Malaya Journal of Matematik, Vol. 7, No. 4, 612-617, 2019

https://doi.org/10.26637/MJM0704/0002

On topological properties of probabilistic neural network

Prosanta Sarkar[1], Sourav Mondal[2], Nilanjan De[3]* and Anita Pal[4]

Abstract

A graphical invariant is a real number related to a graph which is fixed under the graph isomorphism. In chemical graph theory, these invariants are also called topological indices and these are play a vital role to predict various chemical and physical properties of different molecular structures. In this work, we generalized multiplicative version Zagreb indices and compute it for probabilistic neural network. Also, we compute the general Zagreb index or (a,b)-Zagreb index for the same network and compute some other degree based topological indices for some particular values of a and b.

Keywords

Probabilistic neural network, Vertex degree based topological indices, The general Zagreb index, Multiplicative version of general Zagreb index.

AMS Subject Classification

26A33, 30E25, 34A12, 34A34, 34A37, 37C25, 45J05.

[1,2,4] *Department of Mathematics, National Institute of Technology Durgapur, India.*

[3] *Department of Basic Sciences and Humanities (Mathematics), Calcutta Institute of Engineering and Management, Kolkata - 700 040, India.*

***Corresponding author**: [1] prosantasarkar87@gmail.com; [2] souravmath94@gmail.com; [3] de.nilanjan@rediffmail.com; [4] anita.buie@gmail.com

Article History: Received **12** March **2019**; Accepted **26** August **2019**

Contents

1. Introduction

In graph theory, a graph G is defined as the order pair of two sets namely $V(G)$ and $E(G)$ or simply denoted as $G=(V(G),E(G))$, where $V(G)$ and $E(G)$ are the vertex set and edge set of G. Let n and m represents the number of vertices and edges of G respectively that is $|V(G)|=n$ and $|E(G)|=m$. The degree of a vertex $k\in V(G)$ is defined as the number of adjacent vertices of k in G and denoted as $d_G(k)$. In chemical graph theory, a molecular structure can be represented by vertices and edges where vertices denotes atoms and edges denotes the bonds between atoms. A topological index is a real number related to a chemical constitution for correlation of a chemical structure with various physicochemical properties. Different topological indices of a chemical compound help us to predict the behaviour of chemical reactivity or biological activity theoretically. In chemical graph theory, there are various topological indices were introduced by different researchers, among which the Zagreb indices are oldest and extremely studied vertex degree-based topological indices and were introduced by Gutman and Trinajestić in 1972 [1], to study the total π-electron energy (ε) of carbon atoms and are defined as

$$M_1(G)=\sum_{k\in V(G)} d_G(k)^2=\sum_{kt\in E(G)}[d_G(k)+d_G(t)]$$

and

$$M_2(G)=\sum_{kt\in E(G)} d_G(k)d_G(t).$$

See [2–5], for some recent study about this index. In 2011 [6], I. Gutman was first introduce the multiplicative version of Zagreb indices and are defined as

$$\prod\nolimits_1(G)=\prod_{k\in V(G)} d_G(k)^2$$

and

$$\prod\nolimits_2(G)=\prod_{kt\in E(G)} d_G(k)d_G(t).$$

For further study about this index we encourage the reader to [7–10]. Based on first Zagreb index M. Eliasi et al. first investigate the multiplicative sum Zagreb index in 2012 [11] and is defined as

$$\prod_1^*(G)=\prod_{kt\in E(G)}[d_G(k)+d_G(t)].$$

We refer our reader to [12–15], for further study about this index. The forgotten topological index was introduced in the same paper where Zagreb indices were introduced [1], but that time not much studied about this index in 2015, Furtula and Gutman reinvestigate this index again in [16] and is defined as

$$F(G)=\sum_{kt\in E(G)}[d_G(k)^2+d_G(t)^2].$$

The multiplicative version of forgotten topological index is defined as

$$\prod F(G)=\prod_{kt\in E(G)}[d_G(k)^2+d_G(t)^2].$$

Based on multiplicative sum Zagreb index in this paper, we generalised this index as

