Malaya Journal of Matematik, Vol. 7, No. 4, 632-642, 2019
https://doi.org/10.26637/MJM0704/0005

Analysis of thermoelastic characteristics of disk using linear properties of material

Dinkar Sharma[1], Ramandeep Kaur[2]* and Honey Sharma[3]

Abstract
In this paper, finite element method (FEM) is applied on vibrating disk to study thermoelastic characteristics (stress, strain and displacement). Thermoelastic characteristics of disk are examined under two distinct cases of temperature distribution (uniform and steady-state). The material properties young's modulus, coefficient of thermal expansion and density are considered as constant as well as linear function of radius of the disk The materials Aluminimu (Al) and Alumina (Al_2O_3) are considered for construction of functionally graded material (FGM) disk. Further, Poisson's ratio taken as constant because an effect of Poisson's ration on thermoelastic characteristics is negligible. To find solution of governing equation standard discretization approach of finite element method is used. The Graphical results show's significance variation of the Radial stress, Circumferential stress, Radial strain, Circumferential strain and Displacement with respect to normalized radial distance and Kibel Number. The analysis of the results shows that thermoelastic characteristics are not independent of temperature distribution as well as material properties.

Keywords
Functionally Graded Material; Axisymmetric; Thermoelastic field; Finite Element Method, Kibel Number.

AMS Subject Classification
74A15, 80M10, 74S05

[1] *Department of Mathematics, Lyallpur Khalsa Collge, Jalandhar-144001, India.*
[2] *Department of Mathematics, I.K. Gujral Punjab Technical University,Kapurthala-144603, India.*
[3] *Department of Mathematics, Gulzar Group of Institutes, Khanna-141401, India.*
***Corresponding author**: [2] kang.raman1989@gmail.com
Article History: Received **17** May **2019**; Accepted **15** September **2019**

Contents

1 Introduction **633**
2 Mathematical Modelling of FGM Disk **633**
3 Formulation of the Problem **634**
3.1 Uniform temperature distribution in FGM disk (TD-I) 634
3.2 Steady-state temperature distribution in FGM disk (TD-II) 634
4 Boundary Conditions **634**
5 Basic Equation **634**
6 Finite Element Formulation of Problem **634**
6.1 In case of Temperature Distribution I 634
6.2 In case of Temperature Distribution II 634
7 Results and Discussions **635**
7.1 For n=0(Constant Distribution) under Temperature Distribution I and Temperature Distribution II . 635
7.2 For n=1(Linear Distribution) under Temperature Distribution I and Temperature Distribution II 637
7.3 For n=0(Constant distribution) and Temperature Distribution I for different values of Kibel Number ... 638
7.4 For n=0(Constant Distribution) and Temperature Distribution II for different values of Kibel Number ... 639
7.5 For n=1(Linear distribution) and Temperature Distribution I for different values of Kibel Number 640
7.6 For n=1(Linear distribution) and Temperature Distribution II for different values of Kibel Number 641

8 **Conclusion** 641
References 642

1. Introduction

The study of circular disks has received a great deal of attention because of their industrial application in computer memory disk, disk brakes, turbine roots and circular saw blades. FGM is used to make such type of circular disks. FGM's disks maintain structural integrity ensuring thermal conductivity, machinability and toughness in unfavourable environment of high heat, high rate corrosive wear. Therefore, the researcher's give emphasis on study FGM circular disk characteristics. Many numerical as well as analytical methods are developed to study the characteristics.

The finite element method is one of the powerful numerical tool to solve ordinary and partial differential equations. Finite element method is closely related to Galerkin's method and Rayleigh's power method. The literature regarding finite element method is given in [1–4]. The finite element method was used to investigate thermoelastic field in rotating FGM disk, where the material properties of disk vary exponentially in radial direction only[5]. Thermoelastic field studied by finite difference method in FGM disk under mechanical and thermal properties of disk vary as the power function and exponential variation of radius of disk[6]. Wave progagation studied in rotating thermoelastic media which includes the study of phase velocity, attenuation and specific loss with respect to kibel number[7]. Thermal stress discussed in rotating functionally graded hollow circular disk by converting the problem into Fredholm integral equation[8]. Bluk waves studied in rotating thermoelastic media by using voids[9]. A thermoelastic analysis represented by using two types of deflection, small and large, In case of small deflection exact solution was obtained for displacement field and In case large deflection solution was obtained in form of series[10]. Stress, strain and displacement investigated by using finite element method in a rotating homogeneous thermoelastic annular disk under different types of temperature fields[11]. Deformation and stress predicted in rotating disk by finite element method and direct numerical integration of governing differential equation[12]. Analytical as well as numerical solution carried out for rotating annular disk with variable thickness[13]. The effect of temperature distribution, angular speed and thickness showed on displacement, stress and strain for thin FGM annular disk[14]. Stress was investigated for functionally graded rotating disc by using analytical and numerical method[15]. Investigation of thermal stress carried out for functionally graded hollow circular cylinders[16]. Thermoelastic characteristics studied for exponentially vary material properties in disk[17]. A finite element formulation developed for analysis of functionally graded plates and sheets[18]. Analysis of rotating disk carried out by finite element method either in a space-fixed co-ordinate or body-fixed system[20]. Thermoelastic stress, strain and displacement discussed by finite element method for thin circular FGM disk subjected to thermal load[21]. Thermoelastic characteristics analyzed in rotating homogeneous circular disk by using Kibel Number[22]. The thermoelastic response of finite grid disk studied under variable thickness[23]. Stress in functionally graded rotating disks obtained by using finite difference method with non-uniform thickness and variable angular velocity[24]. Displacement and stress calculated in rotating FGM circular disk for a variable speed and mass density studied by using Runge-Kutta method[25].

In the present study, material properties named as young's modulus (E), coefficient of thermal expansion (α) and density (ρ) taken as a constant and linear function of radius r. However the value of Poisson ratio is taken as constant. Two temperature distributions are taken on rotating FGM disk which consists of two types of materials. Matlab programming is used to find the solution of governing equation by FEM. For constant and linear properties of material thermoelastic characteristics values such as stress, strain and displacement are calculated numerically with the help of FEM and then presented graphically.

2. Mathematical Modelling of FGM Disk

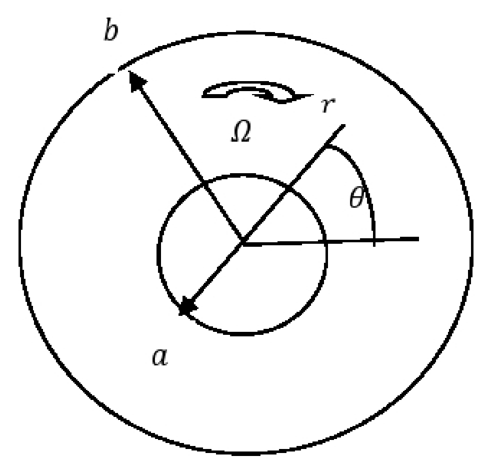

Fig 1. Diagram of rotating FGM disk

As shown in Fig1., We consider concentric circular hole disk which is constructed by using functionally graded materials. The FGM disk consists of two distinct types of materials named as A and B, where A material is used for construction of inner surface and B is used for construction of outer surface construction. The problem is axisymmetric because the distribution of two materials varies continuously along its radial direction. It is assumed that the material properties named as young's modulus (E), coefficient of thermal expansion (α) and density (ρ) of the disk as power function

Table 1. Properties of Al and Al_2O_3

Material	E(MPa)	$\alpha(/^0C)$	$\rho(g/cm^3)$
AI	71	23.1×10^{-6}	2.70
AI_2o_3	380	8.0×10^{-6}	0.96

variation according as:

$$E = E_A + E_B r^n$$
$$\alpha = \alpha_A + \alpha_B r^n$$
$$\rho = \rho_A + \rho_B r^n \tag{2.1}$$

The material properties of A and B are represents by using subscript A and B respectively. The Materials Al and Al_2O_3 are used for construction of inner and outer surface of FGM disk. For ingredient material thermo-mechanical properties are shown in table1.

