi_2 + 2\chi_3 + \chi_4}}$$

6. Conclusion

Hereby this paper develops an inventory model with back-orders using fuzzy parameters as decision variables. Hence the economic order quantity for the inventory system is derived by minimizing the total cost inventory function. The notations are taken as trapezoidal fuzzy numbers, Lagrangian method has been applied and defuzzification has been done by graded mean representation method. The numerical examples are contingently showcasing the difference between increased crisp cases against the fuzzy model.

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