$$\prod_\alpha^*(G)=\prod_{kt\in E(G)}(d_G(k)^\alpha+d_G(t)^\alpha)$$

where, $\alpha\neq 0,\ 1$ *and* $\alpha\in\mathbb{R}$. Based on Randić index Gutman and Lepović introduced the general Randić index in [17] and is defined as

$$R_a=\sum_{hk\in E(G)}\{d_G(h).d_G(k)\}^a.$$

Where, $a\neq 0,\ a\in\mathbb{R}$. In this paper, we generalised second multiplicative Zagreb index and is defined as

$$\prod_2^\alpha(G)=\prod_{kt\in E(G)}(d_G(k)d_G(t))^\alpha$$

where, $\alpha\neq 0,$ *and* $\alpha\in\mathbb{R}$. Followed by first Zagreb index and F-index Li and Zheng in [18], introduced the general first Zagreb index and is defined as

$$M^\alpha(G)=\sum_{h\in V(G)}d_G(h)^\alpha$$

where, $\alpha\neq 0,\ 1$ *and* $\alpha\in\mathbb{R}$. The Symmetric division deg index is defined as

$$SDD(G)=\sum_{hk\in E(G)}[\frac{d_G(h)}{d_G(k)}+\frac{d_G(k)}{d_G(h)}].$$

Based on symmetric division deg index in this paper, we defined the multiplicative symmetric division deg index as

$$\prod SDD(G)=\prod_{kt\in E(G)}[\frac{d_G(k)}{d_G(t)}+\frac{d_G(t)}{d_G(k)}].$$

The redefined version of Zagreb index was introduced by Ranjini et al. [19], In 2013 and is defined as

$$ReZM(G)=\sum_{hk\in E(G)}d_G(h)d_G(k)[d_G(h)+d_G(k)].$$

For some recent study about this index we encourage our reader to [20–22]. Followed by redefined Zagreb index in this paper, we defined multiplicative redefined version of Zagreb index as

$$\prod ReZM(G)=\prod_{kt\in E(G)}d_G(k)d_G(t)[d_G(k)+d_G(t)].$$

Azari et al.[23], in 2011 generalized the Zagreb indices as

$$Z_{a,b}(G)=\sum_{hk\in E(G)}(d_G(h)^a d_G(k)^b+d_G(h)^b d_G(k)^a)$$

and named as the general Zagreb index or (a,b)-Zagreb index. We refer our reader to [24–26], for some recent study about this index. Based on multiplicative version second Zagreb index and multiplicative version sum Zagreb index in this paper, we introduced the multiplicative version of general Zagreb index as

$$\prod Z_{a,b}(G)=\prod_{kt\in E(G)}(d_G(k)^a d_G(t)^b+d_G(k)^b d_G(t)^a).$$

The Table 1 and Table 3, shows the relation between general Zagreb index also known as (a,b)-Zagreb index and the multiplicative version of general Zagreb index to some other vertex degree based topological indices which are mentioned in this work, earlier for some particular values of a and b. Recently, artificial neural networks have been successfully used to many divers field for pattern classification, system modelling and identification for signal processing, image processing, control systems and stock market predictions. An artificial neural network is a computational model which work like the working of a human nervous system. There are various types of artificial neural networks. These kinds of networks are implemented depending on mathematical operations and a set of parameters required to investigate the output. A feed forward neural network is one of the simplest form of artificial neural network, where the data passes through the input nodes and exit on the output nodes in one way. A probabilistic neural network is one type of feed forward neural network, which is widely used in classification and pattern recognition problems. Probabilistic neural networks are more accurate and faster than any multilayer perceptron networks. Probabilistic neural network-based sensor configuration management in a wireless ad hoc network and is also used to modelling structural deterioration of storm water pipes, class prediction of Leukemia and Embryonal Tumor of central nervous system, remote sensing image classification and so fourth. Recently, M. Javaid and J. Cao compute some degree based topological indices of probabilistic neural network in [27] and J.B. Liu et al. studied the topological properties of certain neural networks in [28]. In this paper, we first defined the multiplication version general Zagreb index, the general Zagreb index or (a,b)-Zagreb index and compute these indices for Probabilistic neural network $PNN[x,y,z]$ and derived some other topological indices for some particular values of a and b. A example of a probabilistic neural network $PNN[4,2,3]$ is shown in Figure 1.