3. Formulation of the Problem

We take two different cases of thermal variation as:

3.1 Uniform temperature distribution in FGM disk (TD-I)

: The temperature distribution remains constant in this case therefore temperature is written as:

$$T(r) = T_0, \qquad \frac{dT}{dr} = 0 \tag{3.1}$$

3.2 Steady-state temperature distribution in FGM disk (TD-II)

:The temperature distribution remains steady therefore $\frac{\partial}{\partial t} = 0$, where heat conduction equation as given in [19] and the temperature distribution written as:

$$(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr})T = 0$$

With the help of thermal conduction the solution of this equation is

$$T(r) = bT_0 log(r/a), \frac{dT}{dr} = \frac{bT_0}{r} \tag{3.2}$$

4. Boundary Conditions

To find the boundary conditions of problem, we consider that inside surface of the disk is fixed to a shaft and outside surface is continued at consistent temperature gradient. Hence, the two boundary conditions are:

$$\mu_r = 0, \qquad T = 0, \qquad at r = a \tag{4.1}$$

$$\sigma_r = 0, \qquad \frac{dT}{dr} = T_0, \qquad at r = b \tag{4.2}$$

5. Basic Equation

By using equilibrium equations in polar coordinates for two dimensions, thermal eigenstrain, total strain, Hookes Law[21] for axisymmetric problem we obtained a differential equation as:

$$\begin{aligned} r\frac{d^2F}{dr^2} &+ (1 - \frac{nE_A r^n}{E})\frac{dF}{dr} + (\frac{nE_A r^n \upsilon}{E} - 1)\frac{F}{r} \\ &= (1+\Gamma^2) r^2 \Omega^2 (\frac{nE_A r^n}{E}\rho - 3\rho - n\rho_A r^n - \upsilon\rho) \\ &- Er(T(r)\alpha_A n r^{n-1} + \alpha(r) T''(r) \end{aligned} \tag{5.1}$$

where Ω is angular speed of rotation in the disk, ω is angular frequency of vibration in the disk, $T(r)$ is the change in temperature at any distance r,υ is Poisson's ratio of the material and Γ is kibel number.

6. Finite Element Formulation of Problem

Using equation (5.1), we get

$$\frac{d^2F}{dr^2} + P\frac{dF}{dr} + Q\frac{F}{r} + R\Omega^2(1+\Gamma^2) = S \tag{6.1}$$

where P,Q,R and S are defined as:

6.1 In case of Temperature Distribution I

$$P = (\frac{1}{r} - \frac{nE_A r^{n-1}}{E}),$$

$$Q = (\frac{nE_A r^{n-1}}{E} - \frac{1}{r}),$$

$$R = (3\rho + n\rho_A r^n + \nu\rho - \frac{nE_A r^n \rho}{E}),$$

$$S = -ET_0 \alpha_A n r^{n-1}$$

6.2 In case of Temperature Distribution II

$$P = (\frac{1}{r} - \frac{nE_A r^{n-1}}{E}),$$

$$Q = (\frac{nE_A r^{n-1}\upsilon}{E} - \frac{1}{r}),$$

$$R = (3\rho + n\rho_A r^n + \upsilon\rho - \frac{nE_A r^n}{E}\rho),$$

$$S = -EbT_0(\frac{\alpha(r)}{r} + \alpha_A n r^{n-1} log(\frac{r}{a}))$$

In this problem, a standard discretization approach of finite element is used to solve the differential equation. In this discretization the element size is equal and total domain is divided into N elements and then equation is converted into simultaneous equations:

$$\sum_{j=1}^{2} K_{ij}^{e} F_{j}^{e} = L_{i}^{e} \qquad i = 1,2; \qquad e = 1,2,3,.....N \tag{6.2}$$

The values K_{ij}^{e} and L_{i}^{e}, in each case are given by:

$$K_{ij}^{e} = \int_{r_e}^{r_{e+1}} \frac{d\phi_i^e}{dr}\frac{d\phi_j^e}{dr} dr - \int_{r_e}^{r_{e+1}} (\frac{1}{r} - \frac{nE_A r^{n-1}}{E})\phi_i^e \frac{d\phi_j^e}{dr} dr \tag{6.3}$$

$$- \int_{r_e}^{r_{e+1}} \frac{1}{r}(\frac{nE_A r^{n-1}\upsilon}{E} - \frac{1}{E})\phi_i \phi_j dr$$

$$L_{i}^{e} = \int_{r_e}^{r_{e+1}} \phi_i^e f(r) dr + \phi_i^e(r_{e+1})\frac{d\phi_j^e}{dr}(r_{e+1}) - \phi_i^e \frac{d\phi_j^e}{dr}(r_e) \tag{6.4}$$

where

$$f(r) = (3\rho + n\rho_A r^n + \nu\rho - \frac{nE_A r^n \rho}{E}) r\Omega^2(1+\Gamma^2) \tag{6.5}$$

$$+ET_0\alpha_A n r^{n-1}, forTD-I$$

$$f(r) = (3\rho + n\rho_A r^n + \nu\rho - \frac{nE_A r^n \rho}{E}) r\Omega^2(1+\Gamma^2) \tag{6.6}$$

$$+EbT_0(\frac{\alpha(r)}{r} + \alpha_A n r^{n-1} log(\frac{r}{a})), forTD-II$$

$$\phi_1^e = \frac{r_{e+1-r}}{r_{e+1} - r_e} \qquad \qquad \phi_2^e = \frac{r - r_e}{r_{e+1} - r_e} \tag{6.7}$$

