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On Laplace finite Marchi Fasulo transform of generalized functions

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Abstract

In classical theory the integral transforms are defined on various distribution spaces and most of the integral transforms have been extended to generalized functions.Here we have considered the classical Laplace Finite Marchi Fasulo transform and it has been extended to certain class of generalized functions.

Keywords

Laplace Transform, Finite Marchi Fasulo transform.

AMS Subject Classification

44A10, 42A38, 34M25, 35A22.

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1. Introduction

We present some elements of the theory of generalized functions. The applications of integral transforms are used in the fields of science and engineering. The ordinary and partial differential equations can be solved by integral transform method .Here we use two different integral transforms.The theory of Laplace transform has been developed on various testing function spaces and distribution spaces. Khobragade [1] and Patel [3] used Finite Marchi Fasulo Transform and studied its application.Sajane and Chaudhary [5] studied the inversion formula for extended finite Hankel Laplace transform.Zemanian[9] extended the two sided and right sided Laplace transformation to a class of generalized function.Al-omari SKQ [2] studied the generalized Stieljes and Fourier transforms of certain spaces of generalized functions.Pathak [4]discussed properties of inversion and uniqueness.Sundararanjan[7] studied and discussed the properties of Fourier and Hartly transforms. Most of the integral transforms and their inverses have been defined in Sneddon [6].Distributional aspects of some integral transforms are given by Zemanian[8]. We construct an integral transform whose kernel is the product of the kernels of Laplace and Finite Marchi Fasulo Transform.

2. Preliminaries

Testing function spaces $\pounds_{+,a}$ and $\pounds_{+}(w)$

We shall denote the open set $(0,\infty)\times(-h,h)$ by I . Let $\pounds_{+,a}$ denote the space of all complex valued smooth functions $\varphi(z,t)$ that are infinitely differentiable with respect to z and t on I on which the functionals λ_{a,k_1,k_2} defined by

$$\lambda(\varphi) \triangleq \sup \mid e^{at} D_t^{k_1} \Omega_z^{k_2} \varphi(z,t) \mid$$
$$0<t<\infty$$
$$-h<z<h$$

assumes finite values where k_1, k_2 are non negative integers and $D_t = \frac{\partial}{\partial t}, D_z = \frac{\partial}{\partial z}$

$(\Omega_z^{k_2})(\varphi) = (D_z^2)^{k_2}(\varphi)$

Now $\pounds_{+,a}$ is linear space under the pointwise addition of functions and their multiplication by complex numbers. Each λ_{a,k_1,k_2} is a seminorm on $\pounds_{+,a}$ and $\lambda_{a,0,0}$ is a norm, hence the countable collections $\{\lambda_{a,k_1,k_2}\}_{k_1,k_2=0}^{\infty}$ of seminorms is a countable multinorm on $\pounds_{+,a}$. We assign to $\pounds_{+,a}$ the topology generated by $\{\lambda_{a,k_1,k_2}\}_{k_1,k_2=0}^{\infty}$ making it countably multinormed space. A sequence $\{\varphi_p\}_{p=1}^{\infty}$ converges in $\pounds_{+,a}$ to φ if and only if for each pair of nonnegative integers k_1,k_2 $\lambda_{a,k_1,k_2}(\varphi_p-\varphi)\to 0$ as $p\to\infty$ and a sequence $\{\varphi_p\}_{p=1}^{\infty}$ is a Cauchy sequence in $\pounds_{+,a}$ if and only if $\lambda_{a,k_1,k_2}(\varphi_p-\varphi_q)\to 0$ $\forall k_1,k_2=0,1,2,\ldots$ as $p,q\to\infty$.