2. Main Results

In this section, we compute the multiplicative version general Zagreb index and the general Zagreb index of probabilistic neural network $PNN[x,y,z]$. First we compute multiplicative version of general Zagreb index. The edge sets of $PNN[x,y,z]$ are divided into two sets and the degree of all the vertex are shown in Table 2.

Theorem 2.1. *The multiplicative version general Zagreb index of $PNN[x,y,z]$ is given by*

$\prod Z_{a,b}(PNN[x,y,z])$

$$\begin{aligned} &= \{(yz)^a.(x+1)^b+(yz)^b.(x+1)^a\}^{xyz} \\ &\quad \times\{(x+1)^a.z^b+(x+1)^b.z^a\}^{yz}. \end{aligned} \tag{2.1}$$

Proof. From definition of multiplicative version general Zagreb index, we get

$\prod Z_{a,b}(PNN[x,y,z])$

$$\begin{aligned} &= \prod_{kt\in E(PNN[x,y,z])}(d_G(k)^a d_G(t)^b+d_G(k)^b d_G(t)^a) \\ &= \prod_{kt\in E_1(PNN[x,y,z])}\{(yz)^a.(x+1)^b+(yz)^b.(x+1)^a\} \\ &\quad \times \prod_{kt\in E_2(PNN[x,y,z])}\{(x+1)^a.(z)^b+(z)^a.(x+1)^b\} \\ &= \{(yz)^a.(x+1)^b+(yz)^b.(x+1)^a\}^{|E_1(PNN[x,y,z])|} \\ &\quad \times\{(x+1)^a.(z)^b+(z)^a.(x+1)^b\}^{|E_2(PNN[x,y,z])|} \\ &= \{(yz)^a.(x+1)^b+(yz)^b.(x+1)^a\}^{xyz} \\ &\quad \times\{(x+1)^a.(z)^b+(z)^a.(x+1)^b\}^{yz}. \end{aligned}$$

Hence, the theorem. □

Corollary 2.2. *Using Equation 2.1, computing some other multiplicative version topological indices for some particular values of a and b in the following:*

(i) $\prod_1^*(PNN[x,y,z])$

$$= \prod Z_{1,0}(PNN[x,y,z]) = (x+yz+1)^{xyz}\times(x+z+1)^{yz},$$

(ii) $\prod_2(PNN[x,y,z])$

$$\begin{aligned} &= \frac{1}{2^{yz(x+1)}}Z_{1,1}(PNN[x,y,z]) = \{yz(1+x)\}^{xyz} \\ &\quad \times\{z(x+1)\}^{yz}, \end{aligned}$$

(iii) $\prod F(PNN[x,y,z])$

$$\begin{aligned} &= \prod Z_{2,0}(PNN[x,y,z]) = \{(yz)^2+(x+1)^2\}^{xyz} \\ &\quad \times\{(x+1)^2+z^2\}^{yz}, \end{aligned}$$

(iv) $\prod ReZM(PNN[x,y,z])$

$$\begin{aligned} &= \prod Z_{2,1}(PNN[x,y,z]) = \{yz(x+1)(yz+x+1)\}^{xyz} \\ &\quad \times\{z(x+1)(x+z+1)\}^{yz}, \end{aligned}$$

(v) $\prod_\alpha^*(PNN[x,y,z])$

$$\begin{aligned} &= \prod Z_{\alpha,0}(PNN[x,y,z]) = \{(yz)^\alpha+(x+1)^\alpha\}^{xyz} \\ &\quad \times\{(x+1)^\alpha+z^\alpha\}^{yz}, \end{aligned}$$

(vi) $\prod_2^\alpha(PNN[x,y,z])$

$$\begin{aligned} &= \frac{1}{2^{yz(x+1)}}\prod Z_{\alpha,\alpha}(PNN[x,y,z]) = \{yz(x+1)\}^{\alpha xyz} \\ &\quad \times\{z(x+1)\}^{\alpha yz}, \end{aligned}$$

(vii) $\prod SDD(PNN[x,y,z])$

$$\begin{aligned} &= \prod Z_{1,-1}(PNN[x,y,z]) = \{\frac{yz}{x+1}+\frac{x+1}{yz}\}^{xyz} \\ &\quad \times\{\frac{x+1}{z}+\frac{z}{x+1}\}^{yz}. \end{aligned}$$

Now, we compute the general Zagreb index of probabilistic neural network $PNN[x,y,z]$ and computing some other vertex degree based topological indices for some particular values of a and b. The following Table 3 shows the relations between the general Zagreb index or $(a,b)-$Zagreb index with some other topological indices.