Here the symbol e is used to indicates the element number which is used for disk discritization. The thermoelastic characteristics radial stress(σ_r) , circumferential stress(σ_θ) , radial strain(ε_r), circumferential strain(ε_θ) and displacement(μ_r) are calculated by using below formula:

$$\sigma_r = \frac{1}{r}\sum_{j=1}^{2} F_j^e \phi_j^e, \tag{6.8}$$

$$\sigma_\theta = \sum_{j=1}^{2} F_j^e \frac{d\phi_j^e}{dr} + \rho\Omega^2 r^2(1+\Gamma^2) \tag{6.9}$$

$$\varepsilon_r = \frac{1}{E}\sum_{j=1}^{2} (\frac{\phi_j^e F_j^e}{r} - \upsilon F_j^e \frac{d\phi_j^e}{dr}) - \frac{\upsilon\rho\Omega^2 r^2(1+\Gamma^2)}{E} + \varepsilon^* \tag{6.10}$$

$$\varepsilon_\theta = \frac{1}{E}\sum_{j=1}^{2} (F_j^e \frac{d\phi_j^e}{dr} - \frac{\upsilon}{r} F_j^e \phi_j^e) + \frac{\upsilon\rho\Omega^2 r^2(1+\Gamma^2)}{E} + \varepsilon^* \tag{6.11}$$

$$\mu_r = \frac{r}{E}\sum_{j=1}^{2} (F_j^e \frac{d\phi_j^e}{dr} - \frac{\upsilon}{r} F_j^e \phi_j^e) + \frac{\upsilon\rho\Omega^2 r^3(1+\Gamma^2)}{E} + r\varepsilon^* \tag{6.12}$$

where $\varepsilon^* = \alpha(r) * T(r)$

7. Results and Discussions

In this section, thermoelastic characteristics(stress, strain and displacement) have been represented numerically for a circular FGM disk of materials A and B. For both materials the value of υ is fixed as 0.3. To achieve numerical results, we assumed the inner radius a as 15mm and outer radius b as 150 mm respectively so that $b/a = 10$. The value of angular speed is taken as 1 rad/s. The thermoelastic characteristics have been calculated and represented graphically under effect of two temperature distributions against normalized radial distance ($r^* = \frac{r-a}{b-a}$). Further the vibration analysis made by introducing Kibel Number and thermoelastic characteristics are calculated for different values of r.

7.1 For n=0(Constant Distribution) under Temperature Distribution I and Temperature Distribution II

For n=0: E, α and ρ become constant.

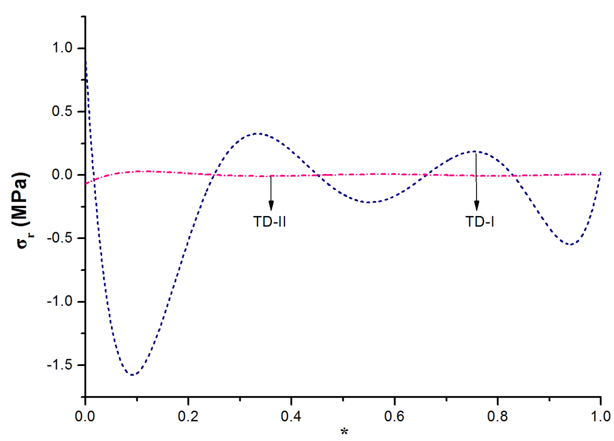

Fig 2. Radial Stress vs. Normalized Radial Distance

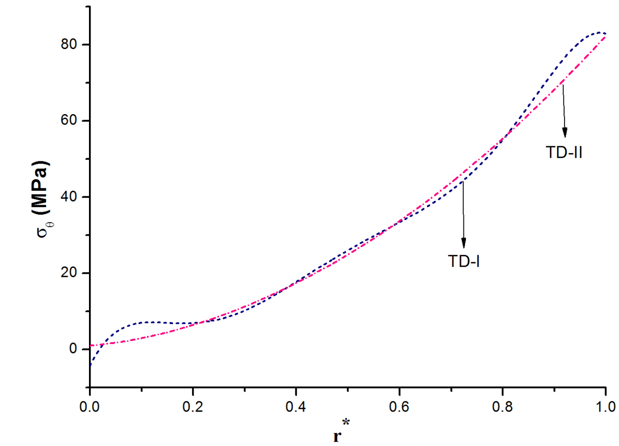

Fig 3. Circumferential Stress vs. Normalized Radial Disatnce

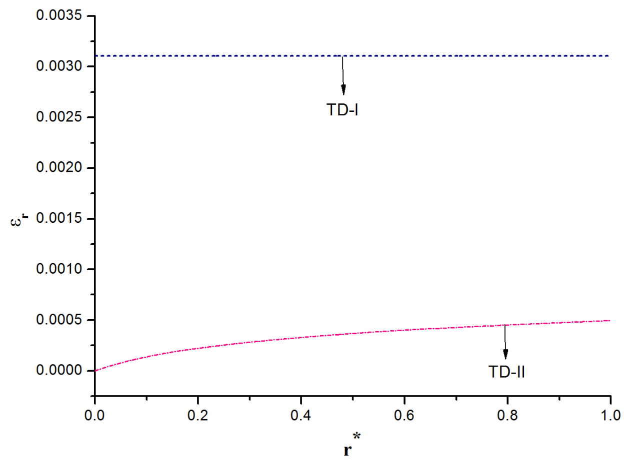

Fig 4. Radial Strain vs. Normalized Radail Disatnce

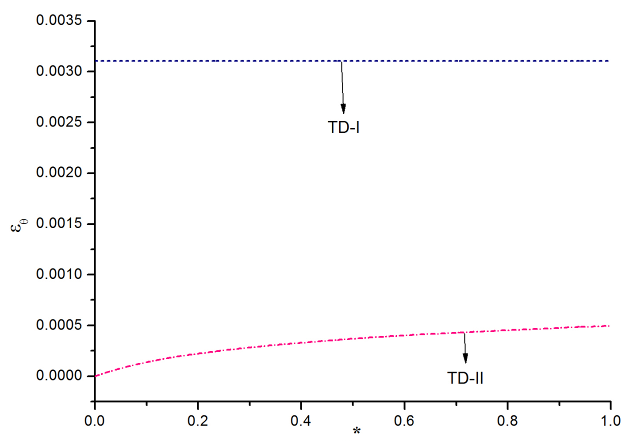

Fig 5. Circumferential Strain vs. Normalized Radial Stress

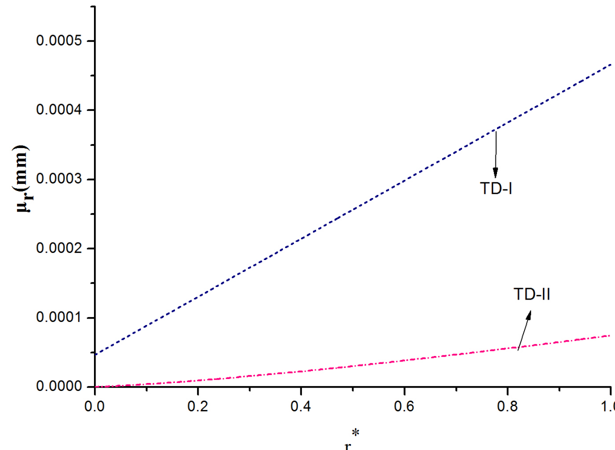

Fig 6. Displacement vs. Normalized Radial Distance

Fig 2. presents radial stress against r^* for different temperature distributions. The maximum and minimum value of radial stress is obtained for temperature distribution I and there is very less variation in values of radial stress for temperature distribution II. The radial stress curve is of variable nature for temperature distribution I. Fig 3. shows behaviour of circumferential stress for two temperature distributions. It is observe that for end values r^* circumferential stress is greater for temperature distribution I. On the other hand, the difference in circumferential stress for two temperature distributions is very less. Fig 4. shows the radial strain corresponding to different values of normalized radial distance. It is clear from figure value of radial strain is maximum for temperature distribution I but radial strain curve is more variable in temperature distribution II. Fig 5. presents circumferential strain against r^* for two temperature distributions. It is recognized that the value of circumferential strain is greater for temperature distribution I.Fig 6. shows the displacement against r^* for two temperature distribution. The value of displacement for temperature distribution I is greater than for temperature distribution II at each of r^*.The displacement curve is more variable in nature for temperature distribution I.