Lemma 2.1. *The space $\pounds_{+,a}$ is complete and therefore it is a Frechet space.*

Proof. Let $\{\varphi_p\}_{p=1}^{\infty}$ be a Cauchy sequence in $\pounds_{+,a}$. Then there exists a smooth function $\varphi(z,t)$ such that for each pair of non negative integers k_1,k_2 $D_t^{k_1}\Omega_z^{k_2}\varphi_p(z,t)\to D_t^{k_1}\Omega_z^{k_2}\varphi(z,t)$ as $p\to\infty$. Moreover given any $\varepsilon>0$ there exists N_{k_1,k_2} such that for every $p,q>N_{k_1,k_2}$

$$|\, e^{at}D_t^{k_1}\Omega_z^{k_2}[\varphi_p(z,t)-\phi_q(z,t)]\,|<\varepsilon$$

for all z.
Taking the limit as $q\to\infty$, we obtain

$$|\, e^{at}D_t^{k_1}\Omega_z^{k_2}[\varphi_p(z,t)-\varphi(z,t)]\,|<\varepsilon \qquad 0<t<\infty, -h<z<h \tag{2.1}$$

Thus as $p\to\infty$ $\lambda_{a,k_1,k_2}(\varphi_p(z,t)-\varphi(z,t))\to 0$ for each k_1,k_2. Since the convergence is uniform and not depending on p such that $|\, e^{at}D_t^{k_1}\Omega_z^{k_2}[\varphi_p(z,t)]\,|<C_{k_1,k_2}$ for all z,t. Therefore (2.1) implies that $|\, e^{at}D_t^{k_1}\Omega_z^{k_2}[\varphi(z,t)]\,|<C_{k_1,k_2}+\varepsilon$ which shows that the limit function $\phi(z,t)$ is a member of $\pounds_{+,a}$ hence $\pounds_{+,a}$ is complete, so it is Frechet space. □

The countable union space $\pounds_{+}(w)$

Let w denote either a finite real number or $-\infty$. We choose the monotonic sequence since the convergence is uniform and $\{a_p\}_{p=1}^{\infty}$ such that $a_p\to w_+$. Then $\{\pounds_{+,a_p}\}_{p=1}^{\infty}$ is a sequence of testing function spaces is such that $\pounds_{+,a_p}\subset\pounds_{+,a_{p+1}}$ for all p and the topology of $\pounds_{+,a_p}$ is stronger than the topology induced on it by $\pounds_{+,a_{p+1}}$.

Definition 2.2. $\pounds_{+}(w)=\bigcup_{p=1}^{\infty}\pounds_{+,a_p}$. *A sequence $\{\varphi_p\}_{p=1}^{\infty}$ converges in $\pounds_{+}(w)$ if and only if it converges in $\pounds_{+,a_p}$ for some p. Then with these properties $\pounds_{+}(w)$ is countable union space.*

$\pounds_{+}(w)$ is a linear space. Further it is complete space, since for fixed $p,\pounds_{+,a_p}$ is complete.

The dual spaces $\pounds'_{+,a}$ and $\pounds'_{+}(w)$

The dual space $\pounds'_{+,a}$ of $\pounds_{+,a}$ is the collection of all continuous linear functionals on $\pounds_{+,a}$. Since $\pounds_{+,a}$ is complete, $\pounds'_{+,a}$ is also complete. If $a\le c$ then $\pounds_{+,c}\subset\pounds_{+,a}$ and the topology of $\pounds_{+,c}$ is stronger than the topology induced on it by $\pounds_{+,a}$. Therefore the restriction of any member $f\in\pounds'_{+,a}$ to $\pounds_{+,c}$ is in $\pounds'_{+,c}$. We assign the customary weak topology generated by the multinorm $\{\xi_\varphi\}\varphi\in\pounds_{+,a}$ to the dual space $\pounds'_{+,a}$, where $\xi_\varphi(f)=|<f,\varphi>|$ $\varphi\in\pounds_{+,a}$. We denote by $\pounds'_{+}(w)$ the dual space of $\pounds_{+}(w)$. Since all $\pounds_{+,a_p}$ are complete and $\pounds_{+}(w)$ is complete, the dual space $\pounds'_{+}(w)$ is also complete.