Theorem 2.3. *The general Zagreb index of $PNN[x,y,z]$ is given by*

$Z_{a,b}(PNN[x,y,z])$

$$\begin{aligned} &= xyz\{(yz)^a.(x+1)^b+(yz)^b.(x+1)^a\} \\ &\quad +yz\{(x+1)^a.z^b+(x+1)^b.z^a\}. \end{aligned} \tag{2.2}$$

Proof. From definition of general Zagreb index, we get

$Z_{a,b}(PNN[x,y,z])$

Table 1. Relations between multiple general Zagreb index with some other multiplicative version topological indices:

Topological index	Corresponding multiple general Zagreb index
Multiplicative sum Zagreb index $\prod_1^*(G)$	$\prod Z_{1,0}(G)$
Second multiplicative Zagreb index $\prod_2(G)$	$\frac{1}{2^m}\prod Z_{1,1}(G)$
Multiplicative version of forgotten topological index $\prod F(G)$	$\prod Z_{2,0}(G)$
Multiplicative redefined Zagreb index $\prod ReZM(G)$	$\prod Z_{2,1}(G)$
General multiplicative sum Zagreb index $\prod_\alpha^*(G)$	$\prod Z_{\alpha,0}(G)$
General second multiplicative Zagreb index $\prod_2^\alpha(G)$	$\frac{1}{2^m}\prod Z_{\alpha,\alpha}(G)$
Multiplicative symmetric division deg index $\prod SDD(G)$	$\prod Z_{1,-1}(G)$

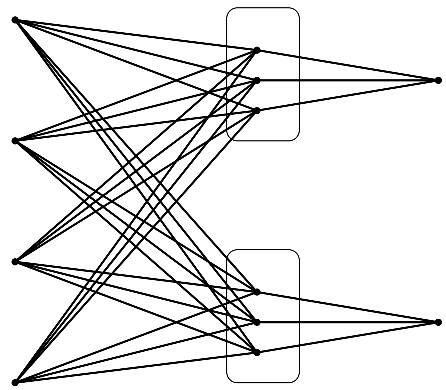

Figure 1. The example of a probabilistic neural network $PNN[4,2,3]$

Table 2. Edge partition of $PNN[x,y,z]$ network

$(d(k),d(t)):\ kt \in E(PNN[x,y,z])$	Total number of edges
(yz,x+1)	xyz
(x+1,z)	yz

Table 3. Relations between generalized Zagreb indices with some other vertex degree based topological indices:

Topological index	Corresponding general Zagreb index
First Zagreb index $M_1(G)$	$Z_{1,0}(G)$
Second Zagreb index $M_2(G)$	$\frac{1}{2}Z_{1,1}(G)$
Forgotten topological index $F(G)$	$Z_{2,0}(G)$
Redefined Zagreb index $ReZM(G)$	$Z_{2,1}(G)$
General first Zagreb index $M^a(G)$	$Z_{a-1,0}(G)$
General Randić index $R_a(G)$	$\frac{1}{2}Z_{a,a}$
Symmetric division deg index $SDD(G)$	$Z_{1,-1}(G)$

$$\begin{aligned}
&= \sum_{kt\in E(PNN[x,y,z])} (d_G(k)^a d_G(t)^b + d_G(k)^b d_G(t)^a) \\
&= \sum_{kt\in E_1(PNN[x,y,z])} \{(yz)^a.(x+1)^b + (yz)^b.(x+1)^a\} \\
&\quad + \sum_{kt\in E_2(PNN[x,y,z])} \{(x+1)^a.(z)^b + (z)^a.(x+1)^b\} \\
&= |E_1(PNN[x,y,z])|\{(yz)^a.(x+1)^b + (yz)^b.(x+1)^a\} \\
&\quad +|E_2(PNN[x,y,z])|\{(x+1)^a.(z)^b + (z)^a.(x+1)^b\} \\
&= xyz\{(yz)^a.(x+1)^b + (yz)^b.(x+1)^a\} \\
&\quad +yz\{(x+1)^a.(z)^b + (z)^a.(x+1)^b\}.
\end{aligned}$$