7.2 For n=1(Linear Distribution) under Temperature Distribution I and Temperature Distribution II

For n=1: E, α and ρ are linear functions of r.

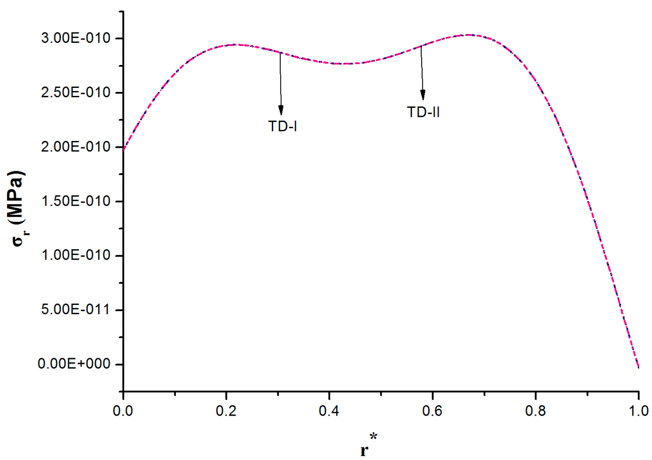

Fig 7.Radial Stress vs. Normalized Radial Distance

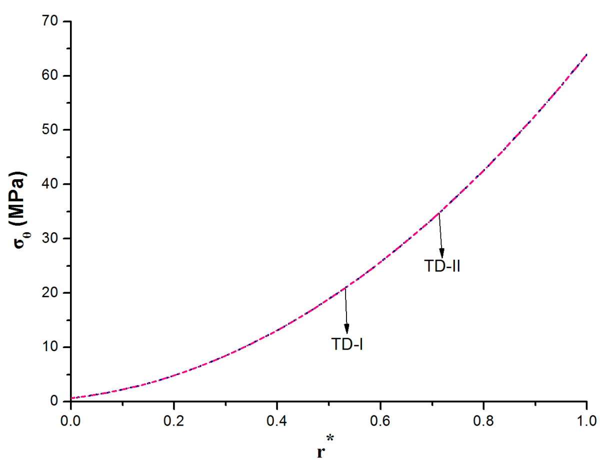

Fig 8. Circumferential Stress vs. Normalized Radial Distance

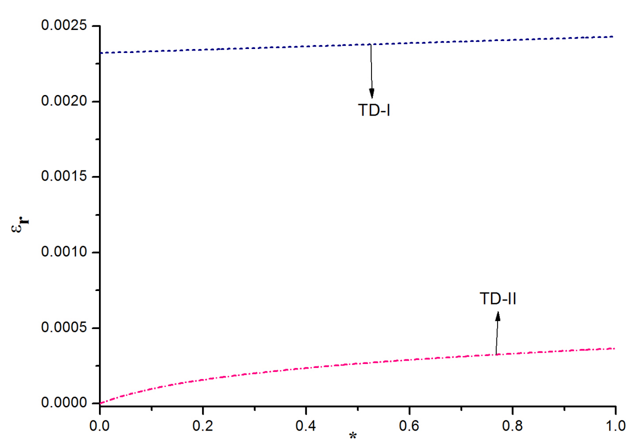

Fig 9. Radial Strain vs. Normalized Radial Distance

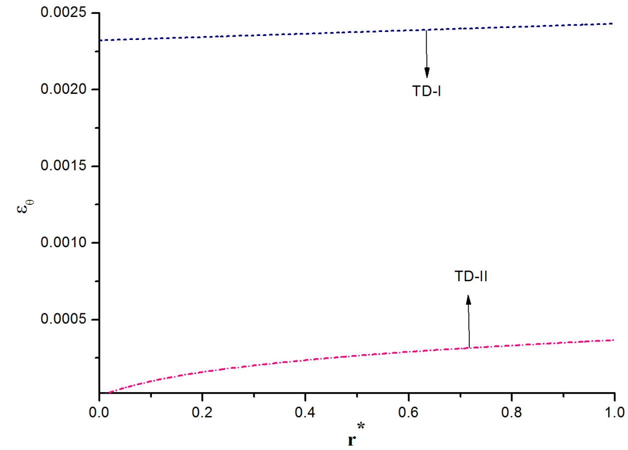

Fig 10. Circumferential Strain vs. Normalized Radial Distance

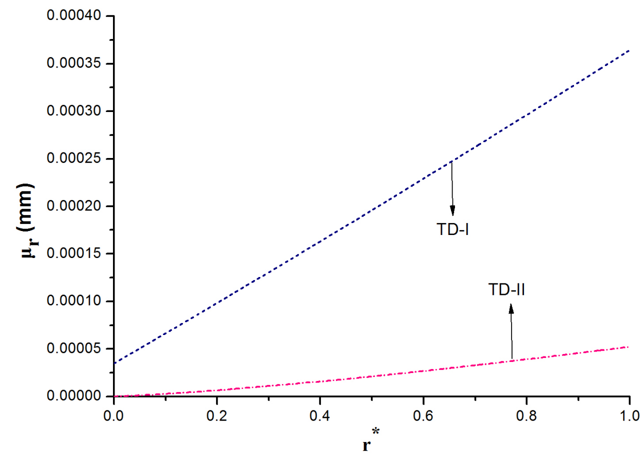

Fig 11. Displacement vs. Normalized Radial Distance

Fig 7. presents the radial stress against different values of r^* for two temperature distributions. The value of radial stress is very less which is near about zero for both temperature distributions. From the curves, it is cleared that there is no effect of temperature field on radial stress when all the material properties are linear functions of r. Fig 8. represents the circumferential stress against r^* for two different temperature fields. The behaviour of both circumferential stress curves are increasing in nature. Also from these curves is observed that circumferential stress is independent of temperature field. Fig 9. shows the radial strain against r^* for different temperature fields. It is noticed that the radial strain is greater for temperature distribution I than temperature distribution II. But radial strain is more variable for temperature distribution II. Fig 10. presents circumferential strain and r^* for two different temperature fields. It is clear from graph the numeric value of circumferential strain is grater in temperature distribution I than from temperature distribution II.Fig 11. shows displacement for different values of r^* for two temperature fields. The displacement numerically greater for temperature distribution II but for both temperature fields displacement increase as the values of normalized radial distance increases.