The properties of testing function spaces and their duals

1. $D\subset\pounds_{+,a}$ and the convergence in $D(I)$ implies the convergence in $\pounds_{+,a}$. Therefore the restriction of any member of $\pounds'_{+,a}$ (I) to $D(I)$ is in $D'(I)$. Similarly $D(I)$ is subspace of $\pounds_{+}(w)$ whatever may be the value of w. The convergence in $D(I)$ implies the convergence in $\pounds_{+}(w)$ and the restriction of any member of $\pounds_{+}(w)$ to $D(I)$ is a member of $D'(I)$.The member of $\pounds'_{+,a}$ (I) and $\pounds'_{+}(w)$ are called distributions.

2. $D(I)$ is dense in $\pounds_{+}(w)$ for every w. Therefore $\pounds'_{+}(w)$ is subspace of $D'(I)$.

3. For each $f\in\pounds'_{+,a}(I)$ there exists a +ve constant c and a nonnegative integer r , such that for all $\varphi\in\pounds_{+,a}$ (I)

$$|<f,\varphi>|\le c \max_{\substack{0\le k_1\le r\\ 0\le k_1\le r}} \lambda_{a,k_1,k_2}(\varphi)$$

4. If $f(z,t)$ is a locally integrable function defined on the interval I such that $\int_{-h}^{h}\int_{0}^{\infty}|\,e^{-at}f(z,t)dtdz\,|$ exists then $f(z,t)$ generates a regular member of $\pounds_{+,a}(I)$ through the definition

$$<f,\varphi>=\int_{-h}^{h}\int_{0}^{\infty}f(z,t)\varphi(z,t)dtdz,$$
$$\varphi(z,t)\in\pounds_{+,a}(I)\ldots(*)$$
$$|<f,\varphi>|=\left|\int_{-h}^{h}\int_{0}^{\infty}\frac{f(z,t)}{e^{at}}e^{at}\phi(z,t)dtdz\right|$$
$$\le\lambda_{a,0,0}\int_{-h}^{h}\int_{0}^{\infty}|\,e^{-at}f(z,t)\,|\,dtdz$$

which exists in view of our assumption . Therefore (*) defines a functional f on $\pounds_{+,a}$ this functional is linear. Further if $\{\phi_p\}_{p=1}^{\infty}$ converges in $\pounds_{+,a}$ to zero then $\lambda_{a,0,0}(\varphi_p)\to 0$ so that $|<f,\varphi_p>|\to 0$ therefore f is continuous on $\pounds_{+,a}$. Similarly if $w<a$, then f generates a regular distribution of $\pounds'_{+}(w)$ through the definition

$$<f,\varphi>=\int_{-h}^{h}\int_{0}^{\infty}f(z,t)\varphi(z,t)dtdz \quad \varphi(z,t)\in\pounds_{+,a}(I)$$

where $\int_{-h}^{h}\int_{0}^{\infty}|\,e^{-at}f(z,t)\,|\,dtdz$ exists.This condition is satisfied if $e^{-\sigma t}f(z,t)$ bounded on $o<t<\infty$ for every choice of σ where $\sigma>w$.

5. For each positive integer n and $Re.s\ge a$ the function $e^{-st}p_n(z)$ is a member of $\pounds_{+,a}$ and is also member of $\pounds_{+}(w)$ if $a>w$.

Proof.

$$e^{at}[D_t^{k_1}\Omega_z^{k_2}e^{-st}p_n(z)]=(-1)^{k_1+k_2}s^{k_1}(a_n^2)^{k_2}e^{-(s-a)t}p_n(Z)$$

(2.2)

for $t \geq 0 \quad -h < z < h$.

The right hand side of above equation is bounded for $Re.s \geq a$ for the positive eigen values a_n. Thus for each k_1, k_2 $\lambda_{a,k_1,k_2}[e^{-st}p_n(z)]$ exists which shows that $e^{-st}p_n(z)$ belongs to $\pounds_{+,a}$. In the similar way we can show that for every $a > w, e^{-st}p_n(z) \in \pounds_{+}(w)$. □