Hence, the result follows as in Theorem 2.3. □

Corollary 2.4. *Using Equation 2.2, computing some other degree based topological indices for some particular values of a and b in the following:*

(i) $M_1(PNN[x,y,z])$

$$= Z_{1,0}(PNN[x,y,z]) = xyz(x+yz+1) + yz(x+z+1),$$

(ii) $M_2(PNN[x,y,z])$

$$= \frac{1}{2}Z_{1,1}(PNN[x,y,z]) = xyz\{yz(1+x)\} + yz\{z(x+1)\},$$

(iii) $F(PNN[x,y,z])$

$$\begin{aligned}= Z_{2,0}(PNN[x,y,z]) &= xyz\{(yz)^2 + (x+1)^2\} \\ &+yz\{(x+1)^2 + z^2\},\end{aligned}$$

(iv) $ReZM(PNN[x,y,z])$

$$\begin{aligned}= Z_{2,1}(PNN[x,y,z]) &= xyz\{yz(x+1)(yz+x+1)\} \\ &+yz\{z(x+1)(x+z+1)\},\end{aligned}$$

(v) $M^a(PNN[x,y,z])$

$$\begin{aligned}= Z_{a-1,0}(PNN[x,y,z]) &= xyz\{(yz)^{a-1} + (x+1)^{a-1}\} \\ &+yz\{(x+1)^{a-1} + z^{a-1}\},\end{aligned}$$

