



A novel method to obtain initial basic solution and optimal solution of pentagonal fuzzy transportation problem

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Abstract

In this paper, we apply the new method to solve the pentagonal fuzzy transportation model. We demonstrate that the special structure of the pentagonal fuzzy transportation model substantially reduces the computational burden. The pentagonal fuzzy solution, that we get by the proposed method are equivalent to the well-known existing methods and it is discussed with the comparison table.

Keywords

Pentagonal fuzzy transportation problem, Pentagonal fuzzy ranking, Pentagonal fuzzy optimal solution.

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1. Introduction

In modern era, rapid changing and competitive business and enterprise are facing a continuous pressure to find a better ways and minimum cost to deliver values to customers becomes stronger. How and when to send the products to the customers in the quantities, they want in a cost-effective manner, become more challenging. Transportation models have been developed to meet this challenge.. Hitchcock[16] developed the original basic transportation problem and also simplex algorithm were developed Primarily by Dentzig[13]

and Charnes & cooper[8] in 1947. Many of the researchers have provided many different methods to solve transportation problem. Few of them are Modified Vogel's Approximation Method for unbalance transportation problem by Balakrishnan.N[3]. An Improved Vogel's Approximation Method by Serder Korukogu and Serkan balli[22]. A new approach for finding an optimal solution for transportation problems by Sudhakar V J et.al[24].

When a decision making problem is formulated as a model for a real world applications, crisp values are used conventionally. In many cases, crisp values possess some imprecision and ambiguity. In such cases, fuzzy numbers have been developed. Coming back to the transportation problem the cost, supply and demand may be uncertain then the model can be formulated as a fuzzy transportation problem. Very few of the research articles are 'A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problem' by P.Pandian and Natarajan. Kaur.A and Kumar.A[21], A new method for solving fuzzy transportation problems using ranking function. Jimenez and Verdegay[19] investigated Solid fuzzy transportation problems by an evolutionary algorithm based parametric approach.

In our article we introduced a novel method to solve the pentagonal fuzzy transportation problem in which supplies, demands, capacities and costs are pentagonal fuzzy numbers. Section 4 deals with our pentagonal fuzzy transportation problem briefly.

2. Preliminaries

Definition 2.1. Fuzzy Set:[17, 18, 26]

If x is a collection of objects denoted by x then a fuzzy set \tilde{B} in x is a set of ordered pairs

$$\tilde{B} = \{(x, \mu_B(x)) / x \in R\}$$

$\mu_B(x)$ is called the membership function of grade of membership of x in \tilde{B} which maps X to the membership space M . The range of the membership function is a subset of the nonnegative real numbers whose supremum is finite generally in the range $[0, 1]$.

Definition 2.2. Fuzzy Number: [17, 18, 26, 27]

A fuzzy number g in the real line R is a fuzzy set $g : R \rightarrow [0, 1]$ that satisfies the following properties.

- (i). g is piecewise continuous.
- (ii). There exists an $x \in R$ such that $g(x) = 1$.
- (iii). g is convex, (i.e), if $x_1, x_2 \in R$ and $\delta \in [0, 1]$ then $g(\delta x_1 + (1 - \delta)x_2) \geq g(x_1) \wedge g(x_2)$.

Definition 2.3. Pentagonal Fuzzy Numbers:[17, 18]

A pentagonal fuzzy number $\tilde{B}_p = (b_1, b_2, b_3, b_4, b_5)$ Where, b_1, b_2, b_3, b_4, b_5 are real numbers. The membership function of the pentagonal fuzzy number is given below.

$$\mu_{\tilde{B}_p(x)} = \begin{cases} 0 & , x < b_1 \\ \frac{1}{2} \left[\frac{x-b_1}{b_2-b_1} \right] & , b_1 \leq x \leq b_2 \\ \frac{1}{2} + \frac{1}{2} \left[\frac{x-b_2}{b_3-b_2} \right] & , b_2 \leq x \leq b_3 \\ 1 - \frac{1}{2} \left[\frac{b_4-x}{b_4-b_3} \right] & , b_3 \leq x \leq b_4 \\ 0 & , x > b_5 \end{cases}$$

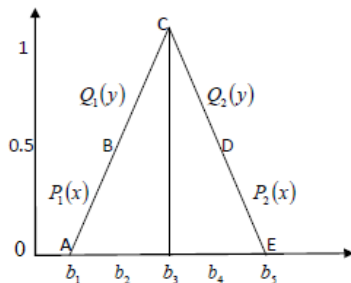


Figure 1. Representation of a Normal Pentagonal Fuzzy Number for $x \in [0, 1]$

Definition 2.4. Canonical Pentagonal Fuzzy Numbers:[17, 18]

\tilde{B}_p is called a Canonical pentagonal fuzzy number, if it is a closed and bounded pentagonal fuzzy number and its membership function is strictly increasing on the interval $[b_2, b_3]$ and strictly decreasing on the interval $[b_3, b_4]$.