3. The Generalized Laplace Finite Marchi Fasulo Transform

We call the generalized function f as Laplace Finite Marchi Fasulo transformable if it belongs to $\pounds'_{+}(w)$ for some real number w. Let σ_f be defined as , $\sigma_f = \inf\{w | f \in \pounds'_{+}(w)\}$ If f is Laplace Finite Marchi Fasulo transformable function, then we see that $\exists \sigma_f$ such that $f \in \pounds'_{+}(w) \forall w < \sigma_f$. Thus for a given Laplace Finite Marchi Fasulo transformable function $f \in \pounds'_{+}(w)$ if D_f denote strip of definition i.e. $D_f = \{(n,s)/\sigma_f < Res, n$ is +ve integer$\}$. Then generalised Laplace Finite Marchi Fasulo transformation $F(n,s)$ of $f(z,t)$ is defined by $\pounds M[f(z,t)] \triangleq F(n,s) = < f(z,t), e^{-st}p_n(z) >$ i.e , it is defined as the application of $f \in \pounds'_{+}(\sigma_f)$ to kernel $e^{-st} \in \pounds'_{+}(\sigma_f)$ or equivalently $f \in \pounds'_{+,a}$ to $e^{-st}p_n(z) \in \pounds_{+,a}$ for any $\sigma_f < a \leq Re.s$

Boundedness property of generalised Laplace Finite Marchi Fasulo transform:

We show that the generalized Laplace Finite Marchi Fasulo transform $F(n,s)$ defined as above is bounded for $(n,s) \in D_f$. by using [9]

Theorem 3.1. *Let $f \in \pounds'_{+,a}$ and $F(n,s) = < f(z,t), e^{-st}p_n(z) >$ For $(n,s) \in D_f$. Then $F(n,s)$ satisfies the inequality $| F(n,s) | \leq CAP(| s | (a_n^2)^r)$*

Proof. Let $f \in \pounds'_{+,a}(I)$. Then by property (3) above there exists a non negative integer r and a positive constant C such that for $Re.s \geq a$

$$\begin{aligned}
| F(n,s) | &= | < f(z,t), e^{-st}p_n(z) > | \\
&\leq C \max_{\substack{0 \leq k_1 \leq r \\ 0 \leq k_2 \leq r}} \lambda_{a,k_1,k_2}[e^{-st}P_n(z)] \\
&\leq C \max_{\substack{0 \leq k_1 \leq r \\ 0 \leq k_2 \leq r}} \sup_{(z,t) \in I} | e^{at} D_t^{k_1} (D_z^2)^{k_2} e^{-st} P_n(z) | \\
&\leq C \max_{\substack{0 \leq k_1 \leq r \\ 0 \leq k_2 \leq r}} \sup_{(z,t) \in I} | e^{at} s^{k_1} a_n^{2k_2} e^{-st} P_n(z) | \\
&= C \max_{\substack{0 \leq k_1 \leq r \\ 0 \leq k_2 \leq r}} \sup_{(z,t) \in I} | e^{at} s^{k_1} a_n^{2k_2} e^{-st} P_n(z) |
\end{aligned}$$

but $| e^{at} e^{-st} P_n(z) | < A$ on $0 < t < \infty, -h < z < h$ so that

$$| F(n,s) | \leq C A \max_{\substack{0 \leq K_1 \leq r \\ 0 \leq k_2 \leq r}} | s^{k_1} a_n^{2k_2} |$$

$| F(n,s) | \leq CA \quad P(|s|(a_n^2)^r)$
where $P(|s|(a_n^2)^r)$ is a polynomial that depends in general on the choice of A . □

Theorem 3.2 (Analyticity theorem)**.** *The generalized Laplace Finite Marchi Fasulo transform is an analytic function of s.*