7.3 For n=0(Constant distribution) and Temperature Distribution I for different values of Kibel Number

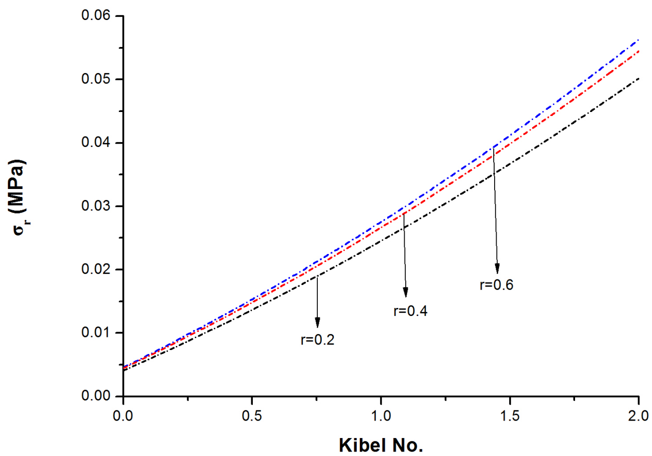

Fig 12. Radial Stress vs. Kibel Number for TD-I

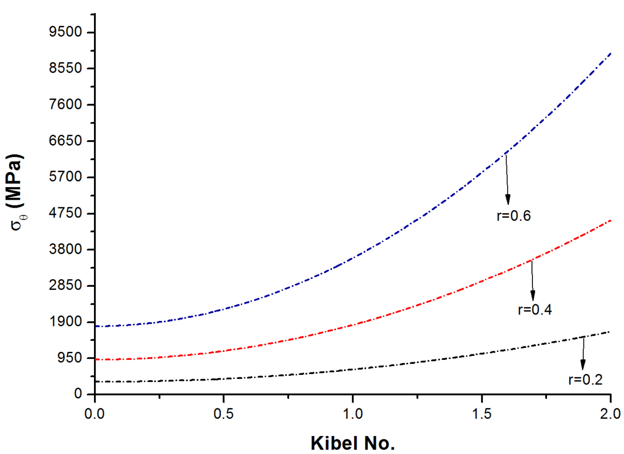

Fig 13. Circumferential Stress vs. Kibel Number for TD-I

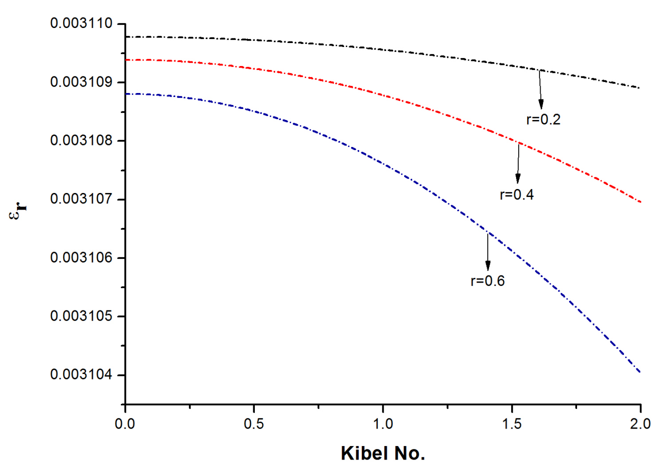

Fig 14. Radial Strain vs. Kibel Number for TD-I

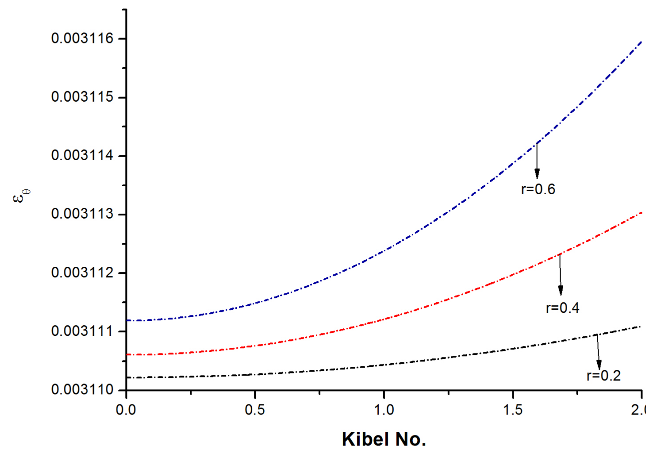

Fig15. Circumferential Strain vs. Kibel Number for TD-I

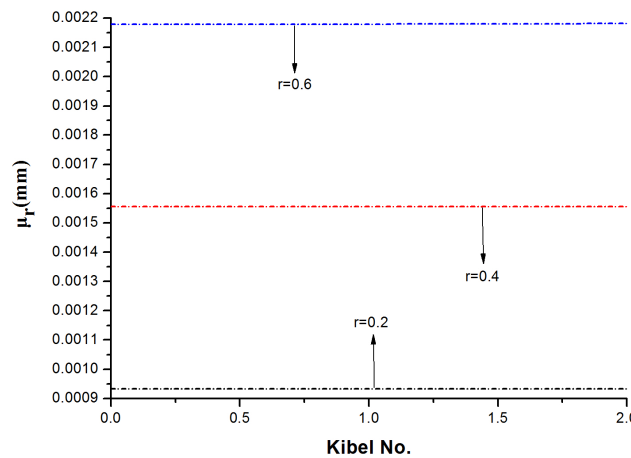

Fig 16. Displacement vs. Kibel Number for TD-I

It is observed from Fig 12 and Fig 13 the value of radial and circumferential stress increases as the value of Kibel Number increases. Also the value of these components is greatest for r=0.6mm and lowest for r=0.2mm. From Fig 14 and 15 it is cleared that the behaviour of radial and circumferential strain is different, radial strain is maximum for r=0.6mm and minimum for r=0.2mm but circumferential strain is opposite in nature. Also radial strain decreases as the value of Kibel Number increases but circumferential strain increases as the value of Kibel Number increases. Fig 16 shows that the displacement is independent of Kibel Number. The maximum value of displacement for r=0.6mm and minimum for r=0.2mm .