2.5 Operations of Pentagonal Fuzzy Numbers [17, 18]

Following are the four operations that can be performed on pentagonal fuzzy numbers, suppose $\tilde{B}_p = (b_1, b_2, b_3, b_4, b_5)$ and $\tilde{C}_p = (c_1, c_2, c_3, c_4, c_5)$ are the two pentagon fuzzy numbers then

Addition: $\tilde{B}_p(+) \tilde{C}_p = (b_1 + c_1, b_2 + c_2, b_3 + c_3, b_4 + c_4, b_5 + c_5)$

Subtraction: $\tilde{B}_p(-) \tilde{C}_p = (b_1 - c_5, b_2 - c_4, b_3 - c_3, b_4 - c_2, b_5 - c_1)$

Multiplication: $\tilde{B}_p(*) \tilde{C}_p = (b_1 * c_1, b_2 * c_2, b_3 * c_3, b_4 * c_4, b_5 * c_5)$

Division: $\tilde{B}_p(/) \tilde{C}_p = (b_1/c_5, b_2/c_4, b_3/c_3, b_4/c_2, b_5/c_1)$

2.6 Ranking of Pentagonal Fuzzy Numbers [11, 17, 18]

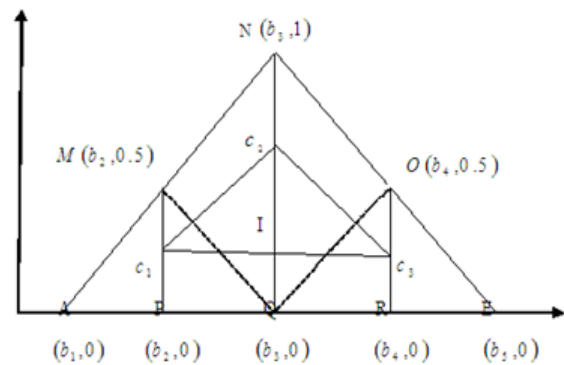


Figure 2. Normalized Pentagonal Fuzzy Number.

Divide the pentagonal into three fuzzy plane figures. In these three plane, first figure is a fuzzy triangular AMQ, second figure is a fuzzy square QMNO and third is again a fuzzy triangle QOE. Let the fuzzy centroid of the three fuzzy plane figures be C_1, C_2, C_3 respectively. The pentagonal fuzzy in centre I of the fuzzy centroid C_1, C_2, C_3 is taken as the point of reference to define the ranking of normalized pentagonal fuzzy numbers. Consider a normalized pentagonal fuzzy number $\tilde{B}_p = (b_1, b_2, b_3, b_4, b_5)$ then fuzzy centroid of the three fuzzy plane figures are

$$C_1 = \{[b_1 + b_2 + b_3]/3, 1/6\}$$

$$C_2 = \{[b_2 + 2b_3 + b_4]/4, 1/2\}$$

$$C_3 = \{[b_3 + b_4 + b_5]/3, 1/6\}$$

respectively



$$I_{\tilde{B}_p} = \left\{ \frac{\alpha_{\tilde{B}_p} \left[\frac{b_1+b_2+b_3}{3} \right] + \beta_{\tilde{B}_p} \left[\frac{b_2+2b_3+b_4}{4} \right] + \gamma_{\tilde{B}_p} \left[\frac{b_3+b_4+b_5}{3} \right]}{\alpha_{\tilde{B}_p} + \beta_{\tilde{B}_p} + \gamma_{\tilde{B}_p}}, \frac{\alpha_{\tilde{B}_p} \left[\frac{1}{6} \right] + \beta_{\tilde{B}_p} \left[\frac{1}{2} \right] + \gamma_{\tilde{B}_p} \left[\frac{1}{6} \right]}{\alpha_{\tilde{B}_p} + \beta_{\tilde{B}_p} + \gamma_{\tilde{B}_p}} \right\}$$

where,

$$\alpha_{\tilde{B}_p} = \frac{\sqrt{(b_4+4b_5-2b_3-3b_2)^2+8}}{12}$$

$$\beta_{\tilde{B}_p} = \frac{\sqrt{(b_1+b_2-b_4-b_5)^2}}{3} \text{ and}$$

$$\gamma_{\tilde{B}_p} = \frac{\sqrt{(4b_1+b_2-2b_3-3b_4)^2+8}}{12} \text{ then}$$

$$R_{\tilde{B}_p} = \sqrt{\tilde{x}_0^2 + \tilde{y}_0^2}$$

Remark[17,18]:

If $\tilde{B}_p = (b_1, b_2, b_3, b_4, b_5)$ and $\tilde{C}_p = (c_1, c_2, c_3, c_4, c_5)$. Then, we have the following comparison.