Proof. Let (n,s) be arbitrary but fixed point in D_f and choose the real number a and r such that $\sigma_f < a \leq Re.s - r \leq Re.s + r$ let Δs be complex increment such that $|\Delta s| < r$. Now for $\Delta s \neq 0$ by definition of $F(n,s)$ as

$$\frac{F(n,s+\triangle s) - F(n,s)}{\triangle s} - < f(z,t), \frac{\partial}{\partial s} e^{-st} P_n(z) > \\ = < f(z,t), \Psi_{\triangle s}(z,t) \quad (3.1)$$

where

$$\Psi_{\triangle s}(z,t) = \{ \frac{1}{\triangle s}[e^{-(s+\triangle s)t} - e^{-st}] - \frac{\partial}{\partial s} e^{-st} \} p_n(z) \quad (3.2)$$

since $\Psi_{\triangle s}(z,t) \in \pounds_{+,a}$ equation (3.2) has meaning . We show that $\Psi_{\triangle s}(z,t) \to 0$ in $\pounds_{+,a}$ as $\triangle s \to 0$ but as $f \in \pounds'_{+,a}$ this implies that $< f, \Psi_{\triangle s} > \to 0$
let c denote the circle with centre at s and radius r_1 where $0 < r < r_1 < Re.s - a$ by equation (3.2)
$D_t^{k_1} \Omega_z^{k_2} \Psi_{\triangle s}(z,t) = (-1)^{k_1+k_2} (a_n^2)^{k_2} p_n(z)$
$\{ \frac{1}{\triangle s}[(s+\triangle s)^{k_1} e^{-(s+\triangle s)t} - s^{k_1} e^{-st}] - s^{k_1} e^{-st} \}$
since $e^{-st}p_n(z)$ is analytic in s using Cauchy's integral formula to the right hand side of the last equation , we obtain

$$D_t^{k_1} \Omega_z^{k_2} \Psi_{\triangle s}(z,t) = (-1)^{k_1+k_2} (a_n^2)^{k_2} p_n(z) [\frac{1}{2\pi i} \frac{1}{\triangle s} \int_c \frac{\xi^{k_1} e^{-\xi t} d\xi}{\xi - (s+\triangle s)} - \int_c \frac{\xi^{k_1} e^{-\xi t} d\xi}{\xi - s} - \frac{1}{2\pi i} \int_c \frac{\xi^{k_1} e^{-\xi t} d\xi}{(\xi - s)^2}]$$

$$D_t^{k_1} \Omega_z^{k_2} \Psi_{\triangle s}(z,t) = (-1)^{k_1+k_2} (a_n^2)^{k_2} p_n(z) \times \frac{1}{2\pi i} \int_c \frac{\xi^{k_1} e^{-\xi t} d\xi}{(\xi - s)(\xi - s - \triangle s)} - \int_c \frac{\xi^{k_1} e^{-\xi t} d\xi}{(\xi - s)^2}$$

$$D_t^{k_1} \Omega_z^{k_2} \Psi_{\triangle s}(z,t) = (-1)^{k_1+k_2} (a_n^2)^{k_2} p_n(z) \frac{\triangle s}{2\pi i} \int_c \frac{\xi^{k_1} e^{-\xi t} d\xi}{(\xi - s)(\xi - s - \triangle s)(\xi - s)^2}$$

now for all $\xi \in C$ and $0 < t < \infty$, $-h < z < h$

$$| e^{at} D_t^{k_1} \Omega_z^{k_2} \Psi_{\triangle s}(z,t) | = (a_n^2)^{k_2} | p_n(z) | \frac{| \triangle s |}{2\pi i} \int_c \frac{| e^{at} \xi^{k_1} e^{-\xi t} || d\xi |}{(\xi - s)(\xi - s - \triangle s)(\xi - s)^2}$$

$$| e^{at} D_t^{k_1} \Omega_z^{k_2} \Psi_{\triangle s}(z,t) | \leq \frac{| \triangle s |}{2\pi i} k (a_n^2)^{k_2} \int_c \frac{| d\xi |}{(r_1 - r) r_1^2} \leq \frac{| \triangle s | k (a_n^2)^{k_2}}{(r_1 - r) r_1}$$

where $| e^{at} \xi^{k_1} e^{-\xi t} || p_n(z)] | \leq k$ k being constant independent of ξ and t. The right hand side of the last equality is independent of z, t and converges to zero as $| \triangle s | \to 0$ This proves that$\Psi_{\triangle s}(z,t)$ converges to zero in $\pounds_{+,a}$ as $\triangle s \to 0$.
Hence if $f \in \pounds'_{+,a}$, $< f, \Psi_{\triangle s} > \to o$ as $\triangle s \to o$. □