7.4 For n=0(Constant Distribution) and Temperature Distribution II for different values of Kibel Number

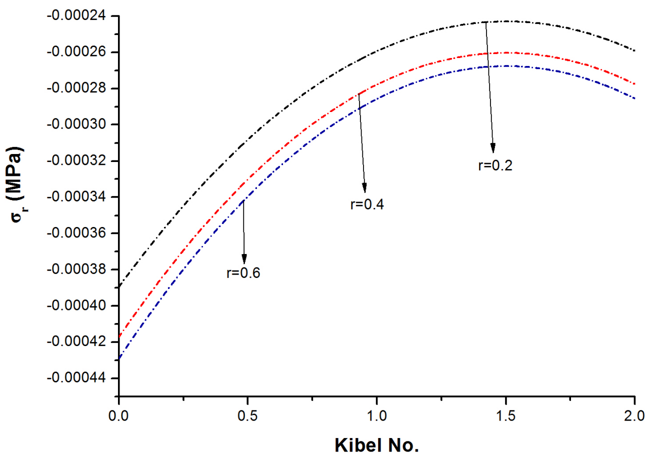

Fig 17. Radial Stress vs. Kibel Number for TD-II

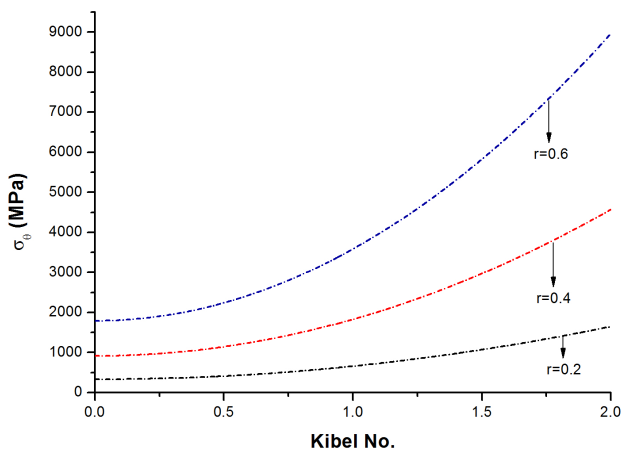

Fig 18. Circumferential Stress vs. Kibel Number for TD-II

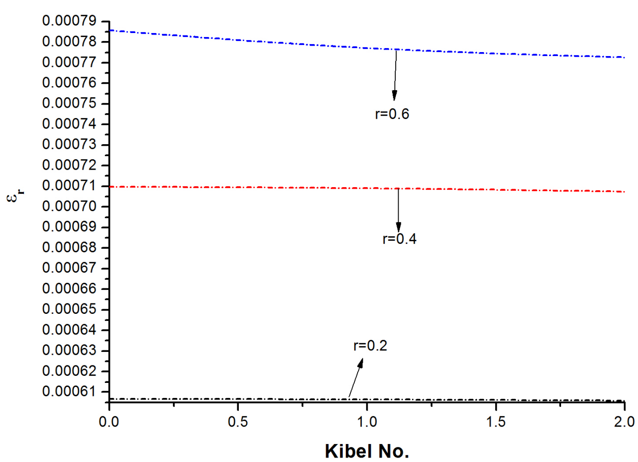

Fig 19. Radial Strain vs. Kibel Number for TD-II

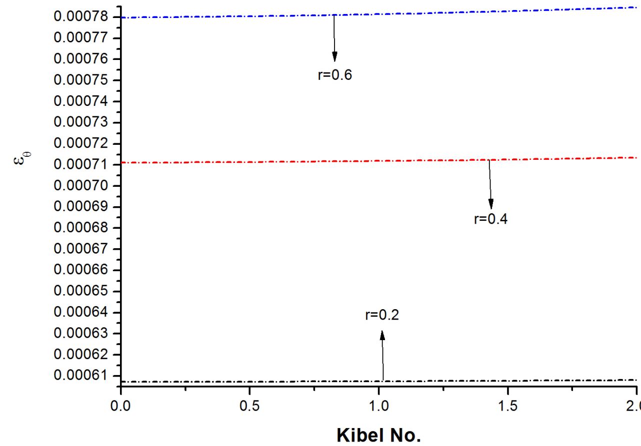

Fig 20. Circumferential Strain vs. Kibel Number for TD-II

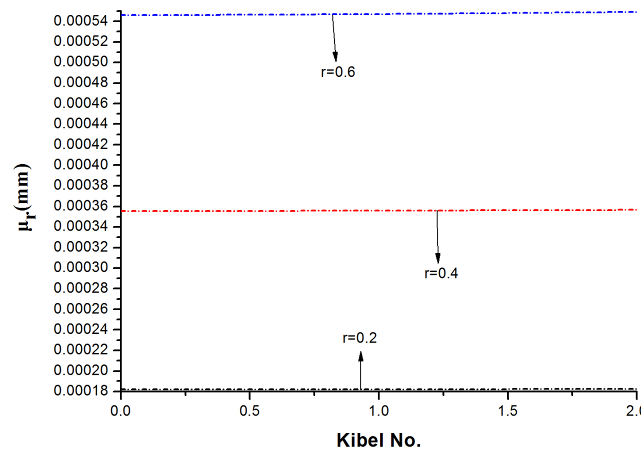

Fig 21. Displacement vs. Kibel Number for TD-II

It is noticed that, behaviour of radial stress curve change when Kibel Number is 1. Radial stress curve changes from increases to decreases at for Kibel Number1. Radial Stress is maximum for *r*=0.6mm and minimum for *r*=0.2mm but circumferential stress is opposite in nature. From Fig 19 and 20 it is observed that radial strain, circumferential strain and displacement are maximum for r=0.6mm and minimum for *r*=0.2mm.