- (i). $\tilde{B}_p \approx \tilde{C}_p \Leftrightarrow R(\tilde{B}_p) = R(\tilde{C}_p)$
- (ii). $\tilde{B}_p \geq \tilde{C}_p \Leftrightarrow R(\tilde{B}_p) \geq R(\tilde{C}_p)$
- (iii). $\tilde{B}_p \leq \tilde{C}_p \Leftrightarrow R(\tilde{B}_p) \leq R(\tilde{C}_p)$

3. Representation of the Fuzzy Transportation Model with Nodes and Arcs

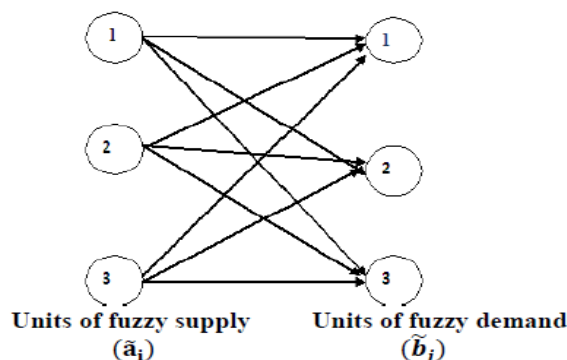


Figure 3. Representation of Fuzzy Transportation Model

4. The Proposed Algorithm

The step by step procedure is given

Step 1: Examine the pentagonal fuzzy transportation is balanced. If so proceed step 2.

Step 2: Mark the pentagonal fuzzy cheapest cost \tilde{C}_{ij} in every rows and column.

Step 3: Assign the lowest pentagonal fuzzy supply or pentagonal fuzzy demand to all marked pentagonal fuzzy cheapest cost in every row and every column.

Step 4: Check for a pentagonal fuzzy cheapest cost for every fuzzy set of $m+n-1$ routes that provides pentagonal fuzzy initial basic feasible solution.

Step 5: Test for the pentagonal fuzzy optimum. If the pentagonal fuzzy value is optimal, stop. Otherwise go to step 6.

Step 6: Now mark all the non-allocated pentagonal fuzzy cost. Now, find cheapest pentagonal fuzzy cost. Try to allocate the pentagonal fuzzy supply /pentagonal fuzzy demand in the constraint.

Step 7: Repeat the step 5.

4.1 Numerical Example of Proposed Method

A military equipment is to be transported from three origins to four destinations. The pentagonal fuzzy supply at the origins, the pentagonal fuzzy demand at the pentagonal fuzzy destinations and the pentagonal fuzzy cost of shipment is shown in the below table. we worked out a pentagonal fuzzy transportation plan so that the pentagonal fuzzy total cost required for shipment is minimum.

	\tilde{D}_1	\tilde{D}_2	\tilde{D}_3	\tilde{D}_4	Supply(\tilde{a}_i)
\tilde{S}_1	(0,1,2,3,4)	(1,2,3,4,5)	(7,9,11,13,15)	(3,5,7,9,11)	(2,4,6,8,10)
\tilde{S}_2	(-1,0,1,2,3)	0	(2,4,6,8,10)	(-1,0,1,2,3)	(1,1,1,1,1)
\tilde{S}_3	(3,4,5,6,7)	(0,4,8,12,16)	(5,10,15,20,25)	(3,6,9,12,15)	(0,5,10,15,20)
Demand(\tilde{b}_j)	(3,5,7,9,11)	(1,3,5,7,9)	(-1,1,3,5,7)	(0,1,2,3,4)	(3,10,17,24,31)

Table 1. Transportation Problem

Step1: $\Sigma(\tilde{a}_i) = \Sigma(\tilde{b}_j) = (3, 10, 17, 24, 31)$

The pentagonal fuzzy transportation problem is balanced.