(vi) $R_a(PNN[x,y,z])$

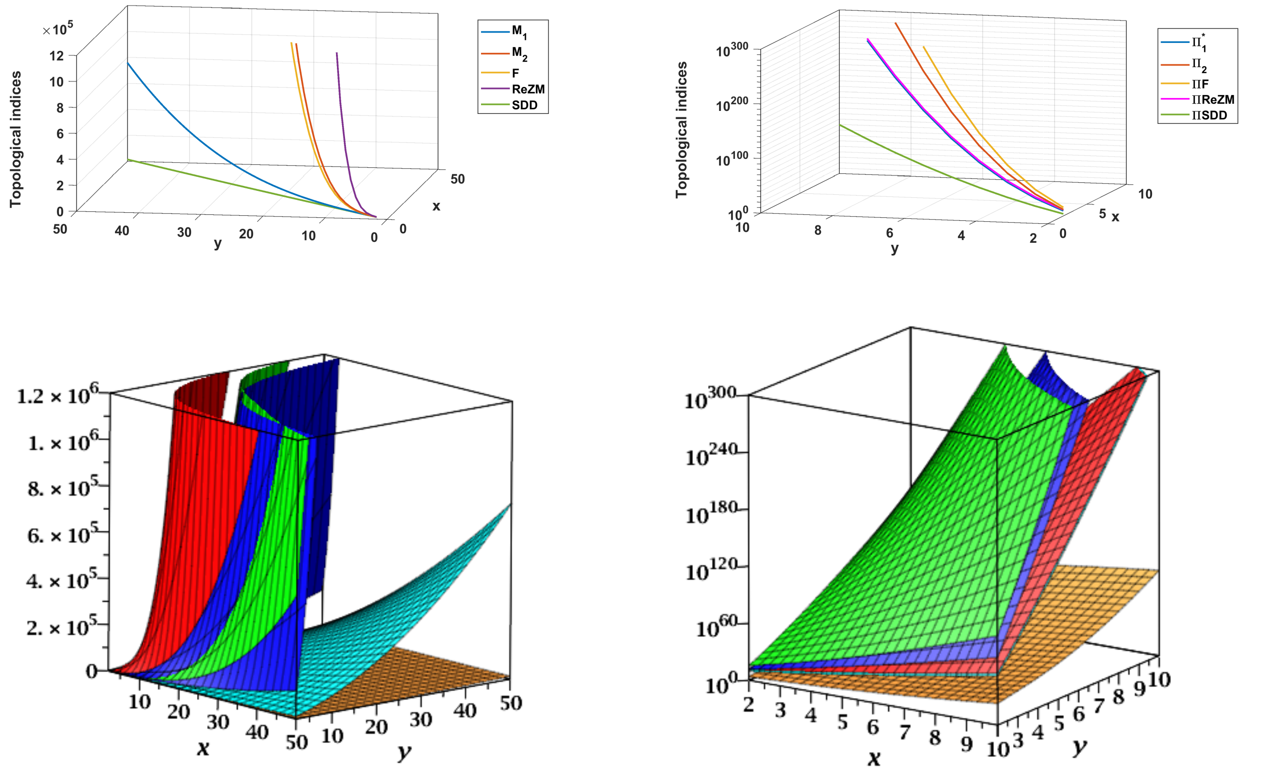

Figure 2. Comparative graphical representations of $M_1,\ M_2,\ F-index,\ ReZM\ and\ SSD$ with $\prod_1^*,\ \prod_2,\ \prod F, \prod ReZM\ and\ \prod SSD$ of probabilistic neural network based on line graphs and their corresponding surface graphs. Here we consider $z=2$ is fixed.

$$= \frac{1}{2}Z_{a,a}(PNN[x,y,z]) = xyz\{yz(x+1)\}^a + yz\{z(x+1)\}^a,$$

(vii) $SDD(PNN[x,y,z])$

$$= Z_{1,-1}(PNN[x,y,z]) = xyz\{\frac{yz}{x+1} + \frac{x+1}{yz}\} + yz\{\frac{x+1}{z} + \frac{z}{x+1}\}.$$

3. Conclusion

In this paper, we compute the general Zagreb index and a new multiplicative version general Zagreb index of probabilistic neural network. Also, we compute some other degree based topological indices for some particular values of a and b. For future study, some other networks can be considered for studying this multiplicative version general Zagreb index.

Acknowledgement

The first author would like to express special thanks of gratitude to CSIR, HRDG, New Delhi, India for their financial support under the grant no. 09/973(0016)/2017-EMR-I.

References

[1] I. Gutman and N. Trinajestić, Graph theory and molecular orbitals total π-electron energy of alternant hydrocarbons, *Chem. Phys. Lett.*, 17,(1972), 535-538.

[2] P. Sarkar, N. De and A. Pal, The Zagreb indices of graphs based on new operations related to the join of graphs, *J. Int. Math. Virtual Inst.*, 7,(2017), 181-209.

[3] M.K. Siddiqui, M. Imran and A. Ahmad, On Zagreb indices, Zagreb polynomials of some nanostar dendrimers, *Appl. Math. Comput.*, 280, (2016), 132-139.

[4] D. Sarala, S.K. Ayyaswamy and S. Balachandran, The Zagreb indices of graphs based on four new operations related to the lexicographic product, *Appl. Math. Comput.*, 309, (2017), 156-169.

[5] H. Deng, D. Sarala, S.K. Ayyaswamy and S. Balachandran, The Zagreb indices of four operations on graphs, *Appl. Math. and Comput.*, 275,(2016), 422-431.

[6] I. Gutman, Multiplicative Zagreb indices of trees, *Bull. Int. Math. Virt. Inst.*, 1, (2011), 13-19.

[7] B. Basavanagoud and S. Patil, Multiple Zagreb indices and coindices of some derived graphs, *Opuscula Math.*, 36, (3), (2016), 287-299.

[8] A. Ghalavand, A.R. Ashrafi and I. Gutman, Extremal graphs for the second multiplicative Zagreb index, *Bull. Int. Math. Virt. Inst.*, 8, (2018), 369-383.

[9] R. Muruganandam, R.S. Manikandan and M. Aruvi, The multiplicative Zagreb indices of products of graphs, *Int. J. Math. Res.*, 8, (1), (2016), 61-69.