7.5 For n=1(Linear distribution) and Temperature Distribution I for different values of Kibel Number

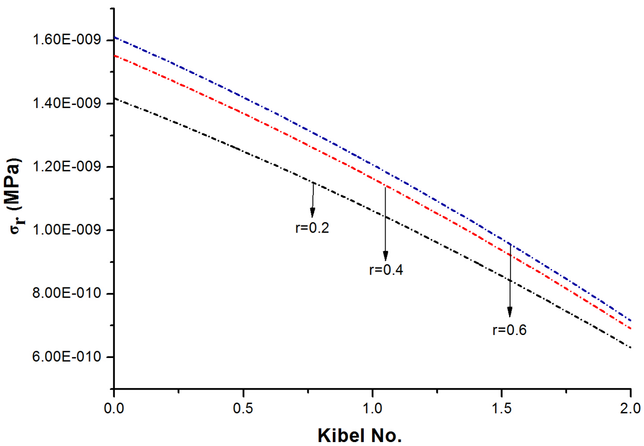

Fig 22. Radial Stress vs. Kibel Number for TD-I

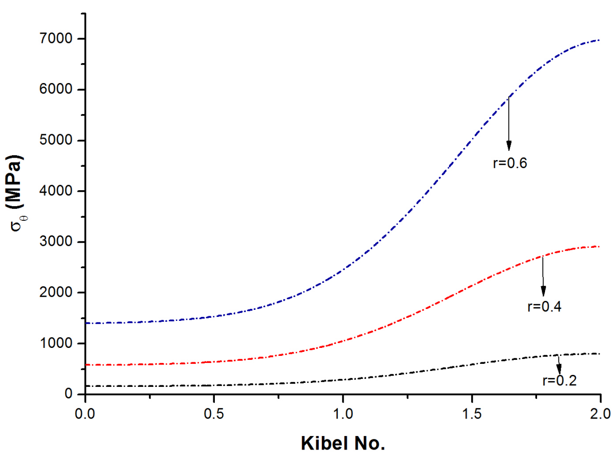

Fig 23. Circumferential Stress vs. Kibel Number for TD-I

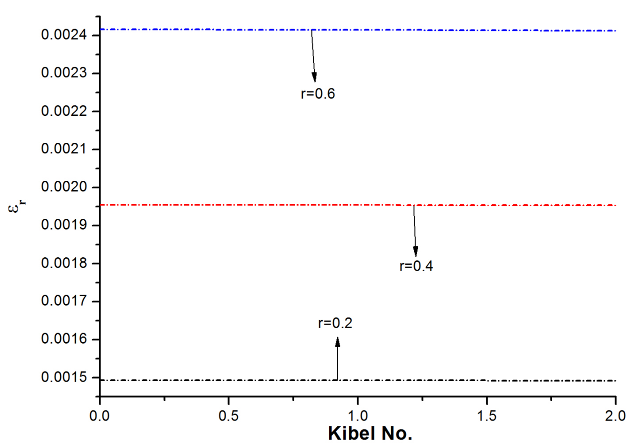

Fig 24. Radial Strain vs. Kibel Number for TD-I

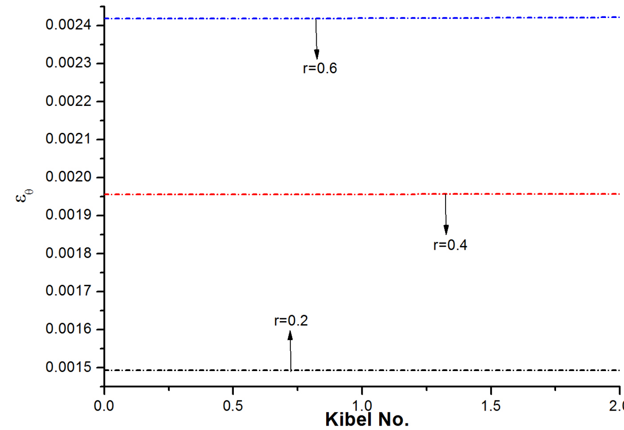

Fig 25. Circumferential Strain vs. Kibel Number for TD-I

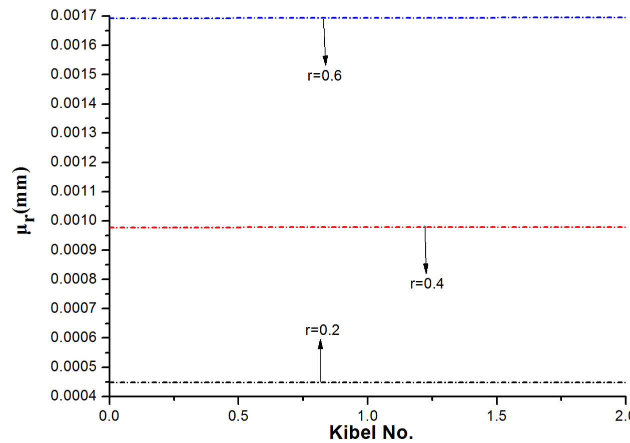

Fig 26. Displacement vs. Kibel Number for TD-I

From Fig 22.it has been observed that the value of radial stress decreases as the value of Kibel Number increases. The greatest value of radial stress is obtained when r=0.6mm and minimum value is obtained when r=0.2mm. Circumferential stress increases as the value of Kibel Number increases as shown in Fig 23. The values of circumferential stress increases after the value of Kibel Number is 1. As shown in Fig 24-26 radial strain, circumferential strain and displacement are independent of Kibel Number. The maximum value of all these components is obtained for r=0.6mm and minimum value is obtained for r=0.2mm.