Lemma 3.3. *Let $\pounds M[f(z,t)] = F(n,s)$ where $(n,s) \in D_f, Re.s > \sigma$ and $\varphi(z,t) \in \pounds_{+,a}$ and $-h < a_1 < b_1 < h$ assume that ,*

$$\varphi(n,s) = \int_{a_1}^{b_1} \int_0^\infty \varphi(z,t) e^{st} \frac{p_n(z)}{\lambda_n} dt dz. \tag{3.3}$$

Then for any fixed real number r with $o < r < \infty$

$$\int_{-r}^{r} < f(u,v), e^{-su} p_n(v) > \varphi(n,s) d\rho = < f(u,v), \int_{-r}^{r} e^{-su} p_n(v) \varphi(n,s) d\rho > \tag{3.4}$$

where $s = \sigma + i\rho$ and σ is fixed and $\sigma > \sigma_f$.

Proof. We know that $e^{-st} p_n(z)$ is a member of $\pounds_{+,a}$ and $\int_{-r}^{r} e^{-su} p_n(v) \varphi(n,s) d\rho$ is also the member of $\pounds_{+,a}$ indeed

$$\begin{aligned} & D_u^{k_1} D_v^{2k_2} \int_{-r}^{r} e^{-su} p_n(v) \varphi(n,s) d\rho \\ & \leq e^{\sigma u} \int_{-r}^{r} s^{k_1} (a_n^2)^{k_2} | p_n(v) || \varphi(n,s) | d\rho \\ & \leq A e^{\sigma u} s^{k_1} (a_n^2)^{k_2} 2r \end{aligned} \tag{3.5}$$

which exists, where A is bound for $\varphi(n,s).p_n(v)$ both sides has sense. If $\varphi(z,t) = o$, the proof is obvious, so assume that $\varphi(z,t) \neq o$. Partition the path of integration on the straight line $s = \sigma - ir$ to $s = \sigma + ir$ into m intervals each of length $\frac{2r}{m}$ and let $s_p = \sigma + i\rho_p$ be any point in path interval

$$\theta_m(u,v) = \sum_{p=1}^{m} e^{-s} p^u p_n(v) \varphi(n, s_p) \frac{2r}{m} \tag{3.6}$$

by applying $f(u,v)$ to (3.6) term by term

$$\begin{aligned} & < f(u,v), \theta_m(u,v) > \\ & = \sum_{p=1}^{m} < f(u,v), e^{-s} p^u p_n(v) > \varphi(n, s_p) \frac{2r}{m} \\ & \to \int_{-r}^{r} < f(u,v), e^{-su} p_n(v) > \varphi(n,s) d\rho \end{aligned}$$

Since $< f(u,v), e^{-su} p^u p_n(v) > \varphi(n,s)$ is a continuous function of ρ choose a such that $\sigma_f < a < \sigma$ since $f \in \pounds_{+,a}$ to show that $\theta_m(u,v)$ converges in $\pounds_{+,a(I)}$ to $\int_{-r}^{r} e^{-su} p_n(v) \varphi(n,s) d\rho$ we will show that for fixed $k_1, k_2 A_m(u,v)$ converges uniformly to zero, as $m \to \infty$ $0 < u < \infty$,$-h < v < h$, where

$$A_m(u,v) = e^{au} D_u^{k_1} \Omega_v^{k_2} [\theta_m(u,v) - \int_{-r}^{r} e^{-su} p_n(v) \varphi(n,s) d\rho]$$

$$\begin{aligned} A_m(u,v) = {} & e^{au} (-1)^{k_1 + k_2} \sum_{p=1}^{m} s_p^{k_1} (a_n^2)^{k_2} e^{-s} p^u p_n(v) \varphi(n,s) \frac{2r}{m} \\ & - (-1)^{k_1 + k_2} \int_{-r}^{r} s^{k_1} (a_n^2)^{k_2} e^{-su} p_n(v) \varphi(n,s) d\rho. \end{aligned} \tag{3.7}$$