[10] M. Ghorbani and N. Azimi, Note on multiplicative Zagreb indices, *Iran. J. Math. Chem.*, 3, (2), (2012), 137-143.

[11] M. Eliasi, A. Iranmanesh and I. Gutman, Multiplicative versions of first Zagreb index, *MATCH Commun. Math. Comput. Chem.*, 68, (2012), 217-230.

[12] V. Božović, Z. K. Vukićević and G. Popivoda, Extremal values of total multiplicative sum Zagreb index and first multiplicative sum Zagreb coindex on unicycle and bycyclic graphs, *MATCH Commun. Math. Comput. Chem.*, 78, (2017), 417-430.

[13] M. Azari and A. Iranmanesh, Some inequalities for the multiplicative sum Zagreb index of graph operations, *J. Math. Inequalities*, 9, (3), (2015), 727-738.

[14] M. Eliasi and A. G. Ghalavand, On trees and the multiplicative sum Zagreb index, *Commun. Comb. Opt.*, 1, (2), (2016), 137-148.

[15] M. Azari, Multiplicative sum Zagreb index of splice, bridge, and bridge-cycle graphs, *Bol. Soc. Paran. Mat.*, (2018), DOI:10.5269/bspm.40503.

[16] B. Furtul and I. Gutman, A forgotten topological index. *J. Math. Chem.*, 53, (2015), 1184-1190.

[17] I. Gutman and M. Lepović, Choosing the exponent in the definition of the connectivity index. *J. Serb. Chem. Soc.*, 66, (9), (2001), 605-611.

[18] X. Li and J. Zheng, A unified approach to the extremal trees for different indices. *MATCH Commun. Math. Comput. Chem.*, 54, (2005), 195-208.

[19] P.S. Ranjini, V. Lokesha and A. Usha, Relation between phenylene and hexagonal squeeze using harmonic index. *Int. J. Graph Theory*, 1, (2013), 116-121.

[20] B. Zhao, J. Gan and H. Wu, Redefined Zagreb indices of some nano structures. *Appl. Math. Nonlinear Sci.*, 1, (1), (2016), 291-300.

[21] Y.C. Kwun, A.R. Virk, W. Nazeer, W. Gao and S.M. Kang, Zagreb polynomials and redefined Zagreb indices of silicon-carbon $Si_2C_3 - I[p,q]$ *and* $Si_2C_3 - II[p,q]$, (2018), *DOI:10.20944/preprints 201806.0038v1.*

[22] R.P. Kumar, D.S. Nandappa and M.R.R. Kanna, Redefined zagreb, Randić, Harmonic, GA indices of graphene. *Int. J. Math. Anal.*, 11, (10), (2017), 493-502.

[23] M. Azari and A. Iranmanesh, Generalized Zagreb index of graphs. *Studia Univ. Babes-Bolyai.*, 56, (3), (2011), 59-70.

[24] P. Sarkar, N. De and A. Pal, The generalized Zagreb index of some carbon structures. *Acta Chem. Iasi*, 26, (1), (2018), 91-104.

[25] P. Sarkar, N. De, I.N. Congul and A. Pal, Generalized Zagreb index of some dendrimer structures. *Uni. J. Math. Appl.*, 1, (3), (2018), 160-165.

[26] P. Sarkar, N. De, I.N. Congul and A. Pal, Generalized Zagreb index of some derived networks. *J. Taibah Uni. Sci.*, (2018), DOI:10.1080/16583655.2018.1535881.

[27] M. Javaid and J. Cao, Computing topological indices of probabilistic neural network, *Neural Comput, Aplic.*, (2017), DOI: 10.1007/s00521-017-2972-1.

[28] J.B. Liu, J. Zhao, S. Wang, M. Javaid and J. Cao, On the topological properties of the certain neural networks, *J. Artificial Intell. Soft Comput. Res.*, 8, (4), (2018), 257-268.

⋆⋆⋆⋆⋆⋆⋆⋆⋆

ISSN(P):2319 − 3786

Malaya Journal of Matematik

ISSN(O):2321 − 5666

⋆⋆⋆⋆⋆⋆⋆⋆⋆