7.6 For n=1(Linear distribution) and Temperature Distribution II for different values of Kibel Number

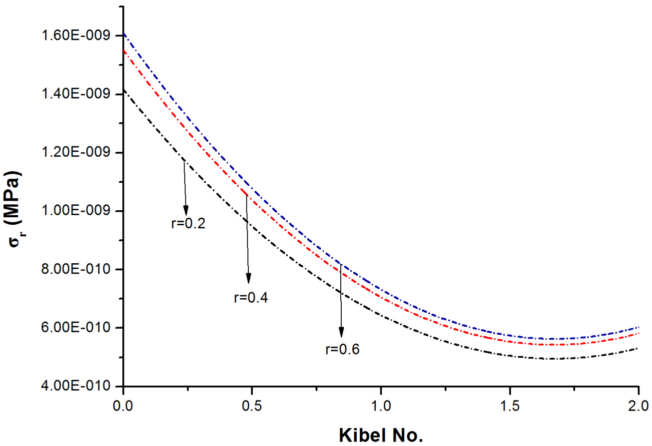

Fig 27. Radial Stress vs. Kibel Number for TD-II

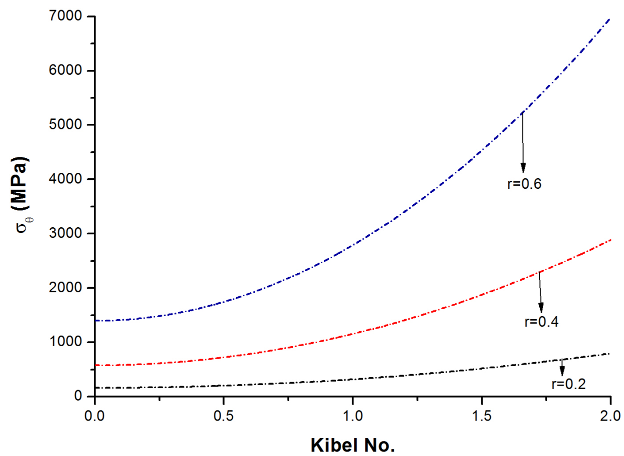

Fig 28. Circumferntial Stress vs. Kibel Number for TD-II

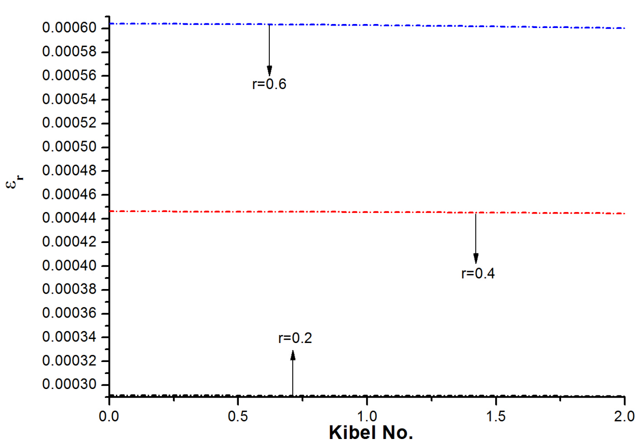

Fig 29. Radial Strain vs. Kibel Number for TD-II

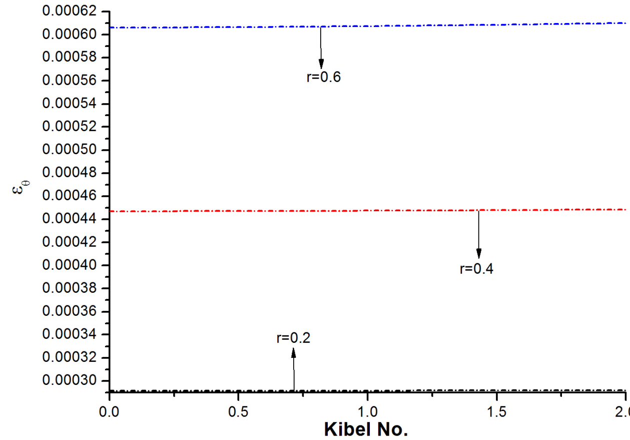

Fig 30. Circumferential Strain vs. Kibel Number for TD-II

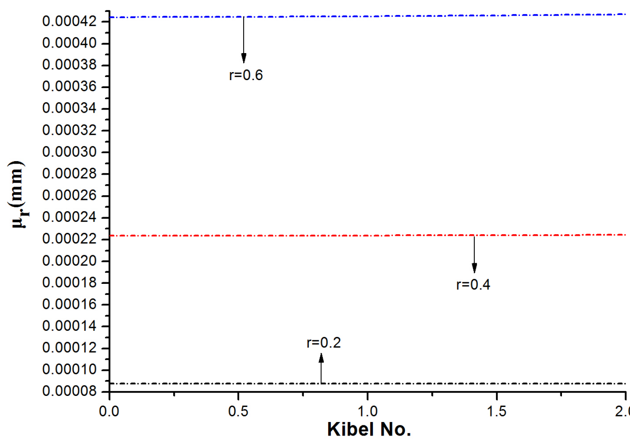

Fig 31. Displacement vs. Kibel Number for TD-II

Fig 27. shown variation in radial stress with Kibel Number for three fixed values of r, it has been observed that radial stress decreases as value of Kibel Number increases but when Kibel Number is 1 the variation in values of radial stress is opposite in nature . Circumferential stress is maximum for$r$=0.6mm and minimum for r=0.2mm as shown in Fig 28. radial strain, circumferential strain and displacement are maximum for r=0.6mm and minimum for r=0.2mm and also independent of Kibel Number as shown in Fig 29-31.

Acknowledgment

One of the author Ms. Ramandeep Kaur is grateful to I.K. Gujral Punjab Technical University, Kapurthala for providing research facilities.

8. Conclusion

A finite element method is developed to study stress, strain and displacement in FGM rotating disk for two temperature fields. For two different temperature distributions two cases for material properties (Constant and Linear) are discussed. It is conclude that thermoelastic characteristics of disk are changed with change made in temperature field. It is noticed that there is very less effect of temperature on circumferential stress. The behaviour of curves of radial, circumferential strain and displacement are same in two temperature cases. Temperature distribution-II is better for construction of disk. It is noticed that when vibration frequency and angular frequency are one, the behaviour of radial stress changes for the different values of r. As Kibel Number is greater than 1 variation is increased circumferential stress curve. The behaviour of radial strain, circumferential strain and displacement is independent of Kibel Number.

References

[1] J.N. Reddy, An introduction to Finite element method, *Tata McGraw-Hill*, (2005).

[2] O.C. Zienkiewicz, R.L. Taylor and J.Z. Zhu, The Finite Element Method its Basis and Fundamentals, *Elsevier*, (2005).

[3] R.D. Cook, D.S. Malkus D, M .E. Pleasha and R.J. Witt, Concept and application of Finite Element Analysis, *Wiley India Edition*, (2001).

[4] D.V. Hutton, Fundamental of Finite Element Analysis, *Tata McGraw-Hill*, (2005).

[5] A.M. Afsar and J. Go, Finite Element Analysis of Thermoelastic Field in a Rotating FGM Disk, *Appl Math Mod*, 34 (2010), 3309–3320.

[6] B. Arnab, S.M.R. Islam, A.A. Khalak and A.M. Afsar, Finite Difference solution of Theroelastic field in circular FGM disk with concentric hole,*Procedia Eng*, 90 (2014), 193–198.

[7] J.N. Sharma and D. Grover, Body Wave Progpagation in Rotating Thermoelastic Media, *Mech Re Commu*, 36 (2009), 715–721.

[8] X.L. Peng and X.F. Li, Thermal Stress in Rotating functionally graded hollow circular disks. *Compos Struct*, 92 (2010), 1896–1904.

[9] J.N. Sharma, D. Gorver and D. Kaur, Mathematical modeling and analysis of Bulk waves in a rotating generalized thermoelastic media with voids, *Appl Math Mod*, 35 (2011), 3396–3407.

[10] M. Bayat, M. Saleem, B.B. Sahari, A.M.S. Hamouda and E. Mahdi, Thermoelastic analysis of a functionally graded rotating disk with small and large deflections, *Thin wall Struct*, 45 (2007), 677–691.

[11] J.N. Sharma, D. Sharma and S. Kumar, Analysis of Stress and Strains in a Rotating Homogeneous Thermoelastic Circular Disk by using Finite Element Method, *Int J Comput Appl*, 35 (2011).

[12] J.F. Durodola and O. Attia, Deformation and stresses in functionally graded rotating disks, *Compos Sci Techol*, 60 (2000), 987–995.

[13] A. Zenkour and D. Mashat, Stress Function of a Rotating Variable-Thickness Annular Disk Using Exact and Numerical Methods, *Eng*, 3 (2011), 422–430.

[14] J.N. Sharma, D. Sharma and S. Kumar, Vibration Analysis of Rotating FGM Thermoelastic Axisymmetric Circular Disk Using FEM, *Int J Comput Methods Eng Sci Mech*, 14 (2013), 262–270.

[15] H. Callioglu H, N.B. Bektas and M. Sayer, Stress analysis of functionally graded rotating discs analytical and numerical solutions, *Acta Mech Sin*, 27 (2011), 950–955.

[16] K.M. Liew, S. Kitipornchai, X.Z. Zhang and C.W. Lim, Analysis of the thermal stress behaviour of functionally graded hollow circular cylinders, *Int J Solids Struct*, 40 (2013), 2355–2380.

[17] J. Go, A.M. Afsar and J.I. Song, Analysis of Thermoelastic Characteristics of a Rotating FGM Circular Disk by Finite Element Method, *Adv Compos Mater*, 19 (2010), 197–213.

[18] R. Naghdabadi and S.A.H. Kordkheili, A finite element formulation for analysis of functionally graded plates and shells, *Arch Appl Mech*, 74 (2005), 375–386.

[19] R.S.Dhaliwal and A. Singh, Dynamic Coupled Thermoelasticity. , *Hindustan Publishing Corporation, New Delh*, 1980.

[20] G.L. Nigh and M.D. Olson, Finite Element Analysis of rotating Disk, *J Sound Vib*, 77, 61–78.

[21] J.N. Sharma, D. Sharma, S. Kumar, Study of vibration analysis of a rotating homogeneous thermoelastic circular disk by using FEM, *Int J Compt Appl*, 35 (2011).

[22] J.N. Sharma, D. Sharma and S. Kumar, Stress and Strain analysis of a rotating FGM thermoelastic circular disk by using FEM, *Int J Pure Appl Math*, 74 (2012), 339–352.

[23] M. Khorsand and Y. Tang, Design functionally graded rotating disks under thermoelastic loads: weight optimization,*Int J Pres Ves Pip*, 161 (2018), 33–40.

[24] Y. Zheng, H. Bahaloo, D. Mousanezhad, E. Mahdi, A. Vaziri and N. Hashemi, Stress analysis in functionally graded disks with non-uniform thickness and variable angular velocity,*Int J Mech Sci*, 119 (2016), 283–293.

[25] T. Dai and H. Dai, Investigation of mechanical behaviour for a rotating FGM circular disk with a variable angular speed, *J Mech Sci Tech*, 29 (2015), 3779–3789.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