Now $| e^{au}.e^{-su} p_n(v) | \leq e^{(a-\sigma)u}.p_n(z) \to 0$ as $| u | \to \infty$ as $a < \sigma$. So given $\varepsilon > 0$ we choose T so large that for all $| u | > T$

$$| e^{au}.e^{-su} p_n(v) | \leq \frac{\varepsilon}{2} [\int_{-r}^{r} | s^{k_1} (a_n^2)^{k_2} \varphi(n,s) | d\rho]^{-1}. \tag{3.8}$$

Since $\varphi(z,t) \neq o$ the r .h. s of (3.8) is finite . Now for all $| u | > T$ the magnitude of the second term on the right hand side of (3.8) is bounded by $\frac{\varepsilon}{2}$. Moreover , for $| u | > T$ the magnitude of first term on right hand side of (3.8) is bounded by

$$\frac{\varepsilon}{2} [\int_{-r}^{r} | s^{k_1} (a_n^2)^{k_2} \varphi(n,s) | d\rho]^{-1} \sum_{p=1}^{m} s^{k_1} (a_n^2)^{k_2} \varphi(n, s_p) \frac{2r}{m}$$

We can now, choose m_0 so large that for all $m > m_0$ the last expression is less than ε Hence for all $| u | > T$ and for all $m > m_0$ $| A_m(u,v) | < \varepsilon$. □

4. Inversion and Uniqueness

Theorem 4.1 (Inversion theorem)**.** *Let $f \in \pounds'_{+,a}$ and let $F(n,s)$ be the distributional Laplace Finite Marchi Fasulo transform of $f(z,t)$. For $(n,s) \in D_f$, in the sense of convergence in $D'(I)$,*

$$f(z,t) = \lim_{r,m \to \infty} \left[\frac{1}{2\pi i} \sum_{n=1}^{m} \frac{2p_n(z)}{\lambda_n} \int_{\sigma - ir}^{\sigma + ir} F(n,s) e^{st} ds \right]$$

where σ is any fixed number such that $\sigma > \sigma_f$.

Proof. Let $\varphi(z,t)$ be an arbitrary member of $D(I)$. To show that

$$\lim_{r,m \to \infty} < \frac{1}{2\pi i} \sum_{n=1}^{m} \frac{2p_n(z)}{\lambda_n} \int_{\sigma - ir}^{\sigma + ir} F(n,s) e^{st} ds, \varphi(z,t) > \to < f(u,v), \varphi(u,v) > \tag{4.1}$$

Let $\varphi \in D(I)$ has support contained in $[A,B]\times[a_1,b_1]$, where $0<A<B<\infty$ and $-h<a_1<b_1<h$.

$$\frac{1}{2\pi}\sum_{n=1}^{m}\frac{p_n(z)}{\lambda_n}\int_{\sigma-ir}^{\sigma+ir}F(n,s)e^{st}ds$$

is locally integrable function on I. Equation (4) can be written without limit notation as

$$\frac{1}{2\pi}\int_{a_1}^{b_1}\int_0^{\infty}[\sum_{n=1}^{m}\frac{2p_n(z)}{\lambda_n}\int_{\sigma-ir}^{\sigma+ir}F(n,s)e^{st}ds]\varphi(z,t)dtdz$$

substituting $s=\sigma+i\rho$ $\quad ds=id\rho$ we get

$$\frac{1}{2\pi}\int_{a_1}^{b_1}\int_0^{\infty}[\sum_{n=1}^{m}\frac{2p_n(z)}{\lambda_n}\int_{-r}^{r}F(n,s)e^{st}d\rho]\varphi(z,t)dtdz$$