Step2:

Finding the pentagonal fuzzy cheapest cost in every row and column

	\tilde{D}_1	\tilde{D}_2	\tilde{D}_3	\tilde{D}_4	Supply(\tilde{a}_i)
\tilde{S}_1	(0,1,2,3,4)	(1,2,3,4,5)	(7,9,11,13,15)	(3,5,7,9,11)	(2,4,6,8,10)
\tilde{S}_2	(-1,0,1,2,3)	0	(2,4,6,8,10)	(-1,0,1,2,3)	(1,1,1,1,1)
\tilde{S}_3	(3,4,5,6,7)	(0,4,8,12,16)	(5,10,15,20,25)	(3,6,9,12,15)	(0,5,10,15,20)
Demand(\tilde{b}_j)	(3,5,7,9,11)	(1,3,5,7,9)	(-1,1,3,5,7)	(0,1,2,3,4)	(3,10,17,24,31)

Table 2. Fuzzy Cheapest Cost in Every Fuzzy Row/Fuzzy Column

Step3:

The algorithm asks for a starting fuzzy trial basic solution which contains $m+n-1=6$ active routes (1,1). This exhausts the fuzzy supply and leaves one more fuzzy unit of the demand to be filled in column 1. Similarly, allocate the fuzzy supply of (1,1,1,1,1) unit in row 2 to the cheapest fuzzy route (2,2). The fuzzy supply of (0,5,10,15,20) in the row 3 is then routed to meet all the remaining unfilled demand. The result is shown in the table 3 given below.



	D_1	D_2	D_3	D_4	supply(\bar{a}_i)
\bar{S}_1	(0,1,2,3,4) (2,4,6,8,10)	(1,2,3,4,5)	(7,9,11,13,15)	(3,5,7,9,11)	(2,4,6,8,10)
\bar{S}_2	(-1,0,1,2,3)	0 (1,1,1,1,1)	(2,4,6,8,10)	(-1,0,1,2,3)	(1,1,1,1,1)
\bar{S}_3	(3,4,5,6,7) (1,1,1,1,1)	(0,4,8,12,16) (0,2,4,6,8)	(5,10,15,20,25) (-1,1,3,5,7)	(3,6,9,12,15) (0,1,2,3,4)	(0,5,10,15,20)
Demand(\bar{b}_j)	(3,5,7,9,11)	(1,3,5,7,9)	(-1,1,3,5,7)	(0,1,2,3,4)	(3,10,17,24,31)

Table 3. Fuzzy Initial Basic Solution

Step4:

Checking for $m+n-1 = 4+3-1 =$ Number of fuzzy allocations = 6

Total fuzzy initial basic feasible cost = (-2,33,112,239,391)

Step5:

Checking for an optimum. As we can see in the above tabular column it is not optimum. We can allocate some of the fuzzy supply /fuzzy demand to the next cheapest cost in the row 1 is (1,2). By assigning to (1,2) there will be a change in (1,3),(2,3),(3,1),(3,3),(3,4). The result shown in the below table 4.

	D_1	D_2	D_3	D_4	supply(\bar{a}_i)
\bar{S}_1	(0,1,2,3,4)	(1,2,3,4,5) (1,3,5,7,9)	(7,9,11,13,15) (-7,-3,1,5,9) (2,4,6,8,10)	(3,5,7,9,11)	(2,4,6,8,10)
\bar{S}_2	(-1,0,1,2,3)	0 (1,1,1,1,1)	(5,10,15,20,25) (1,1,1,1,1)	(-1,0,1,2,3)	(1,1,1,1,1)
\bar{S}_3	(3,4,5,6,7) (3,5,7,9,11)	(0,4,8,12,16) (1,3,5,7,9)	(-1,-5,1,7,13) (-1,1,3,5,7)	(3,6,9,12,15) (0,1,2,3,4)	(0,5,10,15,20)
Demand(\bar{b}_j)	(3,5,7,9,11)	(1,3,5,7,9)	(-1,1,3,5,7)	(0,1,2,3,4)	(3,10,17,24,31)

Table 4. Fuzzy Initial Basic Solution

No other routes are possible .The pentagonal value is optimum.

Pentagonal fuzzy total minimum cost = (-89,-38,100,331,643)

4.2 Comparison of the Numerical Table

Method	Initial Basic Solution	Optimal Solution
North-West Corner Rule	(-2,31,111,235,390)	—
Least-Cost Method	(-1,29,110,236,384)	—
Vogel's Approximation Method	(-2,33,112,239,391)	—
Proposed Method	(-2,33,112,239,391)	(-89,-38,100,331,643)
MODI- Method	—	(-89,-38,100,331,643)

5. Conclusion

The pentagonal fuzzy transportation problem, that we discussed in the section 4 will definitely reduce the computational burden. The table 4.2 says that the pentagonal fuzzy value that we got is equal to the existing methods. This method will serve as a key for decision makers while handling various types of situations and in real life problems.

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