We interchange the order of integration as $\varphi(z,t)$ has bounded support and integrand is continuous function of (z, t , ρ). Therefore last expression takes the form

$$\frac{1}{2\pi}\sum_{n=1}^{m}\int_{-r}^{r}2F(n,s)\int_{a_1}^{b_1}\int_0^{\infty}\varphi(z,t)e^{st}\frac{p_n(z)}{\lambda_n}dtdzd\rho$$

$$=\frac{2}{2\pi}\sum_{n=1}^{m}\int_{-r}^{r}<f(u,v)e^{-su}p_n(v)$$

$$>\int_{a_1}^{b_1}\int_0^{\infty}\varphi(z,t)e^{st}\frac{p_n(z)}{\lambda_n}dtdzd\rho$$

$$=<f(u,v),\frac{2}{2\pi}\sum_{n=1}^{m}\int_{-r}^{r}e^{-su}p_n(v)$$

$$\int_{a_1}^{b_1}\int_0^{\infty}\varphi(z,t)e^{st}\frac{p_n(z)}{\lambda_n}dtdzd\rho>$$

the order of integration for repeated integrals can be changed because again $\varphi(z,t)$ is of bounded support and the integrand is continuous function of (t,z,ρ) , we obtain

$$<f(u,v),\frac{2}{2\pi}\sum_{n=1}^{m}\int_{-r}^{r}e^{-su}p_n(v)$$

$$\int_{a_1}^{b_1}\int_0^{\infty}\varphi(z,t)e^{st}\frac{p_n(z)}{\lambda_n}dtdzd\rho>$$

$$=<f(u,v),\frac{2}{2\pi}\sum_{n=1}^{m}\int_{a_1}^{b_1}\int_0^{\infty}\varphi(z,t)\frac{p_n(z)p_n(v)}{\lambda_n}dtdz$$

$$\times\int_{-r}^{r}e^{\sigma(t-u)}e^{(t-u)i\rho}d\rho>$$

$$=<f(u,v),\frac{2}{2\pi}\sum_{n=1}^{m}\int_{a_1}^{b_1}\int_0^{\infty}\varphi(z,t)T_ne^{\sigma(t-u)}e^{\sigma(t-u)i}dtdz$$

$$\times\frac{e^{i(t-u)r}-e^{-i(t-u)r}}{i(t-u)}>,$$

where $T_n=\sum_{n=1}^{m}\frac{p_n(z)p_n(v)}{\lambda_n}$

$$=<f(u,v),\frac{2}{\pi}\sum_{n=1}^{m}\int_{a_1}^{b_1}\int_0^{\infty}\varphi(z,t)T_ne^{\sigma(t-u)}e^{\sigma(t-u)}$$

$$(\times)\frac{sinr(t-u)}{(t-u)}dtdz>$$

$$\rightarrow<f(u,v),(u,v)>\text{ as } r,m\rightarrow\infty.$$ □

Theorem 4.2 (Uniqueness theorem)**.** *Let £M$[f(z,t)]=F(n,s)$ and £M$[g(z,t)]=G(n,s)$ for all $(n,s)\in D_f$ and $(n,s)\in D_g$ respectively and if $F(n,s)=G(n,s)$ for $(n,s)\in D_f\cap D_g\neq 0$, then $f=g$ in sense of equality in $D'(I)$.*

Proof. In the sense of convergence in $D'(I)$ and in view of inversion theorem

$$f(z,t)=\lim_{r,m\rightarrow\infty}\frac{1}{2\pi i}\sum_{n=1}^{\infty}\frac{p_n(z)}{\lambda_n}\int_{\sigma-ir}^{\sigma+ir}F(n,s)e^{st}ds \quad (4.2)$$

the right hand side of this equation becomes

$$\lim_{r,m\rightarrow\infty}\frac{1}{2\pi i}\sum_{n=1}^{\infty}\frac{p_n(z)}{\lambda_n}\int_{\sigma-ir}^{\sigma+ir}G(n,s)e^{st}ds$$

which by inversion theorem equal to $g(z,t)$.

Hence $f=g$. □

5. Conclusion

In this paper we extended the Laplace finite Marchi Fasulo transform in the distributional space of compact support and hence defined as generalized Laplace finite Marchi Fasulo transform.Some lemma's along with the inversion theorem and analyticity theorem are proved. This plays an important role in solving linear and nonlinear partial differential equations.

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